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Absolutely-p-summing operators in $\mathcal{L}_r$-spaces I

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ABSOLUTELY-p-SUMMING OPERATORS IN $\mathcal{L}_r$-SPACES I

by A. PIETSCH

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The purpose of this paper is to give a uniform presentation of all known results about absolutely-p-summing operators in $\mathfrak{L}_r$-spaces.

§ 1. ABSOLUTELY-p-SUMMING OPERATORS (cf. [11]).

Let E and F be Banach spaces. We denote by $\mathfrak{L}(E,F)$ the set of all bounded linear operators from E into F. An operator $T \in \mathfrak{L}(E,F)$ is called absolutely -p-summing ($1 \leq p < \infty$) if there exists a constant $\rho \geq 0$ such that for every finite set of elements $x_1, \ldots, x_m \in E$ the inequality

$$\{\sum_i \|T x_i\|^p\}^{1/p} \leq \rho \sup_{\|a\| \leq 1} \{\sum_i |\langle x_i, a\rangle|^p\}^{1/p}$$

holds. The set $\mathfrak{P}_p(E,F)$ of all absolutely-p-summing operators $T \in \mathfrak{L}(E,F)$ is a Banach space with norm defined by

$$\pi_p(T) := \inf \rho .$$

It is convenient to put

$$\mathfrak{P}_\infty(E,F) := \mathfrak{L}(E,F) \quad \text{and} \quad \pi_\infty(T) := \|T\| .$$

If $1 \leq p \leq q < \infty$ then

$$\mathfrak{P}_p(E,F) \subset \mathfrak{P}_q(E,F) \quad \text{and} \quad \pi_p(T) \geq \pi_q(T) .$$

§ 2. THE $\mathfrak{L}_r$-SPACES (cf. [8]).

In the following let l_r^n be the Banch space of all n-dimensional real vectors $x = (\xi_i)$ with the norm

$$\|x\|_r := \{\sum_i |\xi_i|^r\}^{1/r} \text{ for } 1 \leq r < \infty \quad \text{and} \quad \|x\|_\infty := \sup_i |\xi_i| .$$

A real Banach space E is called an $\mathfrak{L}_r$-space if for every finite set of elements $x_1, \ldots, x_m \in E$ there exist operators $A \in \mathfrak{L}(E, l_r^n)$ and $X \in \mathfrak{L}(l_r^n, E)$

such that

$$\|x_i - X A x_i\| \leq 1 \text{ for } i=1,\dots,m \quad \text{and} \quad \|X\| \, \|A\| \leq c_E \ ,$$

where the constant $c_E \geq 1$ depends only on E. We note that our definition is a slightly weaker than the definition of J. Lindenstrauss and A. Pełczyński.

All function spaces $L_r(S,\Sigma,\mu)$ are of type $\mathcal{L}_r$. The operators A and X can be constructed as follows. Given $x_1,\dots,x_m \in L_r(S,\Sigma,\mu)$ we find step functions $x_1^o,\dots,x_m^o \in L_r(S,\Sigma,\mu)$ such that $\|x_i - x_i^o\|_r \leq 1/2$. Then there exist disjoint subsets $S_1,\dots,S_n \in \Sigma$ with $\mu(S_i) > 0$ such that the step functions $x_1^o,\dots,x_m^o$ are linear combinations of the corresponding characteristic functions $f_1,\dots,f_n$. Now we define the operators A and X by

$$A\,x := (\mu(S_k)^{-1/r'} \int_S x(s) \, f_k(s) \, d\mu(s))$$

and

$$X(\xi_k) := \sum_k \xi_k \, \mu(S_k)^{-1/r} \, f_k \ .$$

Then we have

$$\|A\| = \|X\| = 1$$

and since $X A f_k = f_k$ the estimate

$$\|x_i - X A x_i\| \leq \|x_i - x_i^o\| + \|X A x_i^o - X A x_i\| \leq 1$$

holds.

Now we show that it is possible to reduce the considerations of absolutely-p-summing operators in $\mathcal{L}_r$-spaces to finite dimensional l_r^n-spaces.

