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ON THE LEFT END POINTS OF BROWNIAN EXCURSIONS

by Martin Barlow

The following question was raised in Paris by P.A. Meyer : Given a Brownian motion B_t with natural filtration $\underline{\underline{B}}_t$, is it possible to find an expansion $(\underline{\underline{F}}_t)$ of $(\underline{\underline{B}}_t)$ such that

a) B is a semimartingale/$(\underline{\underline{F}}_t)$,

b) the set of left end points of the excursions of B from 0 is the union of a sequence $[S_n]$ of stopping times/$(\underline{\underline{F}}_t)$(i.e., is optional/$(\underline{\underline{F}}_t)$)?

Here is a brief proof that this is not possible. Let $Y=|B|$, and suppose such a filtration $(\underline{\underline{F}}_t)$ can be found. We can of course take the S_n disjoint. Let $T_n = \inf\{ t>S_n : B_t=0 \}$: S_n is a predictable stopping time/$(\underline{\underline{F}}_t)$, as the debut of a closed predictable set. Then we have

$$\int_0^t 1_{\{Y_s>0\}} dY_s = \Sigma_n \int_0^t 1_{]S_n,T_n[}(s)dY_s = \Sigma_n (Y_{T_n\wedge t} - Y_{S_n\wedge t}) = Y_t$$

since $Y_{S_n}=Y_{T_n}=0$. By difference, $\int_0^t 1_{\{Y_s=0\}}dY_s = 0$, which is wrong, this stochastic integral being equal to the local time of B at 0.

[Comments from the seminar : This example is closely related to the balayage theory given in this volume. It can be generalized in the following way, using the notations of the last paper by Nicole El Karoui. Assume that for some progressive process Z the Azéma-Yor formula

(1) $$Z_{\tau_t} X_t = \int^t Z_{\tau_s} dX_s + \mathfrak{R}_t \qquad \text{(generalized stoch. integral)}$$

has a non vanishing "remainder" $\mathfrak{R}$. Then the same formula remains valid in any expansion of $(\underline{\underline{F}}_t)$ w.r. to which X still is a semimartingale, and therefore Z cannot be optional in such an expansion. Barlow's example concerns the case where X=B , $Z_t = \limsup_{s\downarrow\downarrow t} \mathrm{sgn}(B_s)$, $Z_{\tau_t}X_t=|B_t|$, and (1) is the Tanaka formula (in this case, the generalized stochastic integral turns out to be a martingale).]

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