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EQUATIONS AUX DERIVEES PARTIELLES

POISSON RELATION FOR THE SCATTERING KERNEL AND INVERSE SCATTERING BY OBSTACLES

L. STOYANOV

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Poisson Relation for the Scattering Kernel and Inverse Scattering by Obstacles

Latchezar Stoyanov*
Department of Mathematics,
The University of Western Australia,
Nedlands, Perth 6009,
Western Australia

1 Introduction

Let K be a compact subset of $\mathbf{R}^n, n \geq 3, n$ odd, with C^∞ boundary ∂K such that

$$\Omega_K = \overline{\mathbf{R}^n \setminus K}$$

is connected. Such a set K is called an *obstacle* in $\mathbf{R}^n$.

Consider the Dirichlet problem:

$$\begin{cases} (\partial_t^2 - \Delta_x)u = 0 \ \text{ in } \ \mathbf{R} \times \Omega, \\ u = 0 \ \text{ on } \ \mathbf{R} \times \partial\Omega, \\ u_{|t=0} = f_1, u_{t|t=0} = f_2. \end{cases}$$

The corresponding *scattering operator* S can be represented as a unitary operator

$$S : L^2(\mathbf{R} \times \mathbf{S}^{n-1}) \longrightarrow L^2(\mathbf{R} \times \mathbf{S}^{n-1})$$

commuting with the translations

$$T_t : L^2(\mathbf{R} \times \mathbf{S}^{n-1}) \longrightarrow L^2(\mathbf{R} \times \mathbf{S}^{n-1}) \ , \ T_t f(s;\omega) = f(s-t;\omega).$$

The linear continuous map

$$S - \mathrm{Id} : C_0^\infty(\mathbf{R} \times \mathbf{S}^{n-1}) \longrightarrow \mathcal{D}'(\mathbf{R} \times \mathbf{S}^{n-1})$$

has a Schwartz kernel

$$s(t,\theta,t',\omega) = s(t-t',\theta,\omega) \in \mathcal{D}'(\mathbf{R}_t \times \mathbf{S}^{n-1} \times \mathbf{R}_{t'} \times \mathbf{S}^{n-1}),$$

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which is called the *scattering kernel*. For fixed ω and θ, s is a distribution on $\mathbf{R}$, $s(t,\omega,\theta) \in \mathcal{D}'(\mathbf{R})$.

Since the singularities of the scattering kernel are observable quantities, it is natural to ask the following

Problem 1. What geometrical information can be obtained about the obstacle K if one knows the singularities of the distribution $s_K(t,\theta,\omega)$ for (almost) all $(\omega,\theta) \in \mathbf{S}^{n-1} \times \mathbf{S}^{n-1}$?

As R. Melrose has kindly informed me recently, in general the singularities of the scattering kernel are not enough to completely recover the obstacle. An example (see Figure 1) which shows this had been given by M. Lifshits and T. Shiota (unpublished; see Lecture 5 in [M3]).

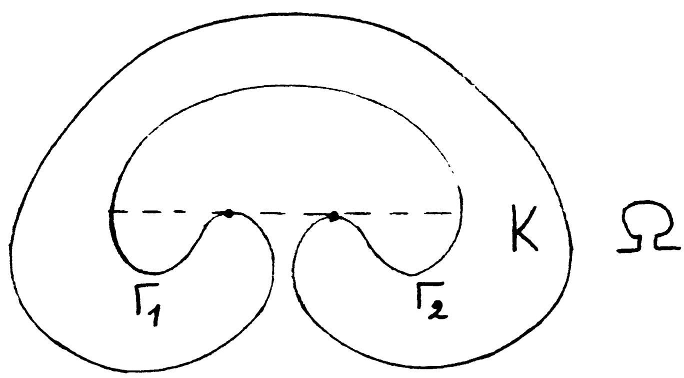

Figure 1

In this example all scattering rays in the exterior of K incoming from infinity and outgoing to infinity after finitely many reflections from ∂K, do not enter these parts of Ω_K that are close to Γ_1 and Γ_2 (see [M3] for more details). Therefore, if one slightly changes ∂K near Γ_1 and Γ_2 only, the new obstacle L obtained in this way will have the same scattering rays and $s_K(t,\theta,\omega)$ and $s_L(t,\theta,\omega)$ will have the same singularities.