<u>Proposition</u> : <u>The following statements are equivalent</u> :

(1) <u>There exists a constant</u> $c_{rs,pq} > 0$ <u>such that</u>

$$\pi_p(T) \le c_{rs,pq}\, \pi_q(T) \quad \text{for all} \quad T \in \mathfrak{L}(l_r^n, l_s^n) \text{ and } n = 1,2,\dots .$$

(2) <u>For every</u> $\mathfrak{L}_r$<u>-space</u> L_r <u>and every</u> $\mathfrak{L}_s$<u>-space</u> L_s <u>the inclusion</u>

$$\mathfrak{P}_q(L_r, L_s) \subset \mathfrak{P}_p(L_r, L_s)$$

<u>holds</u>.

<u>Proof</u> : (1) (2) Let $T \in \mathfrak{P}_q(E,F)$ and $x_1, \dots, x_m \in E$. Then for all $\varepsilon > 0$ there exist $A \in \mathfrak{L}(E, l_r^n)$, $X \in \mathfrak{L}(l_r^n, E)$, $B \in \mathfrak{L}(F, l_s^n)$, and $Y \in \mathfrak{L}(l_s^n, F)$ such that

$$\|x_i - X A x_i\| \le \varepsilon, \quad \|T x_i - Y B T x_i\| \le \varepsilon, \quad \|X\|\,\|A\| \le c_E, \quad \text{and} \quad \|Y\|\,\|B\| \le c_F .$$

Then

$$\|Tx_i\| \le \|T x_i - Y B T x_i\| + \|Y B T x_i - Y B T X A x_i\| + \|Y B T X A x_i\|$$

$$\le \varepsilon(1 + c_F\|T\|) + \|Y B T X A x_i\| ,$$

$$\{\sum_i \|Y B T X A x_i\|^p\}^{1/p} \le \pi_p(Y B T X A) \sup_{\|a\| \le 1} \{\sum_i |\langle x_i, a\rangle|^p\}^{1/p} ,$$

and

$$\pi_p(Y B T X A) \le \|Y\|\, \pi_p(B T X)\, \|A\| \le c_{rs,pq} \|Y\|\, \pi_q(B T X)\, \|A\|$$

$$\le c_{rs,pq} \|Y\|\, \|B\|\, \pi_q(T)\, \|X\|\, \|A\| \le c_{rs,pq}\, c_E\, c_F\, \pi_q(T) .$$

Consequently,

$$\{\sum_i \|T x_i\|^p\}^{1/p} \le \varepsilon(1 + c_F\|T\|) m^{1/p} + c_{rs,pq}\, c_E\, c_F\, \pi_q(T) \sup_{\|a\| \le 1} \{\sum_i |\langle x_i, a\rangle|^p\}^{1/p}.$$

If $\varepsilon \to 0$, we obtain

$$\pi_p(T) \le c_{rs,pq}\, c_E\, c_F\, \pi_q(T) \quad \text{and} \quad T \in \mathfrak{P}_p(E,F) .$$

(2) → (1). Since the sequence space l_r, resp. l_s, is of type $\mathcal{L}_r$, resp. $\mathcal{L}_s$, we have

$$\mathfrak{P}_q(l_r, l_s) \subset \mathfrak{P}_p(l_r, l_s) \quad .$$

Consequently, by the closed graph theorem there exists a constant $c_{rs,pq} > 0$ such that

$$\pi_p(T) \leq c_{rs,pq}\, \pi_q(T) \quad \text{for all } T \in \mathfrak{P}_q(l_r, l_s) \ .$$

Let us consider the operators

$$Q_n(\xi_1, \dots, \xi_n, \dots) := (\xi_1, \dots, \xi_n)$$

and

$$J_n(\xi_1, \dots, \xi_n) := (\xi_1, \dots, \xi_n, 0, \dots) \quad .$$

Then for every $T \in \mathcal{L}(l_r^n, l_s^n)$ since $T = Q_n(J_n\, T\, Q_n)J_n$ we have

$$\pi_p(T) \leq \pi_p(J_n\, T\, Q_n) \leq c_{rs,pq}\pi_q(J_n\, T\, Q_n) \leq c_{rs,pq}\pi_q(T) \ .$$

§ 3. HISTORICAL REMARKS.

The first result about absolutely-p-summing operators in $\mathcal{L}_r$-spaces goes back to A. Grothendieck [5] who showed in 1956 that all bounded linear operators from an $\mathcal{L}_1$-space into an $\mathcal{L}_2$-space are absolutely-1-summing. A simplified proof of this important results was given by J. Lindenstrauss and A. Pełczyński [8].

