Séminaire de théorie des nombres de Grenoble

Helmut Koch

On the Local Langlands Conjecture

Séminaire de théorie des nombres de Grenoble, tome 8 (1979-1980), exp. n° 5, p. 1-14

<http://www.numdam.org/item?id=STNG_1979-1980__8__A5_0>

Séminaire de Théorie des Nombres
21 et 24 janvier 1980
Grenoble

ON THE LOCAL LANGLANDS CONJECTURE

by Helmut KOCH

One of the main questions of algebraic number theory is to find the "right" generalisation of class field theory to nonabelian extensions.

The first who published some ideas in this direction was E. Artin in his Crelle paper of 1931 on the group theoretical structure of the discriminant of an algebraic number field, where he generalised the Führer-dikriminantenproduktformel of class field theory. I recall this formula :

let K be a local field and L/K an abelian extension corresponding to the subgroup A_L of K^X. Let χ be a character of K^X/A_L. The conductor $C(\chi)$ of χ is the minimal number i with $\chi(U_K^i) = \{1\}$, where U_K^i is the i-th group of principal units. Then the discriminant $\vartheta_{L/K}$ of L/K is given by

$$\vartheta_{L/K} = \prod_{\chi} \mathfrak{p}_K ,$$

where $\mathfrak{g}_K$ is the prime ideal of K.
To χ corresponds a character χ' of $G(L/K)$. We define the conductor of χ' by $C(\chi') = C(\chi)$.

Now let L/K be an arbitrary normal extension with Galois group $G(L/K) = G$ and $\bar{\rho} : G \to GL_n(\mathbb{C})$ a representation of the Galois group. Then Artin defines a natural number $C(\rho)$ which is called the Artin conductor of ρ or more exact the exponent of the Artin conductor.

By linearity this notion can be extended to virtuel representations, i.e. linear combinations of irreducible representations with integer coefficients. It has the following properties :

1. $C(\rho_1 + \rho_2) = C(\rho_1) + C(\rho_2)$.

2. Let ρ be induced from the subgroup $H = G(L/F)$ of G and the representation σ of H. Then

$$C(\rho) = \nu_K(\vartheta_{F/K}) \operatorname{Tr}\sigma(1) + f_{F/K} C(\sigma)$$

where $f_{F/K}$ denotes the inertia degree.

3. If G is abelian and χ is a character of G. Then the Artin conductor is the conductor of χ defined above.

By the Theorem of Brauer on induced representations $C(\rho)$ is characterised by the properties 1. - 3.

One has

$$\vartheta_{L/K} = \prod_{\rho \text{ irred}} \mathfrak{p}_K^{C(\rho) \dim \rho}$$

This is of cause not surprising and is the special case of 2. for the regular representation of $G(L/K)$ together with Burnsides Theorem.

But it indicates that representation and there Artin conductors should play an important role in the generalisation of class field theory.

It was only in the sixties that a proposal for such a generalisation was made by Langlands. This is a global theory about the correspondence of Artin L-series and generalisations of classical and Hecke L-series. I shall speak here only about the local aspects of this theory and also about a special case which is however general enough to be a good generalisation of local class field theory.

We need a generalisation of K^{x} and take for it the multiplicative group D_n^{x} of a central division algebra D_n over K of degree n^2. By a standard construction we get D_n in the form

$$D_n = \left\{ \sum_{\nu=0}^{n-1} \alpha_\nu \pi^\nu \mid \alpha_\nu \in K_n \ , \ \pi^n = \pi_K \ , \ \pi\alpha = \theta\alpha\pi \text{ for } \alpha \in K_n \right\} ,$$

where K_n is the unramified extension of K of degree n, θ is a generating automorphism of $G(K_n/K)$ and π_K is a fixed prime element of K.

The ring O of integers of D_n is given by

$$O = \left\{ \sum_{\nu=0}^{n-1} \alpha_\nu \pi^\nu \mid \alpha_\nu \in O_{K_n} \right\} ,$$

where O_{K_n} is the ring of integers of K_n, and the maximal ideal is $\mathfrak{P} = \pi O$.

D_n^{x} is a topological group with the groups $U^i = 1 + \mathfrak{P}^i$, $i = 1, 2, \ldots$, as basis of neidbourhoods of the unit.