However, as we will see below, for some rather large classes of obstacles K the singularities of the scattering kernel provide enough information to completely recover K.

It follows from results of Majda [Ma] and Lax and Phillips [LP2] that for each obstacle K the convex hull of K can be recovered if one knows max sing supp $s_K(t,\theta,\omega)$ for a dense set of (ω,θ)'s in $\mathbf{S}^{n-1} \times \mathbf{S}^{n-1}$. Consequently, an obstacle K is completely determined by the singularities of the scattering kernel, provided one knows in advance that K is convex. Moreover, one can distinguish between convex and strictly convex obstacles by the same data (see [So] and [Y]).

It turns out that for fixed $\omega,\theta \in \mathbf{S}^{n-1}$ the singularities of $s_K(t,\theta,\omega)$ are related to the so called (ω,θ)-rays in Ω_K which are (generalized) geodesics on the manifold with boundary Ω_K incoming from infinity with direction ω and outgoing to infinity with direction θ (see Sect. 2 for the precise definition). The generalized geodesics on a manifold M with boundary were introduced by Melrose and Sjöstrand [MS] as natural projections on M of the integral curves (bicharacteristics) of the generalized Hamiltonian flow on $T^*(M)$ generated by a smooth (Hamiltonian) function – the principal symbol of a certain differential operator on M. Using the work of Melrose and Sjöstrand [MS] and some techniques from Guillemin and Melrose [GM], Majda and Taylor [MaT], Melrose [M1] and Taylor [T], Petkov [P] established that, under certain assumptions about K, we have

$$\text{sing supp } s_K(t,\theta,\omega) \subset \{-T_\gamma : \gamma \in \mathcal{L}_{\omega,\theta}(\Omega_K)\}, \tag{1}$$

where $\mathcal{L}_{\omega,\theta}(\Omega_K)$ is *the set of all* (ω,θ)*-rays in* Ω_K and T_γ is the so called *sojourn time (time delay)* of γ (see Sect. 2). In analogy with the well-known Poisson relation for the Laplacian on Riemannian manifolds, the relation (1) is called *the Poisson relation for the scattering kernel.* In the general case a proof of (1) can be found in Chapter 8 of [PS1].

The behavior of the generalized Hamiltonian flow is rather complicated. In fact, as an example of M. Taylor [T] suggests (see also Sect. 24 in [H]), in general this is not a flow in the usual sense of dynamical systems, since there might exist two different integral curves issued from one and the same point of the phase space. In the case of scattering the following restriction on the obstacle K guarantees ([MS]) that the generalized Hamiltonian flow on $T^*(\Omega_K \times \mathbf{R})$ is well-defined: for each $(x,\xi) \in T^*(\partial K)$ if the curvature of ∂K at x vanishes of infinite order in direction ξ, then ∂K is convex at x in direction ξ. Denote by $\mathcal{K}$ the class of obstacles K that have this property.

Theorem 1. *For each obstacle* $K \in \mathcal{K}$ *there exists a subset* $\mathcal{R}$ *of full Lebesgue measure in* $\mathbf{S}^{n-1} \times \mathbf{S}^{n-1}$ *such that*

$$\text{sing supp } \; s_K(t,\theta,\omega) = \{-T_\gamma : \gamma \in \mathcal{L}_{\omega,\theta}(\Omega_K)\}$$

holds for all $(\omega,\theta) \in \mathcal{R}$.

In the special case when K is a finite disjoint union of convex bodies this statement was proved in [PS2].

Next, we are going to describe a class of obstacles that we call strongly accessible. Though this class is not stable with respect to arbitrary C^2 perturbations, it contains some non-trivial open sets of obstacles in the C^2 topology of Whitney.

For $x \in \partial K$ denote by $N_K(x)$ the *unit normal* to ∂K at x pointing into Ω_K. The obstacle K will be called *regular* if the set of regular points of N_K is dense in ∂K. For $x, \xi \in \mathbf{R}^n$ let $l(x,\xi) = \{x + t\xi : t > 0\}$, $L(x,\xi) = \{x + t\xi : t \in \mathbf{R}\}$. The *convex hull* of K will be denoted by $\text{conv}(K)$.