In 1967, A. Pełczyński [9] and A. Pietsch [11] proved that the absolutely-p-summing operators in Hilbert spaces coincide with the Hilbert-Schmidt operators. This proof used Chintchin's inequality for Rademacher functions. Finally, D.J.H. Garling [3] determined the exact value of the π_p-norm of diagonal operators in l_2.

Important progress was made in 1969, when L. Schwartz [13], [14], [15] remarked, in his theory of p-radonifying operators, that it is possible to use in place of Rademacher functions general sequences of independant and equidistributed random variables. By his method S. Kwapień [7] and P. Saphar [12] proved the fundamental theorems on absolutely-p-summing operators in $\mathcal{L}_r$-spaces.

§ 4. A PROBABILITY LEMMA.

For $1 \leq s \leq 2$ let μ_s be the probability measure on the real line which is uniquely determined by its characteristic function

$$e^{-|\alpha|^s} = \int_{\mathbb{R}} e^{i\alpha\beta}\, d\,\mu_s(\beta) \quad .$$

If $1 \leq s < 2$ and $1 \leq p < s$ or if $s = 2$ and $1 \leq p < \infty$ then the moments

$$c_{sp} := \{\int_{\mathbb{R}} |\beta|^p\, d\,\mu_s(\beta)\}^{1/p} > 0$$

exist (cf. [4]).

Let μ_s^n be the n-dimensional product measure of μ_s then the following probability lemma holds. It was used in functional analysis at first by J. Bretagnolle, D. Dacunha-Castelle and J.D. Krivine [1].

<u>Lemma</u> : <u>If $y \in \mathbb{R}^n$ then</u>

$$\{\int_{\mathbb{R}^n} |\langle y,b\rangle|^p\, d\,\mu_s^n(b)\}^{1/p} = c_{sp}\, \|y\|_s \quad .$$

<u>Proof</u> : We consider on the probability space $[\mathbb{R}^n, \mu_s^n]$ the independant random variables

$$f_i(b) := \beta_i \quad \text{for} \quad i = 1,\ldots,n \; .$$

Then

$$\hat{f}_i(\alpha) = e^{-|\alpha|^s} \quad ,$$

where $\hat{f}_i$ is the characteristic function of f_i .

Consequently, the random variable

$$f(b) := \langle y,b\rangle = \sum_i \eta_i f_i(b)$$

has the characteristic function

$$f(\alpha) = e^{-\|y\|_s^s |\alpha|^s} .$$

The same characteristic function corresponds to the random variable

$$\varphi(\beta) := \|y\|_s \beta$$

which is defined on the probability space $[\mathbb{R}, \mu_s]$.
Therefore, the two random variables f and φ are equidistributed and we have

$$\int_{\mathbb{R}^n} |\langle y,b\rangle|^p d\mu_s^n(b) = \int_{\mathbb{R}} |\beta|^p d\mu_s(\beta) \|y\|_s^p .$$

§ 5. ABSOLUTELY-p-SUMMING OPERATORS IN l_r^n-SPACES.

We begin the central part of this paper with some few lemmata.

Lemma 1 : Let $T \in \mathcal{L}(E, l_s^n)$. If $1 < s < 2$ and $1 \le p < s$ or if $s = 2$ and $1 \le p < \infty$ then

$$\pi_p(T) \le c_{sp}^{-1} \{\int_{\mathbb{R}^n} \|T'b\|^p d\mu_s^n(b)\}^{1/p} .$$

Proof : It follows from the probability lemma that if $x_1, \dots, x_m \in E$ then

$$\{\sum_i \|T x_i\|_s^p\}^{1/p} = c_{sp}^{-1} \{\int_{\mathbb{R}^n} \sum_i |\langle T x_i, b\rangle|^p d\mu_s^n(b)\}^{1/p}$$

$$\le c_{sp}^{-1} \{\int_{\mathbb{R}^n} \|T'b\|^p d\mu_s^n(b)\}^{1/p} \sup_{\|a\| \le 1} \{\sum_i |\langle x_i, a\rangle|^p\}^{1/p} .$$

Lemma 2 : Let $T \in \mathcal{L}(l^n_{s'},F)$. If $1 < s < 2$ and $1 \leq p < s$ or if $s = 2$ and $1 \leq p < \infty$ then

$$c_{sp}^{-1} \{\int_{\mathbb{R}^n} \|T x\|^p \, d\, \mu_s^n(x)\}^{1/p} \leq \pi_p(T) \quad .$$

