SEMINAIRE

Equations aux Dérivées Partielles

2001-2002

Pierre-Emmanuel Jabin and Benoît Perthame

Kinetic methods for Line-energy Ginzburg–Landau models

Séminaire É. D. P. (2001-2002), Exposé n° II, 10 p.

<http://sedp.cedram.org/item?id=SEDP_2001-2002____A2_0>

U.M.R. 7640 du C.N.R.S.
F-91128 PALAISEAU CEDEX
Fax : 33 (0)1 69 33 49 49
Tél : 33 (0)1 69 33 49 99

Kinetic methods for Line–energy Ginzburg–Landau models

Pierre-Emmanuel Jabin and Benoît Perthame
Département de Mathématiques
et Applications, UMR8553,
Ecole Normale Supérieure, 45, rue d'Ulm,
75230 Paris Cedex 05, France

Abstract. A class of variational problems arising in thin micromagnetic film or in the gradient theory of phase transitions exhibit an hyperbolic behavior, a surprising property being given their natural elliptic structure. These two–dimensional Ginzburg–Landau problems are, for instance, characterized by energy density concentrations on a one–dimensional set - comparable to a steady shock wave. Here we review how methods based on kinetic formulations can help to understand some features of this broad and fascinating class of problems. Especially we deduce a general regularity result and also we characterize the zero-energy states and the domains where they can occur.

Key words. Ginzburg–Landau energy, vortices, kinetic formulation, averaging lemmas, Sobolev spaces.

AMS Class. Numbers. 35B65, 35J60, 35L65, 74G65, 82D30.

Contents

1 Typical examples **II-1**

2 Kinetic formulation **II-3**

3 A generic regularity result **II-5**

4 Vortices and zero energy states **II-6**

1 Typical examples

Among the wide subject of Ginzburg-Landau variational problems, a typical problem is to study the limit as the parameter ε vanishes, for divergence free functions in $\mathbb{R}^2$ with a finite Ginzburg-Landau energy. Namely, we consider functions $u_\varepsilon \; : \; \Omega \subset \mathbb{R}^2 \to \mathbb{R}^2$ (Ω a smooth domain of $\mathbb{R}^2$) such that

$$\operatorname{div} u_\varepsilon = 0 \quad \text{in } \Omega, \qquad u_\varepsilon \cdot n = 0 \quad \text{on } \partial\Omega, \tag{1.1}$$

where n denotes the outer unit normal to the boundary $\partial\Omega$ of $\Omega \subset \mathbb{R}^2$, and, in this *weakly constrained* case the energy is

$$E_\varepsilon(u_\varepsilon) \;=\; \varepsilon \int_\Omega |\nabla u_\varepsilon|^2 + \frac{1}{\varepsilon}\int_\Omega (1-|u_\varepsilon|^2)^2. \tag{1.2}$$

Roughly speaking, these models introduced in Jin & Kohn [18], Ambrosio, De Lellis & Mantegazza [5], DeSimone, Kohn, Müller & Otto [10] come through dimensional reduction of a three dimensional Ginzburg–Landau–type model in a thin film and singularly depend on the small parameter ε proportional to the film thickness. They arise in many physical situations like smectic liquid crystals, soft ferromagnetic films, in blister formation or — more abstractly — in the gradient theory of phase transition (see [11] and the references therein). Due to the variety of these situations, to the complexity of the reduction and the necessity of mathematical simplifications, several other models are of interest. For instance Rivière & Serfaty [24] consider the *strongly constrained* case where the constraint is given by

$$|u| = 1 \quad \text{in } \Omega, \tag{1.3}$$

whereas the functional is

$$E^2_\varepsilon(u) \;=\; \varepsilon \int_\Omega |\nabla u|^2 + \frac{1}{\varepsilon}\int_{\mathbb{R}^2} |\nabla^{-1} \operatorname{div} u|^2, \tag{1.4}$$

where for the last term, u has been trivially (that is, by zero) extended on all $\mathbb{R}^2$. More recently a model retaining the three dimensional aspect of these problems has been studied by Allouges, Rivière & Serfaty [2].