Definition. A regular obstacle $K \in \mathcal{K}$ will be called *strongly accessible* if there exist an open subset $\partial K^{(n)}$ of ∂K and an open subset V of the *sphere bundle* $S(\partial K)$ of ∂K such that the following conditions are satisfied:

SA1: $\partial K = \partial K^{(n)} \cup \partial K^{(t)}$, where $\partial K^{(t)} = \; \text{pr}_1(V)$, $\text{pr}_1 : S(\partial K) \longrightarrow \partial K$ being the natural projection on the first factor.

SA2: For each $x \in \partial K^{(n)}$ we have $l(x, N_K(x)) \cap K = \emptyset$.

SA3: For each connected component K' of K either $\partial K' \subset \partial K^{(t)}$ or the set $\partial K^{(n)} \cap \partial K'$ is connected and contains extremal points of K.

SA4: $L(x, \xi) \cap K = \{x\}$ for all $(x, \xi) \in V$.

SA5: For every $x \in \partial K$ the segment $l(x, N_K(x)) \cap \mathrm{conv}(K)$ does not contain focal points of the Gauss map N_K.

SA6: $\mathrm{conv}(K) \setminus K \subset \overline{\left(\bigcup_{x \in \partial K^{(n)}} l(x, N_K(x))\right) \bigcup \left(\bigcup_{(x,\xi) \in V} L(x, \xi)\right)}$.

One can check that many familiar obstacles are strongly accessible. For example if a regular obstacle K is connected and satisfies SA2 and SA5 with $\partial K^{(n)} = \partial K$, it is clearly strongly accessible (see Figure 2a). Also every obstacle K which is a finite disjoint union of convex bodies in $\mathbf{R}^n$ is strongly accessible provided it satisfies the condition (H) introduced by M.Ikawa: the convex hull of the union of any two convex components of K has no common points with any other convex component of K.

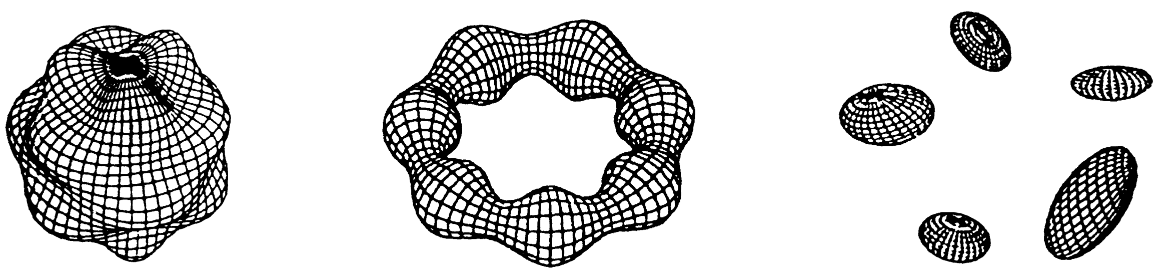

Figure 2. Some strongly accessible obstacles[1]

It turns out that within the class $\mathcal{K}$ the strongly accessible obstacles K can be completely recovered if one knows the scattering length spectrum of Ω_K.

Theorem 2. *Let K be strongly accessible and $L \in \mathcal{K}$. Assume that there exists a subset $\mathcal{R}$ of full Lebesgue measure in $\mathbf{S}^{n-1} \times \mathbf{S}^{n-1}$ such that*

$$\{T_\gamma : \gamma \in \mathcal{L}_{\omega,\theta}(\Omega_K)\} = \{T_\gamma : \gamma \in \mathcal{L}_{\omega,\theta}(\Omega_L)\} \quad , \quad (\omega, \theta) \in \mathcal{R}. \tag{2}$$

Then $K = L$.

It should be stressed that, according to the above theorem, one can not only recover K among the strongly accessible obstacles but also in the (large) class of obstacles $\mathcal{K}$.

[1] Pictures made by Maple

It is quite reasonable to expect that the above statement remains true under weaker assumptions about K.