Proof : The main theorem of absolutely-p-summing operators (cf. [8], [11]) implies that there exists a measure μ on the closed unit ball U^n_s of l^n_s such that

$$\|T x\| \leq \{\int_{U^n_s} |<x,a>|^p d\,\mu(a)\}^{1/p} \text{ for all } x\in E \text{ and } \mu(U^n_s)^{1/p} = \pi_p(T) \quad .$$

Therefore, it follows from the probability lemma that

$$\{\int_{\mathbb{R}} \|T x\|^p d\,\mu^n_s(x)\}^{1/p} \leq \{\int_{\mathbb{R}^n} \int_{U^n_s} |<x,a>|^p d\,\mu(a)\, d\,\mu^n_s(x)\}^{1/p}$$

$$\leq \int_{U^n_s} c^p_{sp} \|a\|^p_s \, d\,\mu(a)\}^{1/p}$$

$$\leq c_{sp}\, \pi_p(T) \quad .$$

Now we obtain the following lemma, which was proved by S. Kwapień [7], immediately.

Lemma 3 : Let $T \in \mathcal{L}(E,l^n_s)$. If $1 < s < 2$ and $1 \leq p \leq q < s$ or if $s = 2$ and $1 \leq p \leq q < \infty$ then

$$\pi_p(T) \leqq c_{sq}\, c^{-1}_{sp}\, \pi_q(T') \quad .$$

In particular,

$$\pi_p(T) \leq \pi_p(T') \quad .$$

Proof : Applying lemma 1 to $T \in \mathcal{L}(E,l^n_s)$ and lemma 2 to $T' \in \mathcal{L}(l^n_{s'},E')$ we obtain

$$\pi_p(T) \leq c^{-1}_{sp}\{\int_{\mathbb{R}^n} \|T' b\|^p \, d\,\mu^n_s(b)\}^{1/p}$$

$$\leq c^{-1}_{sp}\{\int_{\mathbb{R}^n} \|T' b\|^q \, d\,\mu^n_s(b)\}^{1/q} \leq c_{sq}\, c^{-1}_{sp}\, \pi_q(T') \, ,$$

Remark (of C. Sunyack) : Let $T \in \mathcal{L}(l^n_{s'}, l^n_s)$. Then by lemma 3

$$\pi_p(T) = \pi_p(T') \quad ,$$

and from the inequality in the proof of lemma 3 we obtain the equality

$$\pi_p(T) = c_{sp}^{-1}\{\int_{\mathbb{R}^n} \|T' b\|^p d\mu_s^n(b)\}^{1/p} = c_{sp}^{-1}\{\int_{\mathbb{R}^n} \|T x\|^p d\mu_s^n(x)\}^{1/p} \quad .$$

In particular, if I_n is the identity operator of the Hilbert space l^n_2 then (cf. [3])

$$\pi_p(I_n) = \left(\frac{\Gamma(\frac{n+p}{2}) \ \Gamma(\frac{1}{2})}{\Gamma(\frac{n}{2}) \ \Gamma(\frac{1+p}{2})} \right)^{1/p} \quad .$$

The next lemma was proved by J.S. Cohen [2] and P. Saphar [12].

Lemma 4 : Let $T \in \mathcal{L}(E, l^n_s)$. Then

$$\pi_s(T) \le \pi_s(T') \quad .$$

Proof : If $e_1, \dots, e_n$ are the usual unit vectors we have

$$\|T x\|_s = \{\sum_k |<T x, e_k>|^s\}^{1/s} = \{\sum_k |<x, T' e_k>|^s\}^{1/s}$$

and

$$\{\sum_k \|T' e_k\|^s\}^{1/s} \le \pi_s(T') \sup_{\|y\|_s \le 1} \{\sum_k |<y, e_k>|^s\}^{1/s} = \pi_s(T') \quad .$$

Consequently, if $x_1, \dots, x_m \in E$ then

$$\{\sum_i \|T x_i\|_s^s\}^{1/s} = \{\sum_{ik} |<x_i, T' e_k>|^s\}^{1/s}$$

$$\le \{\sum_k \|T' e_k\|^s\}^{1/s} \sup_{\|a\| \le 1} \{\sum |<x_i, a>|^s\}^{1/s}$$

$$\le \pi_s(T') \sup_{\|a\| \le 1} \{\sum_i |<x_i, a>|^s\}^{1/s} \quad .$$

The proofs of the following propositions are obtained by different combinations of lemma 3 and 4.