Many analytical methods arise to study these kinds of problems. Especially, variational analysis arises naturally, SBV-type spaces, geometrical measure theory... These reflects the ellipticity of the problem. More surprisingly entropies, compensated compactness, kinetic formulations and averaging lemmas also help to provide pieces of informations. These reflects the hyperbolic feature of the limit $\varepsilon \to 0$.

In this paper we wish to illustrate a particular point which is why *kinetic formulations* arise and what kind of information it can provide. Let us insist that alternative tools have also been used in the above mentioned papers and most of the results explained here can be derived differently. It seems however that the kinetic struture which arises here is fascinating enough to have a look at it, and especially to further investigate in these terms the difference between the two models mentioned above and also their variants.

Before doing that, we would like however to spend some time to explain the hyperbolic aspect in the limit $\varepsilon \to 0$ in these problems following [10]. This formally appears very naturally because in this limit we obtain

$$|u| = 1, \qquad \operatorname{div} u = 0, \tag{1.5}$$

in Ω. In two dimensions this is typically a scalar conservation law which can be also seen as

$$\frac{\partial \cos(\theta)}{\partial x_1} + \frac{\partial \sin(\theta)}{\partial x_2} = 0.$$

The question is to prove (or as conjectured for (1.1), (1.2), rather to disprove) that it is the entropy solution. Another related point of view is to transform the problem in

$$u = \nabla^T \psi(x), \quad |\nabla \psi| = 1.$$

Then the question is to know weither (or rather why not in the case of (1.1), (1.2)) ψ is the viscosity solution to the above eikonal equation. Also notice that this one-dimensional aspect of the singular set makes this problem very different form the usual two dimensional Ginzburg-Landau problem [7] where point vortices appear, or of the three dimensional vortex tubes as in Aftalion and Jerrard [1].

The paper is organized as follows. The second section explains why a kinetic formulation is natural here, the third section shows the regularity that can be deduced from averaging lemmas, the last section is devoted to the characterization of zero energy states.

2 Kinetic formulation

The kinetic formulation arises naturally in the limit as ε vanishes in the problem (1.1), (1.2). The original motivation comes from an argument developed in [10] and which is based on a family of entropies adapted to the limit as written in (1.5) (different entropies are also built in [18] which produce somewhat different properties). And, the kinetic formulations introduced in Lions, Perthame and Tadmor [20], [21] aim exactly to represent a full family of entropies by a single generating 'equilibrium' function, denoted by χ below, by means of integration of an extra variable. This has the advantage to replace an infinite family of inequalities by a single equation in a higher dimension space. The extra variable, denoted by ξ in this paper, is homogeneous to a velocity and is thus called the kinetic variable. In the context of Line–energy Ginzburg–Landau models it can be introduced drectly through a simple and general lemma which proof can be found in Jabin and Perthame [15].

Lemma 2.1 *For any smooth function u defined on Ω, and with the notation*

$$\chi(\xi, u) = \mathbb{I}_{\{\xi \cdot u > 0\}}, \tag{2.1}$$

we have, in the sense of distribution in ξ,

$$|u|\, \xi \cdot \nabla_x \chi(\xi, u) + |\xi|^2 \left[\nabla_x |u| - \mathrm{div} u \ \frac{u}{|u|}\right] \cdot \nabla_\xi \chi = 0. \tag{2.2}$$

The equation (2.2) above enters the class of classical kinetic equations. It is very close to the Vlasov equation describing the motion of particles in a force fields, see Glassey [13] for instance.

Coming back to the variational problem (1.1), (1.2), the term $\mathrm{div} u$ disappears in the above formula, and $|u| \approx 1$, therefore we can expect that simplications arise then. This is indeed the case and we have

Theorem 2.2 *Any sequence u_ε which satisfies (1.1) and with finite energy in (1.2), is relatively compact in* $\mathrm{L}^2(\Omega)$. *After extraction of a subsequence, the limit u satisfies (1.5) and*

$$\xi \cdot \nabla_x \chi(\xi, u) = \sum \Big(\frac{1}{\beta} \partial_{\xi_i} (|\xi|^2 \partial_{\xi_j} (m^\beta_{ikj} \xi_k)) - \frac{3+\beta}{\beta} \xi_j \partial_{\xi_i} (m^\beta_{jki} \xi_k) \Big) \tag{2.3}$$