As a consequence of Theorems 1 and 2 one gets that if K is strongly accessible, $L \in \mathcal{K}$ and there exists a subset $\mathcal{R}$ of full Lebesgue measure in $\mathbf{S}^{n-1} \times \mathbf{S}^{n-1}$ such that

$$\text{sing supp} s_K(t, \theta, \omega) = \text{sing supp} s_L(t, \theta, \omega)$$

holds for all $(\omega, \theta) \in \mathcal{R}$, then $K = L$.

The proof of Theorem 2 is purely geometrical. Form the relation (2), we derive that $\partial K = \partial L$ using some special kind of (ω, θ)-rays in Ω_K and Ω_L.

In connection with Theorem 2 it is interesting to consider the following

Problem 2. Let $K \in \mathcal{K}$ be an obstacle and let $\mathcal{R}$ be a subset of full Lebesgue measure of $\mathbf{S}^{n-1} \times \mathbf{S}^{n-1}$. Consider the map

$$\mathcal{R} \ni (\omega, \theta) \mapsto F(\omega, \theta) = \{T_\gamma : \gamma \in \mathcal{L}_{\omega,\theta}(\Omega_K)\}.$$

Eventually shrinking the domain $\mathcal{R}$ of F, we may assume that the set $F(\omega, \theta)$ is finite for all $(\omega, \theta) \in \mathcal{R}$. Suppose we are given the map F, i.e. we know the set $\mathcal{R}$ and for each $(\omega, \theta) \in \mathcal{R}$ we know the finite set $F(\omega, \theta)$. Assuming we know in advance that the obstacle K is strongly accessible, can we then reconstruct K from F ?

It seems that the answer to the above question should be affirmative. As another consequence of Theorem 1, one can show the following.

Proposition 1. *Let $K \in \mathcal{K}$, U_0 be an open ball containing K and C be the boundary sphere of U_0. For almost all $(x, \omega) \in S^*_C(\Omega)$ the generalized geodesic $\gamma(x, \omega)$ in Ω_K issued from x in direction ω leaves U_0 (and goes to infinity) after finitely many reflections at ∂K.*

The detailed proofs of Theorems 1, 2 and Proposition 1 can be found in [St2]. In Sect. 3 below the idea of the proof of Theorem 1 is presented.

2 Scattering Rays

Let K be an obstacle in $\mathbf{R}^n$ and $\Omega = \Omega_K$ the corresponding exterior domain with smooth boundary $\partial\Omega$. Let

$$S^*(\Omega) = \{(x, \xi) \in T^*(\Omega) : |\xi| = 1\}$$

be the *cosphere bundle* of Ω and

$$F_t : S^*(\Omega) \longrightarrow S^*(\Omega)$$

be the *generalized geodesic flow* in Ω (R.Melrose, J.Sjöstrand [MS]).

A curve γ in Ω of the form

$$\gamma(t) = \mathrm{pr}_1(F_t(\rho)), t \in \mathbf{R},$$

is called a *generalized geodesic* in Ω. Here

$$\mathrm{pr}_1 : T^*(\Omega) \longrightarrow \Omega$$

is the natural projection.

Let $\omega, \theta \in \mathbf{S}^{n-1}$. A generalized geodesic $\gamma(t)$ in Ω is called an *(ω, θ)-ray* in Ω if there exist $a < b$ such that $\dot{\gamma}(t) = \omega$ for $t < a$ and $\dot{\gamma}(t) = \theta$ for $t > b$. If γ has no gliding (geodesic) segments on $\partial\Omega$, it is called a *reflecting (ω, θ)-ray*. If moreover γ has no segments tangent to ∂K, then γ will be called *ordinary*.

Fix an open ball U_0 containing K and let ρ_0 be its radius. Given a vector $\xi \in \mathbf{S}^{n-1}$, denote by Z_ξ the tangent plane to U_0 such that K is contained in the half-space H_ξ determined by Z_ξ and having ξ as an inner normal.

The *sojourn time* of γ is defined by $T_\gamma = T'_\gamma - 2\rho_0$, where T'_γ is the length of this part of γ which is contained in $H_\omega \cap H_{-\theta}$ (Guillemin [G]).