<u>Proposition 1</u> : <u>Let</u> $T \in \mathcal{L}(l_r^n, l_s^n)$. <u>If</u> $2 < r < \infty$, $1 \le s < 2$, <u>and</u> $1 \le p < r'$ <u>then</u>

$$\pi_1(T) \le c_{r'p} \, c_{r'1}^{-1} \, \pi_p(T) \quad .$$

<u>Proof</u> : Applying lemma 3 to $T' \in \mathcal{L}(l_{s'}^n, l_{r'}^n)$ we obtain

$$\pi_1(T') \le c_{r'p} \, c_{r'1}^{-1} \, \pi_p(T) \quad .$$

On the other hand by lemma 3 in the case $1 < s < 2$, and by lemma 4 in the case $s = 1$,

$$\pi_1(T) \le \pi_1(T') \ .$$

<u>Theorem 1</u> (P. Saphar [12]) : <u>Let</u> $T \in \mathcal{L}(l_r^n, F)$. <u>If</u> $2 < r < \infty$ <u>and</u> $1 \le p < r'$ <u>then</u>

$$\pi_1(T) \le c_{r'p} \, c_{r'1}^{-1} \, \pi_p(T) \quad .$$

<u>Proof</u> : Without loss of generality we may assume that the Banach space F has the extension property. Consequently (cf. [10]), for all $\varepsilon > 0$ there exists a factorization

$$T : l_r^n \xrightarrow{A} l_\infty^m \xrightarrow{D} l_p^m \xrightarrow{Y} F$$

such that

$$\|A\| \le 1, \ \|Y\| \le 1, \text{ and } \pi_p(D) \le \pi_p(T) + \varepsilon \quad .$$

Now it follows by proposition 1 that

$$\pi_1(T) \le \|Y\| \, \pi_1(D\,A) \le c_{r'p} \, c_{r'1}^{-1} \, \pi_p(D\,A)$$

$$\le c_{r'p} \, c_{r'1}^{-1} \, [\pi_p(T) + \varepsilon] \ .$$

Proposition 2 (S. Kwapień [7]) : Let $T \in \mathcal{L}(l_r^n, l_2^n)$. If $1 < r \le \infty$ then

$$\pi_1(T) \le c_{2r'}\, c_{21}^{-1}\, \pi_{r'}(T) \quad .$$

Proof : Applying lemma 3 to $T \in \mathcal{L}(l_r^n, l_2^n)$ and lemma 4 to $T' \in \mathcal{L}(l_2^n, l_{r'}^n)$ we obtain

$$\pi_1(T) \le c_{2r'}\, c_{21}^{-1}\, \pi_{r'}(T') \quad \text{and} \quad \pi_{r'}(T') \le \pi_{r'}(T) \quad .$$

The case $r = 1$, which is not dealt with in proposition 2, is identical with the fundamental theorem of A. Grothendieck [5].

Proposition 2^G : Let $T \in \mathcal{L}(l_1^n, l_2^n)$. Then

$$\pi_1(T) \le c_G \,\|T\| \; .$$

Remark : If the constant c_G is the best possible then

$$\pi/2 \le c_G \le \sinh \pi/2 \; .$$

Theorem 2 (S. Kwapień) : Let $T \in \mathcal{L}(l_r^n, F)$. If $r = 1$, resp. $1 < r \le 2$, then

$$\pi_1(T) \le c_G\, \pi_2(T), \text{ resp. } \pi_1(T) \le c_{2r'}\, c_{21}^{-1}\, \pi_2(T) \; .$$

Proof : For all $\varepsilon > 0$ there exists a factorization

$$T : l_r^n \xrightarrow{A} l_\infty^m \xrightarrow{D} l_2^m \xrightarrow{Y} F$$

such that

$$\|A\| \le 1, \; \|Y\| \le 1, \text{ and } \pi_2(D) \le \pi_2(T) + \varepsilon \; .$$

Now it follows by proposition 2 that

$$\pi_1(T) \le \|Y\| \pi_1(DA) \le c_{2r'}\, c_{21}^{-1}\, \pi_{r'}(DA)$$

$$\le c_{2r'}\, c_{21}^{-1}\, \pi_2(DA) \le c_{2r'}\, c_{21}^{-1}\, [\pi_2(T) + \varepsilon] \; .$$

The proof in the case $r=1$ is the same.