where the sum is taken for i, j, k equal to $1, 2$ and β is any real number with $0 < \beta \le 1$, the $m^\beta_{ijk}(\xi, x)$ are measures such that

$$\int_\Omega \sup_{\xi \in \mathbb{R}^2} |m^\beta_{ijk}(\xi, x)|\, dx \le \mathrm{liminf} E_\varepsilon(u_\varepsilon)/2, \quad \sum_i m^\beta_{iij} = 0, \quad \sum_k \xi_k \nabla_\xi m^\beta_{ijk} = 0, \tag{2.4}$$

$$\int_{\Omega \times B(R)} |\nabla_\xi m^\beta_{ijk}(\xi, x)|\, dx\, d\xi \le C(R)\ \mathrm{liminf} E_\varepsilon(u_\varepsilon), \quad \text{for all } R > 0. \tag{2.5}$$

The righthand side measure here appears as a measure of the singular set (shock location) where the above limit is not strong enough (derivatives do not converge strongly). For that reason it is called the kinetic entropy defect measure and very similar to that appering int he kinetic formulation of conservation laws (see [20], [22]). This singular set is expected to be one dimensional as built in several examples ([18]) and as it is proved generally in [3]. Such a property is not proved but a corollary is as follows

Proposition 2.1 *Additionally, we have for any bounded open subset $\mathcal{O}$ of Ω and any $r > 1$*
i) $u \in W^{s,q}$ for all $0 \le s < \frac{1}{3}$, $q < \frac{3}{2}$.
ii) If ∇u_ε is uniformly bounded in $L^r(\mathcal{O})$, then $m^\beta_{ijk} = 0$ on $\mathcal{O}$.
iii) If $\nabla_x |u_\varepsilon|$ is uniformly bounded in $L^r(\mathcal{O})$, then $\sum_j m^\beta_{ijj} = 0$.
4) If $|u_\varepsilon| \to 1$ in $L^\infty(\mathcal{O})$, then $\frac{1}{\beta} m^\beta_{ijk} = m_{ijk}$ does not depend on β, $\sum_j m^\beta_{ijj} = 0$ and

$$\xi \cdot \nabla_x \chi(\xi, u) = \sum_{i,j,k} |\xi|^2 \xi_k \partial_{\xi_i} \partial_{\xi_j} m_{ikj} \ \text{on } \mathcal{O}. \tag{2.6}$$

We refer again to [15] for a proof of these results. We would like however to explain, in next section, the point (i) of this proposition. Here it is improved compared to the original result written in [15] and this regularity was obtained in [16]. Of course it is certainly not optimal and many reasons lead to think that very particular spaces related to spaces of functions with bounded variations play a role here ([6], [18]).

Also we would like to comment that the case of (1.3), (1.4) leads to a much stronger kinetic structure and a sign of the kinetic defect mesure can be proved. This makes the case closer to Scalar Conservation laws and is related to the fact that then u is the classical entropy solution to the hyperbolic equation (1.5), or in other words that ψ is the viscosity solution, see Ambroso, Lecumberry & Rivière [4].

3 A generic regularity result

One of the important tools in kinetic equations consists in deriving regularity in x for averages in ξ through the so called *averaging lemmas.* This idea applies to equations arising from kinetic physics but also to macroscopic equations with a kinetic formulation as scalar conservation laws [20] and systems like isentropic (barotropic) gas dynamics with $\gamma = 3$ [21], or more generally to a class introduced by Brenier and Corrias [8]. The basic averaging lemmas apply to solutions of a transport equation, that we only state in the stationary case although the evolution is more relevant for hyperbolic balance laws,

$$\xi \cdot \nabla_x f(x, \xi) = \Delta_x^{\alpha/2} \, g(x, \xi), \quad x \in \mathbb{R}^d, \ \xi \in \mathbb{R}^d. \tag{3.1}$$

Now we choose a $\phi \in C_c^\infty(\mathbb{R}^d)$ and define

$$u(x) = \int_{\mathbb{R}^d} f(x, \xi)\phi(\xi) d\xi. \tag{3.2}$$

Assume that

$$\begin{aligned} &g \in L^p(\mathbb{R}^d, \ W_\xi^{\beta,p}(\mathbb{R}^d)), \quad 1 < p \le 2, \ \beta \le \frac{1}{2}, \\ &f \in L^q(\mathbb{R}^d, \ W_\xi^{\gamma,q}(\mathbb{R}^d)), \quad 1 < q \le 2, \ 1 - \frac{1}{q} < \gamma \le \frac{1}{2}. \end{aligned} \tag{3.3}$$