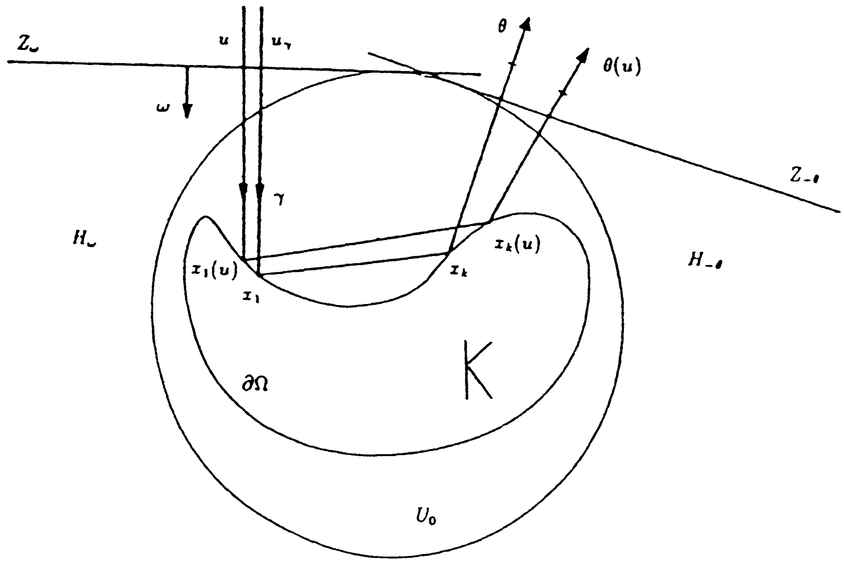

Figure 3

Let γ be an ordinary reflecting (ω, θ)-ray and $x_1, \ldots, x_k$ be its successive (transversal) reflection points. Then (cf. [G])

$$T_\gamma = \langle \omega, x_1 \rangle + \sum_{i=1}^{k-1} \|x_i - x_{i+1}\| - \langle \theta, x_k \rangle.$$

Let u_γ be the orthogonal projection of x_1 on the hyperplane Z_ω. Consider a small open neighbourhood $W = W_\gamma$ of u_γ in Z_ω. For $u \in W$ there are unique $\theta(u) \in \mathbf{S}^{n-1}$ and points

$x_1(u), \ldots, x_k(u) \in \partial K$ which are the successive reflection points of a reflecting $(\omega, \theta(u))$-ray in Ω passing through u. Thus, one gets a smooth map

$$J_\gamma : W_\gamma \longrightarrow \mathbf{S}^{n-1} \;,\; J_\gamma(u) = \theta(u).$$

The ray γ is called *non-degenerate* if $\det dJ_\gamma(u_\gamma) \neq 0$.

Finally denote by $\mathcal{L}_{\omega,\theta}(\Omega)$ the *set of all* (ω, θ)*-rays* in Ω.

3 Idea of the proof of Theorem 1

Let $K \in \mathcal{K}$ be an obstacle in $\mathbf{R}^n$ and $\Omega = \Omega_K$. According to some of the results mentioned in Sect. 1 and to those in [PS2], it follows that to prove Theorem 1 it is sufficient to establish the following

Lemma 1. *For almost all* $(\omega, \theta) \in \mathbf{S}^{n-1} \times \mathbf{S}^{n-1}$ *there do not exist* (ω, θ)*-rays in* Ω *containing non-trivial gliding segments on* $\partial\Omega$.

Since this statement is of a local nature, it is enough to consider (ω, θ) close to a fixed $(\omega_0, \theta_0) \in \mathbf{S}^{n-1} \times \mathbf{S}^{n-1}$ for which there exists an (ω_0, θ_0)-ray γ_0 in Ω containing a non-trivial gliding ray on $\partial\Omega$. Fix γ_0 with this property and take a ball U_0 containing the obstacle K as in Sect. 2. Set $Z = Z_{\omega_0}$, $Z' = Z_{-\theta_0}$. Let l be one of the gliding segments of γ_0 (by assumption there is at least one such segment). Fix an arbitrary interior point x_0 of l and a hyperplane Z'' in $\mathbf{R}^n$ passing through x_0 and transversal to l at x_0 (see Figure 4).