D_n^{x} denotes the set of equivalence classes of irreducible representations ρ such that $\operatorname{Ker}\rho$ is closed and has finite index in D_n^{x}. We call this representations irreducible finite representations. The maximal number j with $\rho(1+\mathfrak{P}^j) \neq \{1\}$ is called the index $j(\rho)$ of ρ. If $\rho(1+\mathfrak{P}) = \{1\}$ we put $j(\rho) = 0$.

On the other hand let $G_K = G(\overline{K}/K)$ be the Galois group of a separable algebraic closure $\overline{K}$ of K. A continuous representation $\rho : G_K \longrightarrow GL_n(C)$ factors through a finite factor group $G(L/K)$ and the Artin conductor of ρ as representation of $G(L/K)$ is independent of the choice of L. Therefore we have the notion of Artin conductor of ρ as representation of G_K. In the following it is in fact more convenient to work with a modification of the Artin conductor : the Swan conductor $j(\rho)$ which is defined as follows

$$j(\rho) = C(\rho) - \dim\rho \quad \text{if } C(\rho) > 0$$

$$j(\rho) = 0 \quad \text{if } C(\rho) = 0 \ .$$

In this connection it is also convenient to introduce the Swan different exponent $j(L/K)$ of an extension L/K :

$$j(L/K) = d(L/K) - e(L/K) + 1 \ ,$$

where $d(L/K)$ is the different exponent and $e(L/K)$ the ramification index of L/K .

Let R_n be the set of equivalence classes of irreducible representations ρ of G_K with $\dim\rho \,|\, n$.

Now the local Langlands conjecture can be formulated as follows : there is a one to one correspondence Φ_n of R_n and D_n^x with the following properties :

1. Φ_1 is induced by the reciprocity map. (In the following, we identify $\chi \in R_1$ and $\Phi_1\chi$) .

2. If $\chi \in R_n$ is one dimensional then $\Phi_n\chi = \chi N_{D_{n/K}}$, where N_{D_n} denotes the reduced norm,

3. $\Phi_n(\rho\otimes\chi) = \Phi_n\rho \otimes \Phi_n\chi$,

4. $j(\Phi_n(\rho)) = j(\rho)^{n/\dim\rho}$,

5. $\Phi_n\rho\,|\,K^x = (\det\rho)^{n/\dim\rho}$,

6. For the ϵ-factors associated to ρ and $\Phi_n\rho$ we have $\epsilon(\rho)^{n/\dim\rho} = \epsilon(\Phi_n\rho)$.

For $n = 2$, Φ_2 is unique and exists. This was shown for residue characteristic $p \neq 2$ in Jacquet-Langlands [3] and for $p = 2$ in almost all cases by Tunnell [9] with a global argument, finaly by Gerardin and Kutzko in all cases with a local argument (International Congress of Mathematicians Helsinki 1978).

For $n > 2$, Φ_n is not uniquely determined by 1. - 6.

If $p \nmid n$, then one can construct a map $\Phi'_n : R_n \to D^x_n$ which satisfies 1. - 4. but not 5. , 6. through it may be not too difficult to satisfy also 5. and 6. (see [5]). After some minutes I shall come back to this case.

The real difficult case is the wild case $p \mid n$. In the moment it seems that there is no method for finding Φ_n in this case. But one can formulate the following weaker conjecture, which I shall call the numerical Langlands conjecture : let

$$R_n(j) = \left\{ \rho \in R_n \mid j(\rho)n/\dim \rho = j \ , \ (\Phi_1 \det \rho)^n/\dim \rho(\pi_K) = 1 \right\} ,$$

$$D^x_n(j) = \left\{ \rho \in \hat{D}^x_n \mid j(\rho) = j \ , \ \rho(\pi_K) = 1 \right\} ,$$

where π_K is a fixed prime element of K .

$$\operatorname{card} R_n(j) = \operatorname{card} D^x_n(j) \tag{1}$$

THEOREM 1. - (1) <u>is true for</u> $p \nmid n$.

THEOREM 2. - (1) <u>is true for</u> $n = p$.

In the following I shall speak mainly about the proof of theorem 2. (for the details see [6]), but first I say some words about the tame case $p \nmid n$.

The representations in R_n and in D^x_n can be parametrised by the same objects, the so called admissible pairs $(L/K, \chi)$, where L/K is an extension of degree a divisor n and χ is a character of L^x with $[L^x : \operatorname{Ker} \chi]$ finite. The following two conditions must be fulfilled.