Theorem 3.1 *[16] (Case $0 \le \alpha < 1$) Let f, g satisfy (3.1) and (3.3), then we have for $s' < s = \theta(1-\alpha)$ and $r' < r$ with $\frac{1}{r} = \frac{\theta}{p} + \frac{1-\theta}{q}$ and*

$$\theta = \frac{1 + \gamma - 1/q}{1 + \gamma - \beta + 1/p - 1/q}. \tag{3.4}$$

$$\|u\|_{W_{loc}^{s',r'}} \le C \left(\|g\|_{L_x^p W_\xi^{\beta,p}} + \|f\|_{L_x^q W_\xi^{\gamma,q}} \right).$$

For $\gamma = 0$, $\beta \le 0$, we are in a case included in standard averaging lemmas (see in particular [12]). However our statement here is not optimized and thus does not recover the limiting case ($s' = s$, $r' = r \dots$) as can be done following for instance Bouchut in [9], see also Perthame and Souganidis [23]. Also a very surprising phenomena arises for β or γ larger than $1/2$. Then the corresponding formulas for the win of regularity do not yield the optimal gain of regularity and the usual method of proof does not work. In [16] we gave an improved regularity result which uses a frequency decomposition also in the ξ variable, which is not necessary in the case mentioned above.

The new point here compared to [12] is to take into account for possible regularity of f in the ξ variable. This wins extra x regularity by the same process which leads to loose x derivatives to pay for negative ξ derives on the righthand side g. Concerning the applications to macroscopic laws as Brenier-Corrias systems or line-energy Ginzburg-Landau system, it allows to win regularity compared to previously known, and in the case of Scalar Balance laws to simplify the proof).

We do not wish to prove Theorem 3.1, and we rather refer to [16], we just indicate that its proof relies, following [23] in a perturbation of the transport operator which allows to recover an invertible operator in H^1 (product Hardy space) and to use classical Lions-Peetre interpolation being given that in L^2 Fourier analysis allows to prove regularizing effects simply as in the original proof of averaging lemmas in [14].

We just indicate how it is possible to derive from it the point (i) in Proposition 2.1. We define, for $\xi \in \mathbb{R}$, the function $f(x,\xi)$ in (2.1) and use its kinetic formulation (2.3). First of all, we notice that u being in the unit sphere, the localization ϕ in (3.2) does not appear and we can equivalently use

$$u(x) = \int_{\mathbb{R}^d} f(x,\xi)\xi \, d\xi, \qquad f(x,\xi) = \chi(\xi; u(x)).$$

Next, we know, from the estimate (2.5), that the right hand side is the derivative in ξ of a bounded Radon measure in (x,ξ). A measure belongs to any Sobolev space $W^{-\alpha,(1-\alpha/(d+2))^{-1}}$ with $\alpha > 0$. Therefore, we may equivalently write the equation (3.1) in place of (2.3), and choose in (3.1), (3.3) any $\alpha > 0$, $\beta = -\alpha - 1$, $p = (1 - \alpha/(2d+1))^{-1}$. On the other hand, since the derivative in ξ of f is a bounded measure and f belongs to L^∞, by interpolation we know that f belongs to $L^q_x W^{\gamma,q}_\xi$ for any $q < 2$ and $\gamma < \frac{1}{2}$. Applying Theorem 3.1, with $\gamma \approx \frac{1}{2}$, $q \approx 2$, $\alpha \approx 0$, $p \approx 1$, $\beta \approx -1$, we immediately deduce the regularity result (i) in Proposition 2.1.

4 Vortices and zero energy states

In this section we consider again a family u_ε in the variational problem (1.1), (1.2). We wish more precisely to study the possible limits u of sequences $\{u_\varepsilon\}_{\varepsilon\downarrow 0}$ such that $\lim_{\varepsilon\downarrow 0} E_\varepsilon(u_\varepsilon) = 0$ ("zero–energy states"). We follow the presentation in Jabin, Otto and Perthame [17].

