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## On Brownian Local Time

by M.T. Barlow

Let  $B$  be a Brownian motion starting at 0, and  $L_t^a$  denote its local time - as usual we take a version of  $L$  which is jointly continuous in  $(a,t)$ . Recently, Perkins has proved that, for fixed  $t$ , the process  $a \rightarrow L_t^a$  is a semimartingale relative to the excursion fields. It is natural to ask about  $L_T^a$ , where  $T$  is a stopping time: in this note we give an example to show that  $L_T^a$  may be very far from being a semimartingale.

Given a stopping time  $T$  (which will be defined later) let

$$M = \inf \{ a : L_T^a > 0 \} = \inf_{s \leq T} B_s ,$$

$$Y_a = L_T^{M+a} ,$$

$\underline{Y}_a$ ,  $a > 0$ , be the (usual augmentation of the) natural filtration of  $Y$

We will choose  $T$  so that, for some fixed  $x > 0$ , if  $R = \inf\{a: Y_a = x\}$ , then the process  $(t, \omega) \rightarrow Y_{R+t}(\omega)$  is  $B([0, \infty)) \otimes \sigma(R)$  measurable with positive probability. Since  $Y$  is never of finite variation, it follows that  $Y$  is not a semimartingale  $\underline{Y}_t$ .

Let  $\psi : C[0, \infty) \rightarrow [0, 1]$  be injective and measurable. Set  $S = \inf \{ t : |B_t| = 1 \}$ , and let  $\varepsilon, x$  be positive reals. On  $\{B_S = 1\}$  let  $T = S$ , and on  $\{B_S = -1\}$  define

$$U = \inf \{ a : L_S^a \geq x \} ,$$

$$V = \psi ( L_S^{U+} )$$

$$W = \max \{ a < 1 : a + n\epsilon = U - \epsilon V \text{ for some } n \geq 0 \}$$

Thus  $-(1 + \epsilon) \leq W < -1$  , and  $U - W = \epsilon(V + n)$  for some  $n(\omega) \geq 0$ . Now on  $\{B_S = -1\}$  let  $T = \inf \{ t > S : B_t = U \text{ or } W \}$ . Then, if

$$F = \{B_S = -1\} \cap \{B_T = W\} \cap \{L_T^a < x , \text{ for } a \geq U\} ,$$

it is evident that  $\epsilon, x$  may be chosen so that  $P(F) > 0$ . However, on  $F$   $V = [R/\epsilon]$  ( $[x]$  denotes the fractional part of  $x$ ), and thus if  $X_{R+} = \psi^{-1}([R/\epsilon])$ ,  $1_F Y_{R+} = 1_F X_{R+}$ . Thus  $T$  has the required properties, and it is clear, from, for example, the characterization of semimartingales as stochastic integrators, that  $Y$  is not a semimartingale.

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