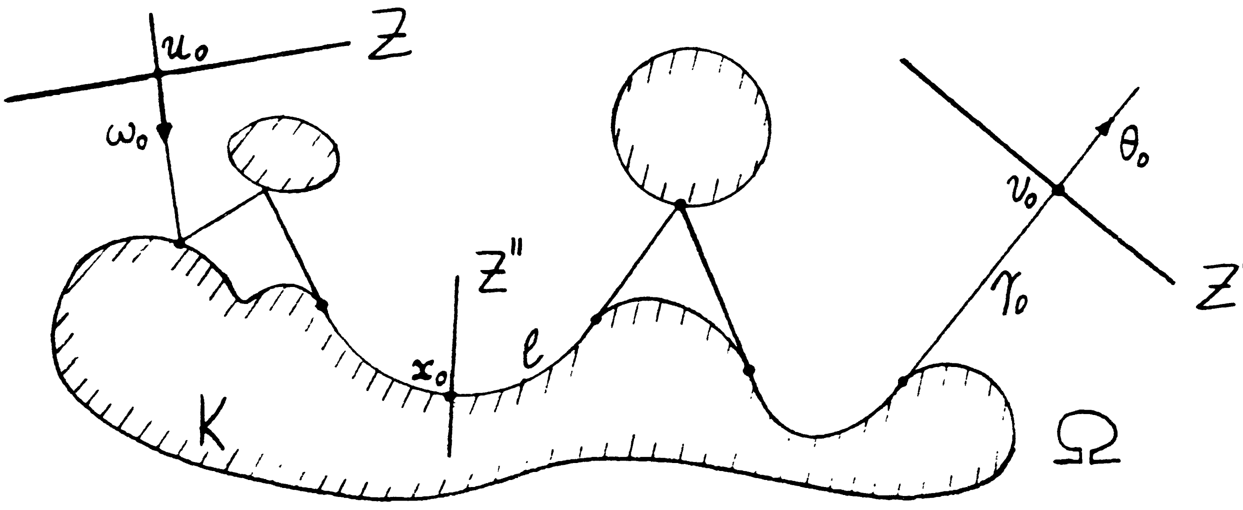

Figure 4

Denote by u_0 the first intersection point of γ_0 with Z and by v_0 the last intersection point of γ_0 with Z'. Locally, near (u_0, ω_0), we can identify $S^*(Z)$ with $Z \times \mathbf{S}^{n-1}$. Given $(u, \xi) \in S^*(Z)$

close to (u_0, ω_0), consider the generalized geodesic $\gamma(u,\xi)$ in Ω issued from (u,ξ) and denote by $\mathcal{P}(u,\xi) = (v,\eta)$ the (unique) point on $\gamma(u,\xi)$ such that v is its last intersection point with Z'. In this way we get a local map

$$S^*(Z) \ni (u,\xi) \mapsto \mathcal{P}(u,\xi) \in S^*(Z')$$

which is also a local homeomorphism (cf. [MS]). Unfortunately $\mathcal{P}$ is not smooth and it is not even clear whether it is Hölder continuous (see [St1]) for a partial result in this direction).

Fix $\omega \in \mathbf{S}^{n-1}$ close to ω_0 and consider the Lagrangian submanifold

$$\mathcal{L} = \mathcal{L}_\omega = \{(u,\omega) \in S^*(Z) : u \in Z\}.$$

Denote by $\mathcal{T}$ *the set of all* $(u,\xi) \in S^*(Z)$ such that the generalized geodesic $\gamma(u,\xi)$ contains a non-trivial gliding segment on $\partial\Omega$ intersecting transversally the hyperplane Z''. Our aim is to prove that the set of those $\theta \in \mathbf{S}^{n-1}$ for which there exists $(u,\omega) \in \mathcal{T}$ with $\mathcal{P}(u,\omega) = (*,\theta)$ has Lebesgue measure zero in $\mathbf{S}^{n-1}$. That is, we want to show that $\mathrm{pr}_2(\mathcal{P}(\mathcal{T}))$ has Lebesgue measure zero in $\mathbf{S}^{n-1}$, where $\mathrm{pr}_2 : S^*(Z') \longrightarrow \mathbf{S}^{n-1}$ is the natural projection on the second factor. This follows easily from the following lemma and Sard's theorem.

Lemma 2. *There exists a countable family* $\{\mathcal{I}_k\}$ *of smooth* $(n-2)$*-dimensional (isotropic) submanifolds of* $S^*(Z')$ *such that* $\mathcal{P}(\mathcal{T}) \subset \cup_k \mathcal{I}_k$.