Roughly speaking, we prove that the limit u is either a constant vector field (and the domain is either the whole space or a strip) or a vortex (and the domain is either the whole space or a disk), but more general conclusions can also be carried out locally. This conclusions are not at all obvious when considered directly at the level of (1.1), (1.2). It is interesting to notice that the variant model (1.3), (1.3) does not lead to same zero energy states. Namely vortices do not exist in this model due to the stronger constraint $|u_\varepsilon| = 1$, a feature that changes completely the possible singularities, see Lecumberry and Rivière [19].

A first step towards the result is the following obvious consequence of Section §2. The only variant in the kinetic formulation is that now we may, without loosing information, consider that the kinetic variable ξ belongs to the sphere.

Proposition 4.2 *Consider any zero–energy state u, then it satisfies*

$$\begin{aligned} |u(x)| &= 1 \quad \textit{for a. e. } x \in \Omega, && (4.1)\\ \operatorname{div} u &= 0 \quad \textit{distributionally in } \mathbb{R}^2 \textit{ if trivially extended}, && (4.2)\\ \xi \cdot \nabla\chi(\cdot,\xi) &= 0 \quad \textit{distributionally in } \Omega \textit{ for all } \xi \in S^1. && (4.3) \end{aligned}$$

Notice that the statement (4.2) is another way to state that $u \cdot n = 0$ on $\partial\Omega$, as it obviously follows from (1.1).

In [17], we derive from this equation the following characterization of *zero energy states*

Theorem 4.1 *(Case $\Omega = \mathbb{R}^2$) Consider any measurable function u, satisfying (4.1) and (4.3) with $\Omega = \mathbb{R}^2$. Then either u is constant, that is, there exists an $u_0 \in S^1$ such that*

$$u(x) \;=\; u_0 \quad \textit{for a. e. } x \in \mathbb{R}^2,$$

or u is a vortex, that is, there exists a point $\mathcal{O} \in \mathbb{R}^2$ and a sign $\alpha \in \{-1, 1\}$ such that

$$u(x) \;=\; \alpha \frac{(x-\mathcal{O})^\perp}{|x-\mathcal{O}|} \quad \textit{for a. e. } x \in \mathbb{R}^2,$$

where $\perp$ denotes the counter clockwise rotation by $\frac{\pi}{2}$.

Theorem 4.2 *(General Ω) Let Ω satisfy the following property*

$$\begin{array}{l}\Omega \neq \mathbb{R}^2 \textit{ is connected, } C^2, \textit{ and either } \Omega \textit{ is a strip or } \exists\, \tilde{y},\ \tilde{z}\ \in \partial\Omega \\ \textit{such that the normal lines issued from } \tilde{y},\ \tilde{z} \textit{ are different and} \\ \textit{intersect in } \Omega \textit{ before crossing } \partial\Omega.\end{array} \tag{4.4}$$

Assume that the measurable function u satisfies (4.1), (4.2) and (4.3). Then Ω is either a disk and u is a vortex, or Ω is a strip and u is a constant.

We point out that any regular, simply connected domain satisfies Property (4.4). In any simply connected domain there is always a ball of radius at least the minimal radius of curvature of the boundary. But also, for instance, the domain included between two balls which do not have the same center is possible by our condition.

Again, we do not wish to give here a full proof of this theorem and rather refer to [17] for the details. One of the technical difficulty arises from the only L^∞ regularity of u which does not allow to have well defined traces a priori. We can formally explain the argument towards Theorem 4.1 assuming they exist.

Formal proof of theorem 4.1. As a *first step* we notice that, being given a point x_0, and denoting by $L(x_0)$ the line passing by x_0 and orthogonal to $u(x_0)$, then for $x \in L(x_0)$, $u(x)$ is orthogonal to $L(x_0)$. To see this, we can think ofthe direction ξ_0 of $L(x_0)$ as the discontinuity line in ξ of $\chi(x_0; \xi)$, where its value is 1/2 say. By the method of characteristics, $\chi(x_0 - s\xi_0; \xi_0)$ is constant along this line, and thus it is the '1/2'-discontinuity level set i.e. $u(x_0 - s\xi_0)$ is perpendicular to ξ_0 i.e. to $L(x_0)$.