1. If $\chi = \chi' N_{L/L'}$ on L^x with $K \subset L' \subset L$, then $L' = L$.
2. Let U^1_L be the group of principal units of L .

If $\chi = \chi' N_{L/L'}$ on U_L^1 then L/L' is unramified.

Two pairs $(L_1/K,\chi_1)$, $(L_2/K,\chi_2)$ are called equivalent if there exists an isomorphism $\eta : L_1 \to L_2$ such that $\chi_1 = \chi_2 \eta$.

Given an equivalence class of admissible pairs $(L/K,\chi)$ it is easy to associate to it a representation ρ in $R_n : \rho = \mathrm{ind}_{G_L}^{G_K} \chi$. The conditions 1. , 2. mean that ρ is irreducible and that our map $\{(L/K,\chi)\} \to R_n$ is injective.

So the sole problem is that the map is also surjective. To prove this one must know that the representations in R_n are induced from characters. One can deduce this from Clifford-Mackey theory.

Now we want to associate to $(L/K,\chi)$ a representation in D_n^x . Since the degree of L/K divides n there is an injection $L \hookrightarrow D_n$.

Let D_L be the centralicer of L in D_n . Then $\chi' = \chi N_{D_L/L}$ is a character of D_L^x . Furthermore there is an induction process which is much more complicated as the usual one. It leads from χ' to a representation of D_n^x , which we associate to $(L/K,\chi)$. For the details see [5].

Now we come to the main question of this talk, the Langlands correspondence in the case $n = p$.

1. - Representations of D_p^x .

These representations were constructed by Howe [2] and Tunnell [9] saw how to compute the number

$D(j) = \mathrm{card}\, D_p^x(j)$ if $(j,p) = 1$.

This idea is so simple and nice that I want to reproduce it here :

from the results of Howe follows, that $\dim\rho = \frac{q^p-1}{q-1} \cdot q^{(j-1)(p-1)/2}$ for $\rho \in \hat{D}_p^\times(j)$. The dimension of a representation depends only on its index j .

Now a representation $\rho \in \hat{D}_p^\times(j)$ is the same as a representation of the group $D_p^\times/U^{j+1}(\pi_K)$ which is not representation of $D_p^\times/U^j(\pi_K)$.

We consider Burnside's formula for $D_p^\times/U^{j+1}(\pi_K)$ and get

$$\operatorname{card} D_p^\times/U^{j+1}(\pi_K) = D(j)\left(\left(\frac{q^p-1}{q-1}\right) q^{(j-1)(p-1)/2}\right)^2 + \operatorname{card} D_p^\times/U^j(\pi_K)$$

and therefore

$$D(j) = p(q-1)^2 q^{j-1} .$$

It remains the case $p|j$. One says that $\rho \in \hat{D}_p^\times$ is <u>in general position</u> or in french <u>primordial</u> if there is no character χ of $D_p^\times$ such that $j(\rho\otimes\chi) < j(\rho)$. From Howes results one can compute $\dim\rho$ for a representation in general position. Furthermore it is easy to show that

$$j(\rho\otimes\chi) \leq \max\{j(\rho), j(\chi)\} \quad \text{with equality}$$

if $j(\rho) \neq j(\chi)$. I said already that we get each χ in the form $\chi' N_{D_p/K}$. Then $j(\chi) = pj(\chi')$. Hence we obtain the representations which are not in general position with index j by tensoring representations of index smaller j with characters of index j . Taking this in mind one can again use the idea of Tunnell to compute the number $A(j)$ of representations in $D_p(j)$ which are in general position.

2. - <u>Representations</u> ρ <u>of</u> R_p <u>with</u> $p|j(\rho)$.

We have the same situation as with the representations of $D_p^\times$ with $p|j(\rho)$. One says that ρ is in general position if there is no character χ of $K^\times$ such that $j(\rho\otimes\chi) < j(\rho)$. Concerning the computation of $j(\rho\otimes\chi)$ one has $j(\rho\otimes\chi) \leq \max\{j(\rho), pj(\chi)\}$ with equality if $j(\rho) \neq pj(\chi)$. Here it is essential that ρ is irreducible. Concerning the nature of representations in general position one has the following :

PROPOSITION 2.1 - Let $p \mid j(\rho)$. Then ρ is in general position if and only if ρ is induced from a character of the multiplicative group of K_p , the unramified extension of K of degree p .