Define the local maps

$$\mathcal{P}' : S^*(Z) \longrightarrow S^*(Z'') \ , \ \mathcal{P}'' : S^*(Z'') \longrightarrow S^*(Z')$$

in the same way as we defined $\mathcal{P} : S^*(Z) \longrightarrow S^*(Z')$. Then $\mathcal{P} = \mathcal{P}'' \circ \mathcal{P}'$. Denote by M the set of glancing points of $S^*(Z'')$. Then M is a symplectic submanifold of $S^*(Z'')$ with dimension $2n-4$. Clearly

$$\mathcal{P}'(\mathcal{T}) \subset M.$$

To explain the idea of the proof of Lemma 2 we will proceed for a moment as if the maps $\mathcal{P}', \mathcal{P}''$ were smooth and symplectic. Then

$$\mathcal{L}' = \mathcal{P}'(\mathcal{L})$$

would be a Lagrangian submanifold of $S^*(Z'')$. Though the intersection $M \cap \mathcal{L}'$ might have singularities and so it would not be a submanifold, it could be shown that locally $M \cap \mathcal{L}' \subset \mathcal{L}''$ for some Lagrangian submanifold $\mathcal{L}''$ of M. Consequently, $\mathcal{P}''(\mathcal{L}'')$ would be an isotropic submanifold of $S^*(Z')$ with dimension $n-2$. Moreover

$$\mathcal{P}(\mathcal{T}) \subset \mathcal{P}''(\mathcal{L}' \cap M) \subset \mathcal{P}''(\mathcal{L}''),$$

which would prove the statement.

However, as we mentioned above, the map $\mathcal{P}$ (and $\mathcal{P}'$ and $\mathcal{P}''$ as well) is not smooth. Actually, we only want some information about the set $\mathcal{P}(\mathcal{T})$, so the above argument can be still applied if one can change $\mathcal{P}$ outside $\mathcal{T}$ and get a smooth and symplectic map near $\mathcal{T}$. It is even enough to find a representation of $\mathcal{T}$ as a countable union

$$\mathcal{T} = \bigcup_{\alpha} S_\alpha, \tag{3}$$

such that on each set S_α, $\mathcal{P}$ coincides with a smooth symplectic map $\mathcal{P}_\alpha$ defined (locally) on $S^*(Z)$. It turns out that such a representation of $\mathcal{T}$ exists. In what follows we briefly describe the construction of the sets S_α.

Let α be a symbol of the type

$$\alpha = (m; k_1, \dots, k_m; l_1, \dots, l_m; q_0, q_1, \dots, q_m), \tag{4}$$

where $m = m(\alpha) \geq 1$, $k_i \geq 3, l_i \geq 3, q_i \geq 0$ are integers. For such an α denote by S_α *the set of those* $\rho = (u, \xi) \in S^*(Z)$ such that the part of the ray $\gamma(u,\xi)$ between the hyperplanes Z and Z' contains exactly m gliding segments $\delta_i = [x_i, y_i]$ $(i = 1, \dots, m)$ with the following properties:

(i) for every $i = 1, \dots, m$, the ray $\gamma(u, \xi)$ is tangent to ∂K at x_i (resp. at y_i) of order exactly k_i (resp. l_i);

(ii) $\gamma(u, \xi)$ has exactly q_0 transversal reflections from ∂K between u and x_1, q_m transversal reflections between y_m and Z' and q_i transversal reflections between y_i and x_{i+1} for each $i = 1, \dots, m-1$.

It follows from a result in [MS] that (3) holds, where α runs over the (countable) set of all symbols of type (4). Given α and $(u_0, \xi_0) \in S_\alpha$, one changes slightly the generalized geodesic flow near $\gamma(u_0, \xi_0)$ and defines the local smooth and symplectic map $\mathcal{P}_\alpha$ in such a way that it coincides with $\mathcal{P}$ on S_α. In the same way one gets $\mathcal{P}'_\alpha$ and $\mathcal{P}''_\alpha$. The detailed constructions are however rather lengthy and technical and cannot be presented here.

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e-mail address: stoyanov@maths.uwa.edu.au