In a *second step*, we assume that u is not constant on $\mathbb{R}^2$ and we prove that it cannot be constant up to a sign. Indeed, assume we had $u(x) = \pm u_0 \in S^1$ for every $x \in \mathbb{R}^2$. Then for every couple x, y such that $x - y$ is not orthogonal to u_0, we would have $u(x) = u(y)$ thanks to the method of characteristics which implies that

$$\chi(x - s\xi; \xi) = \chi(x; \xi). \tag{4.5}$$

Indeed it is enough to choose s and ξ such that $y = x - s\xi$. From this it follows that $u(x)$ is in fact constant.

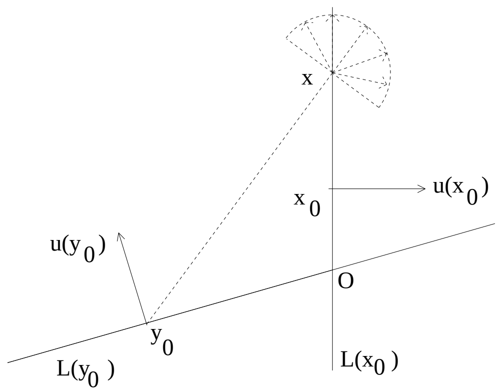

Figure 1: Values of $u(x)$ on the line $L(x_0)$ compatible with $u(y_0)$. Following the characteristic from y_0 to any x, we indeed deduce that $u(x)$ belongs to the drawn half-sphere.

In a *third step*, we assume that u is not constant on $\mathbb{R}^2$ and we precise the 'signs' of $u(x)$ along with the construction in figure (4). From the second step, there exist two points x_0 and y_0 in $\mathbb{R}^2$ with

$$u(x_0) \neq u(y_0) \quad \text{and} \quad u(x_0) \neq -u(y_0). \tag{4.6}$$

We denote $L(x_0)$ and $L(y_0)$ the lines orthogonal to $u(x_0)$ (resp. $u(y_0)$) and passing through the points x_0 (resp. y_0). According to (4.6), these lines intersect in a point that we denote by $\mathcal{O}$, the origin after translation. Notice that this proves that the function u is not continuous, since we know from the first step that $u(\mathcal{O}) \perp L(x_0)$ and $u(\mathcal{O}) \perp L(y_0)$, this remark motivates the technical questions on traces of u which are solved in [17] and some non-trivial counterexample on averaging lemmas in [16]. Let us continue the argument still forgetting about regularity issues. We claim, again as a trivial consequence of the method of characterisitcs (4.5) that on the four half lines $L(x_0)$, $L(y_0)$ delimited by $\mathcal{O}$, $u(x)$ is constant and takes opposite values on both side of $\mathcal{O}$, which determines the right sign because we already know from the first step that $u(x) \in S^1$ and $u(x) \perp L$. To do that, it is enough to join a given point y_1 of a half $L(y_0)$ line to any of the point x of $L(x_0)$ and use $\xi = (x - y_1)/|x - y_1|$, then we notice that $\chi(u(y_1), \xi)$ is constant for all such ξ (the point $y_1 = y_0$ is chosen in figure (4)).

In a *fourth step*, we assume that u is not constant on $\mathbb{R}^2$ and we prove that it is a vortex. Consider now a third point $z_0 \in \mathbb{R}^2$. Then, we have $u(z_0) \neq \pm u(x_0)$ (otherwise we have $u(z_0) \neq \pm u(y_0)$ and this does not change the end of the argument). As before, let $L(z_0)$ be the line through z_0 normal to $u(z_0)$. Then earlier arguments apply, and they imply that $L(z_0)$ intersect $L(x_0)$, and that this again occurs at the origin. Indeed, reproduce the construction in step 3 and notice that the intersection point separates again $L(x_0)$ in two segments where u is constant and takes opposite values. A careful analysis of the values (and their orientation) based again on the method of characteristics now leads to the conclusion that u is a vortex of center $\mathcal{O}$. This completes the proof of Theorem 4.1.

References

[1] A. Aftalion and R. L. Jerrard, Shape of vortices for a rotating Bose Einstein condensate. Preprint cond-mat/0204475.

[2] F. Alouges, T. Rivière and S. Serfaty, Néel walls and cross-tie walls for micromagnetic materials having a strong planar anisotropy. ESAIM:COCV (2002), volume in memory of J.-L. Lions.