From the conductor formula for induced representation which I mentioned at the beginning of my talk we get $j(\rho) = pj(\chi)$. Therefore it is clear that a representation of the form $\rho = \operatorname{ind}_{G_{K_p}}^{G_K} \chi$ satisfies $p \mid j(\rho)$.

The other direction of the proposition is more difficult. One has to consider the case of induced and primitive representation separately and the most essential part of the proof is to show that a primitive representation ρ with $p \mid j(\rho)$ is always not in general position. This case is reduced to the induced case by the following theorem on primitive representations ([4]) :

Let ρ be a primitive representation and let G_L be the kernel of the corresponding projective representation. Furthermore let T/K be the maximal tamely ramified sub extension of L/K . Then the restriction of ρ to G_T is irreducible and induced.

W. Zink has proposition 2.1. generalised to the following

THEOREM. - If $\rho \in \hat{G}_K$ then $\dim\rho \mid j(\rho)$ if and only if ρ is induced from an unramified extension E/K and a representation of G_E which is not in general position.

Now it is easy to compute the number $A(j)$ of $\rho \in R_n(j)$ which are in general position. One finds $A(j) = B(j)$. Thus we are reduced to the case $p \nmid j(\rho)$ and we have to show that $\operatorname{card} R_p(j) = p(q-1)^2 q^{j-1}$.

We consider first induced and then primitive representations the formulas which one get look much more complicate than the formula above.

But I don't see any method to count induced and primitive representations together, the methods in both cases are different.

3. - Induced representations $\rho \in R_p$ with $p \nmid j(\rho)$.

Induced representations of degree p are of the form

$$(E/K,\chi) = \mathrm{ind}_{G_E}^{G_K} \chi \ ,$$

where $[E:K]=p$ and χ is a finite character of $E^{\times}$ which we identify with the corresponding character of G_L .

First of all we need a criterion for the irreducibility and equivalence of such representations. It is more convenient to work with normal extensions, therefore we make a base change F/K which garantees that EF/F is normal. It is easy to see that the maximal extension of K of exponent $p-1$ is sufficient. This F/K is the tamely ramified abelian extension $F = K_{p-1}(\sqrt[p-1]{\pi_K})$.

Mackeys irreducibility criterion for induced representations than leads to the following

Criterion.

$(E/K,\chi)$ is irreducible if, and only if, the character $\tilde{\chi}$ of $H=EF$ with

$$\tilde{\chi}(x) = \chi N_{H/E}\, \sigma x/x \ , \text{ where } (\sigma) = G(H/F) \ ,$$

is nontrivial.

If $p \nmid j(E/K,\chi)$, then this is the case if, and only if,

$$j(\chi) > d(E/K)/_{p-1} - 1 = j(E/K)/_{(p-1)} \ .$$

In this connection one finds also easily

$$j(\tilde{\chi}) = a(\tilde{\chi}) - 1 = e(F/K)a(\chi) - (p-1)d(H/F) - 1 \ . \qquad (1)$$

Now we come to the question of equivalence. There is a general criterion which is similar to the irreducibility criterion above : two representations $(E/K,\chi)$ and $(E'/K,\chi')$ of G_K of degree p are equivalent if, and only if, there exists $\eta \in G_K$ with

$$\chi\eta N_{\eta^{-1}EE'/\eta^{-1}E} = \chi' N_{\eta^{-1}EE'/E'} \; . \tag{2}$$

We consider the question of equivalence in more detail. If E and E' are conjugate over K: $E' = \eta E$. Then $(E'/K,\chi')$ is equivalent to $(E/K,\chi'\eta)$. Therefore this case is settled by the following :

PROPOSITION 3.1. - _If_ $(E/K,\chi)$ _is equivalent to_ $(E'/K,\chi')$ _and_ E,E' _are conjugate over_ K _then there is an isomorphism_ η : $E \to E'$ _such that_ $\chi = \chi'\eta$.

This looks trivial but is trivial only in the case that E/K is normal. Nevertheless the more interesting case is the case that $(E/K,\chi)$ is equivalent to $(E'/K,\chi')$ with E' not conjugate to E over K . If such an $(E'/K,\chi')$ exists we say that $(E/K,\chi)$ is of the second kind. The following two propositions express the main properties of representations of the second kind.