Proposition 3 : Let $T \in \mathcal{L}(l_2^n, l_s^n)$. If $1 \le s \le p < \infty$ then

$$\pi_s(T) \le c_{2p}\, c_{2s}^{-1}\, \pi_p(T) \quad .$$

Proof : Applying lemma 4 to $T \in \mathcal{L}(l_2^n, l_s^n)$ and lemma 3 to $T' \in \mathcal{L}(l_{s'}^n, l_2^n)$ we obtain

$$\pi_s(T) \le \pi_s(T') \quad \text{and} \quad \pi_s(T') \le c_{2p}\, c_{2s}^{-1}\, \pi_p(T) \quad .$$

Theorem 3 (S. Kwapień [7]) : Let $T \in \mathcal{L}(E, l_s^n)$. If $1 \le s \le 2$ and $2 \le p < \infty$ then

$$\pi_2(T) \le c_{2p}\, c_{2s}^{-1}\, \pi_p(T) \quad .$$

Proof : If $x_1, \dots, x_m \in E$ we define the operator $X \in \mathcal{L}(l_2^m, F)$ by

$$X(\xi_i) := \sum_i \xi_i\, x_i \quad .$$

Then

$$\|X\| = \sup_{\|a\| \le 1} \{\sum_i |\langle x_i, a\rangle|^2\}^{1/2} \quad .$$

Consequently, by proposition 3 we have

$$\begin{aligned}
\{\sum_i \|T x_i\|^2\}^{1/2} &= \{\sum_i \|T X e_i\|^2\}^{1/2} \\
&\le \pi_2(TX) \sup_{\|f\|_2 \le 1} \{\sum_i |\langle e_i, f\rangle|^2\}^{1/2} \\
&\le \pi_s(TX) \le c_{2p}\, c_{2s}^{-1}\, \pi_p(TX) \\
&\le c_{2p}\, c_{2s}^{-1}\, \pi_p(T) \sup_{\|a\| \le 1} \{\sum_i |\langle x_i, a\rangle|^2\}^{1/2} \quad .
\end{aligned}$$

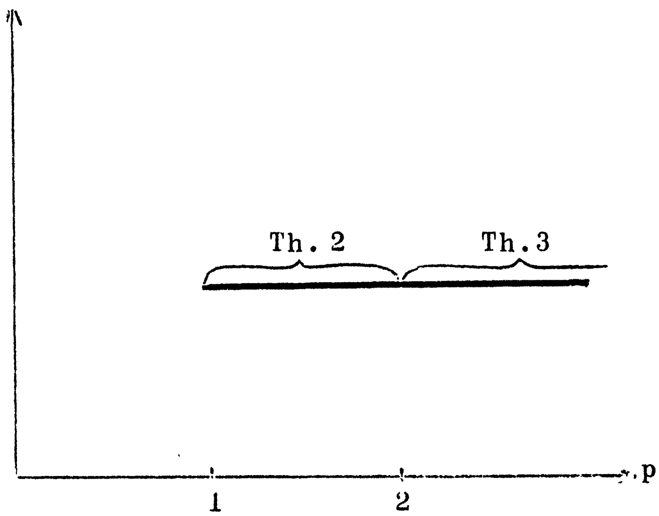
Because of symmetry it seems very probable that we have :

<u>Theorem 4</u> (CONJECTURE) : <u>Let</u> $T \in \mathcal{L}(E, l_s^n)$. <u>If $2 < s < p \leq q < \infty$ then, with a constant</u> $c_{s,pq} > 0$,

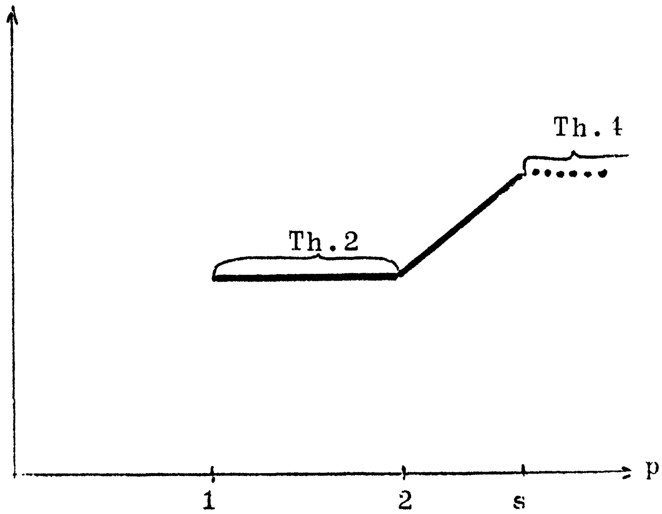
$$\pi_p(T) \leq c_{s,pq}\, \pi_q(T) \quad .$$

Finally, we illustrate the results in the following diagrams where the ordinate is a symbolic measure of the largeness of $\wp_p(L_r, L_s)$.