[3] L. Ambrosio, B. Kirchheim, M. Lecumberry and T. Rivière, On the rectifiability of defect measures arising in a micromagnetics model. Preprint 2002.

[4] L. Ambrosio, M. Lecumberry and T. Rivière, A viscosity property of minimizing micromagnetic configurations. Preprint.

[5] L. Ambrosio, C. De Lellis and C. Mantegazza, Line energies for gradient vector fields in the plane. *Calc. Var. PDE*, **9** (1999), 327–355.

[6] P. Aviles and Y. Giga, On lower semicontinuity of a defect energy obtained by a singular limit of the Ginzburg-Landau type energy for grasient fields. *Proc. Roy. Soc. Edinburgh*, **129A** (1999), 1–17.

[7] F. Béthuel, H. Brézis and F. Hélein, Ginzburg-Landau vortices. *Progress in Nonlinear Differential Equations and their Applications*, Birkhauser (1994).

[8] Y. Brenier and L. Corrias, A kinetic formulation for multi-branch entropy solutions of scalar conservation laws. *Ann. Inst. H. Poincaré, Analyse non-linéaire*, **15** (1998), 169–190.

[9] F. Bouchut, F. Golse and M. Pulvirenti, Kinetic equations and asymptotic theory. *Series in Appl. Math.*, Gauthiers-Villars (2000).

[10] A. Desimone, R.V. Kohn, S. Müller and F. Otto, A compactness result in the gradient theory of phase transitions. To appear in *Proc. Roy. Soc. Edinburgh.*

[11] A. Desimone, R.V. Kohn, S. Müller and F. Otto, Magnetic microstructures, a paradigm of multiscale problems. To appear in *Proceedings of ICIAM*, (1999).

[12] R.J. DiPerna, P.-L. Lions and Y. Meyer, L^p regularity of velocity averages. *Ann. I.H.P. Anal. Non Linaire* , **8**(3–4) (1991), 271–287.

[13] R.T. Glassey, *The Cauchy problem in kinetic theory*, SIAM publications, Philadelphia (1996).

[14] F. Golse, P.-L. Lions, B. Perthame and R. Sentis, Regularity of the moments of the solution of a transport equation. *J. Funct. Anal.*, **26** (1988), 110-125.

[15] P.-E. Jabin and B. Perthame, Compactness in Ginzburg-Landau energy by kinetic averaging. *Comm. Pure Appl. Math.*, **54** (2001), 1096–1109.

[16] P.-E. Jabin and B. Perthame, Regularity in kinetic formulations via averaging lemmas. ESAIM:COCV (2002), volume in memory of J.-L. Lions.

[17] P.E. Jabin, F. Otto and B. Perthame, Line–energy Ginzburg–Landau models: zero–energy states. *Ann. Sc. Norm. Sup. di Pisa, Cl. Scienze*, to appear.

[18] W. Jin and R. V. Kohn, Singular perturbation and the energy of folds. *J. Nonlinear Sci*, **10** (2000), 355–390.

[19] M. Lecumberry and T. Rivière, Regularity for micromagnetic configurations having zero jump energy. To appear in *Calc. of Var. and PDE* (2002).

[20] P.-L. Lions, B. Perthame and E. Tadmor, A kinetic formulation of multidimensional scalar conservation laws and related equations. *J. Amer. Math. Soc.* **7**, (1994), 169–191.

[21] P.-L. Lions, B. Perthame and E. Tadmor, Existence of entropy solutions to isentropic gas dynamics system in Eulerian and Lagrangian variables. *Comm. Math. Phys.* , **163** (1994), 415–431.

[22] B. Perthame, Kinetic formulations. Oxford University Press (to appear).

[23] B. Perthame and P.E. Souganidis, A limiting case for velocity averaging. *Ann. Sci. École Norm. Sup.(4)*, **31** (1998), 591–598.

[24] T. Rivière and S. Serfaty, Limiting domain wall energy for a problem related to micromagnetics. *Comm. Pure Appl. Math.*, **54** (2001), 294–338.

[25] T. Rivière and S. Serfaty, Compactness, kinetic formulation, and entropies for a problem related to micromagnetics. To appear in *Comm. in PDE.*