PROPOSITION 3.2. - _Let_ F _be a normal extension of_ K _such that_ $([F:K],p) = 1$, $H = EF$, $H' = E'F$ _are normal over_ F , $L = HH'$, _as above_ $U = \operatorname{Ker}\chi \subset H^{x}$. _Then_

1. L/H _corresponds to_ U _in the sense of class field theory, therefore_ L/H _is independent of the choice of_ E' , χ'
2. $j(L/F) = j(p)(p-1)e(F/K)$
3. _Let_ $\beta : G(F/K) \to Z/pZ^{x}$ _be given by_ $\tau\sigma\tau^{-1} = \sigma^{\beta(\bar\tau)}$ _for_ $\tau \in G(H/K)$, $\sigma \in G(H/F)$ _and let_ $\beta' : G(H'/K) \to Zp/Z^{x}$ _be defined in the same manner, then_ $\beta' = \beta^{-1}$.

Proof. -

1. If $(E/K,\chi)$ is equivalent to $(E'/K,\chi')$, then, if we go over from $(E/K,\chi)$ to $(\eta^{-1}E/K,\chi\eta)$, we have by (2)

$$\chi N_{L/E}x = \chi' N_{L/E'}x \quad \text{for} \quad x \in L^x .$$

Let $\sigma \in G(L/F)$ such that the restriction of σ to H generates the group $G(H/F)$ and that the restriction of σ to H' is trivial. We put $x = \sigma y/y$ with some $y \in L^x$. Then

$$\chi N_{H/E}\sigma(N_{L/H}y)/N_{L/H}y = \chi' N_{L/E'}\sigma y/y = 1 .$$

It follows $N_{L/H}L^x \subset \operatorname{Ker}\tilde{\chi} \subset H^x$, where $\left[H^x : N_{L/H}L^x\right] = p$.

Since $(E/K,\chi)$ is irreducible, we have $\operatorname{Ker}\tilde{\chi} \neq H^x$, therefore $N_{L/H}L^x = \operatorname{Ker}\tilde{\chi}$.

2. $d(L/F) = pd(H/F) + d(L/H) = pd(H/F) + (p-1)a(\tilde{\chi})$.

Therefore by (1)

$$j(L/F) = d(L/F) - (p^2-1) = p(J(H/F)+p-1) - (p^2-1) + (p-1)(e(F/K)a(\chi))$$
$$- (p-1)(j(H/F)+p-1) - 1 = j(\rho)(p-1)e(F/K) .$$

We omit the proof of 3. which is more difficult. Now we formulate a conversion of the last proposition :

PROPOSITION 3.3. - Let E/K be a ramified extension of degree p, $H = EF$ and $U \subset H^x$ a subgroup with the properties

(i) $(H^x : U) = p$

(ii) $\tau x \equiv x\beta^{(\tau)-1}$ (mod U) for $\tau \in G(H/K)$, $x \in H^x$.

(iii) The index a of U satisfies the conditions

$$a \equiv -j(H/F)/(p-1) \quad (\text{mod}\, e = e(F/K))$$

$$a \not\equiv 0 \quad (\text{mod}\, p) .$$

Then it exists a character χ of E^x such that $U = \operatorname{Ker}\tilde{\chi}$.

The extension L corresponding to U is normal over K by (ii).

If $G(L/F) = Z/pZ \times Z/pZ$ then $(E/K,\chi)$ is of the second kind. The two last propositions reduce the counting of induced representation to the counting of field extension with certain given properties which is possible by class field theory and is similar to the procedure of Tunnell in the case $p=2$. The result is the following :

Let $I(j)$ *be the number of induced representations in* $R_p(j)$. *Then*

$$I(j) = p(q-1)^2 q^{j-1}(1-A(j)q^{[r/p(p-1)]-[r/(p+1)]})$$

where $r = (p-1)j \not\equiv 0 \pmod p$

$$A(j) = 1 \quad \text{if} \quad r < p(p+1)v_K(p) \ , \ r \not\equiv 0 \pmod{p+1}$$

$$A(j) = \left[\frac{p}{2}\right] \frac{(q+1)}{(p+1)} \quad \text{if} \quad r < p(p+1)v_K(p) \ , \ r \equiv 0 \pmod{p+1}$$

$$A(j) = 0 \quad \text{if} \quad r > p(p+1)v_K(p) \ .$$

4. - *Primitive representations* $\rho \in R_p$ *with* $p \nmid j(\rho)$.