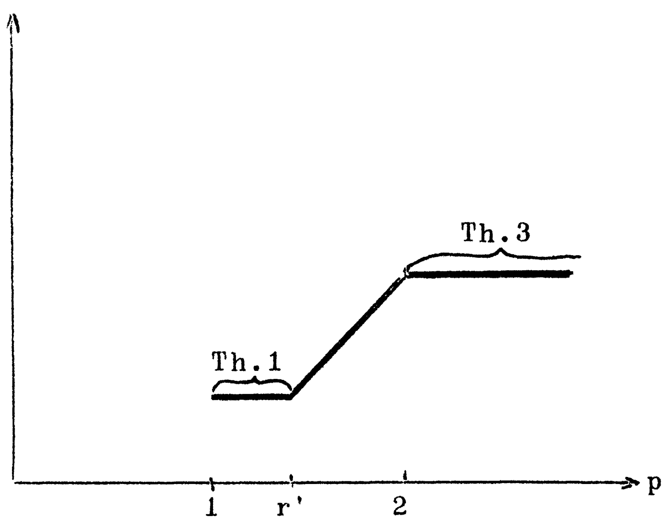
$\wp_p(L_r, L_s)$, $1 \leq r \leq 2$, $1 \leq s \leq 2$



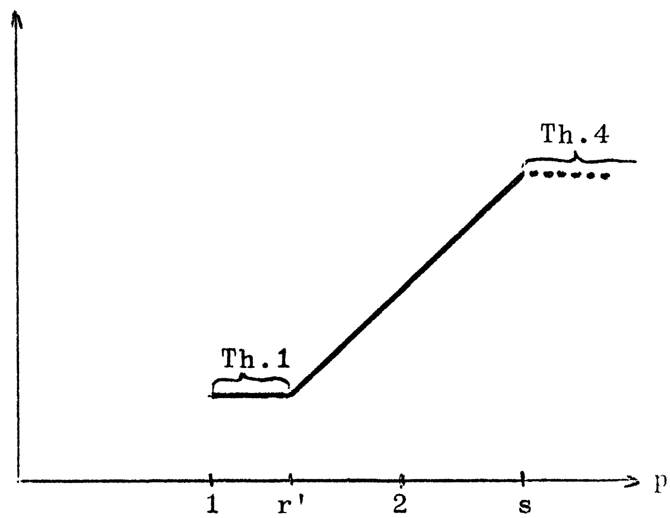
$\wp_p(L_r, L_s)$, $1 \leq r \leq 2$, $2 \leq s \leq \infty$



$\wp_p(L_r, L_s)$, $2 \leq r \leq \infty$, $1 \leq s \leq 2$



$\wp_p(L_r, L_s)$, $2 \leq r \leq \infty$, $2 \leq s \leq \infty$



Remarks :

(1) If the spaces L_r and L_s are infinite dimensional then "$\nearrow$" means that $\mathcal{P}_p(L_r, L_s)$ is strictly increasing (cf. part II).

(2) $\mathcal{P}_p(L_r, L_s)$ depends continuously on p if and only if it is constant since B. Maurey proved, assuming approximations property, the following results.

If $\mathcal{P}_p(E,F) = \bigcap_{\varepsilon>0} \mathcal{P}_{p+\varepsilon}(E,F)$ then $\mathcal{P}_p(E,F) = \mathcal{P}_{p+\varepsilon}(E,F)$ for $0 < \varepsilon < \varepsilon_0$.

If $\mathcal{P}_p(E,F) = \bigcup_{\varepsilon>0} \mathcal{P}_{p-\varepsilon}(E,F)$ then $\mathcal{P}_p(E,P) = \mathcal{P}_{p-\varepsilon}(E,F)$ for $0 < \varepsilon < \varepsilon_0$.

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