Primitive representations are best classified by considering first projective representations and then going over to linear representation :

$$\begin{array}{ccc} \rho : G_K & \longrightarrow & GL_n(C) \\ & \searrow & \downarrow \\ & & PGL_n(C) = GL_n(C)/C^{\times} \end{array}$$

Projective primitive representations are projective representations whose liftings are primitive representations. They were classified in my paper [4] which I mentioned above, but I could not calculate the conductor of these representations, which is defined to be the minimum of the conductors of the liftings. This then seemed to be the most difficult question in the representation theory of local Galois groups. Buhler [1] and Zink [12] were able to compute this conductor in the case $n=p$ but the proof is rather complicate. I am going now to give a very simple proof :

THEOREM. - *Let* $\rho : G_K \longrightarrow PGL_p(\mathbb{C})$ *be a primitive representation and let* $G_L = \mathrm{Ker}\,\rho$, *the maximal tamely ramified sub extension of* L/K . *Then*

$$j(\rho) = \frac{j(L/T)}{(p-1)e(T/K)} .$$

Proof. - I mentioned already above that the restriction of a primitive representation to the ramification group remains irreducible. Let $\bar{\rho}$ be a lifting of ρ with $j(\bar{\rho}) = j(\rho)$ and $\bar{\rho}_T$ the restriction to $G(L/T)$. Then it is easy to see from the definition of the Artin conductor that

$$j(\bar{\rho}_T) = e(T/K)j(\bar{\rho})$$

$G(L/T)$ is the direct product of two cyclic groups of order p . Furthermore $\bar{\rho}_T$ is an induced representation of the second kind which is induced from each of the sub extension of L/T of degree p as was proved in my paper [4]. Therefore we can apply proposition 3.2 with $F = T$ (T plays here the role of K),

$$j(L/T) = (p-1)j(\bar{\rho}_T) .$$

This proves the formula.

On the basis of this formula one can compute the number of primitive projective representations with given Swan-conductor j , which was essentially already done by W. Zink [11].

The transition to linear representation is easy on the basis of the following

THEOREM. - *Let* ρ *be a primitive representation of dimension* p *and* φ *a character of* G_K . *Then* $\rho \otimes \varphi$ *is equivalent to* ρ *if, and only if,* $\varphi = 1$.

For $p=2$ this was proved by Tunnell by global argument. Already after my talk in Grenoble, E.-W. Zink found that the theorem can be generalised to the following

THEOREM OF ZINK. - Let G be a finite group and let φ be a one dimensional representation of G. We denote the kernel of φ by H. Then for all irreducible representations ρ of G the following statements are equivalent :

(i) $\rho \otimes \varphi$ is equivalent to ρ ,

(ii) ρ is induced from a representation of H .

BIBLIOGRAPHY

[1] BUHLER J.P. , Icosahedral Galois representations, Springer Lecture Notes 654, Chapter 2 (1978).

[2] HOWE R. , Kirillov Theory for Compact p-adic Groups, Pac. J. Math. 73, (1977), pp. 365-381.

[3] JACQUET H. , Langlands, R.P. , Automorphic Forms on GL(2), Springer Lecture Notes 114, Berlin (1970).

[4] KOCH H. , Classification of the Primitive Representations of the Galois Group of Local Fields, Inv. Math. 40, (1977), pp. 195-216.

[5] KOCH H. , ZINK E.-W. , Zur Korrespondenz von Darstellungen der Galoisgruppen und zentralen Divisionsalgebren über lokalen Körpern (Der zahme Fall), Preprint R-03/79, Akademie der Wissenschaften der DDR, ZIMM, Berlin (1979).

[6] KOCH H. , On the local Langlands conjecture for central division algebras of index p . (To appear).

[7] LANGLANDS R.P. , Problems in the Theory of Automorphic Forms, Lectures on Modern Analysis and Applications, Springer Lecture Notes 170, (1970).

[8] SERRE J.P. , Corps locaux, Hermann, Paris 1962.

[9] TUNNELL J.B. , On the local Langlands conjecture for GL(2), Inv. Math. 46, (1978), pp. 179-200.

[10] WEIL A. , Exercises dyadiques, Inv. Math. 27, (1974), pp. 1-22.

[11] ZINK E.-W. , Counting primitive projective representations of local Galois groups, Preprint (1978).

[12] ZINK E.-W. , Ergänzungen zu Weil's Exercises dyadiques, to appear in Math. Narch.