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ON THE CONSTRUCTION OF SOME SEMIGROUPS

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We study the properties and the construction of semigroups S, each left ideal of which contains a unique right identity.

Denote by $I(S)$ the set of all idempotents of S (elements of $I(S)$ by e, with indices if needed), by $L(x)$ $(R(x))$ the set of all elements of S generating left (right) principal ideal $(x)_L$ $((x)_R)$. Then, $I(S)$ is a commutative subsemigroup of S, $I(S)$ is a well ordered set according to the relation

$$e_k \leqslant e_i \iff e_i\, e_k = e_k\, e_i = e_k \ .$$

In S, each element x belongs to some $L(e_i)$ and to some $R(e_k)$ class. Denote

$$L^*(e) = \{L(e) \cap R(e_i) \ ; \quad e_i \leqslant e\} \ .$$

Then $L^*(e)$ is a subsemigroup of S with the two-sided identity e. Further, $\mathfrak{L}(e) = \{L(e_i) \ ; \quad e_i \leqslant e\}$ is a subsemigroup of S with the two-sided identity e. The same for $R^*(e)$, $\mathfrak{R}(e)$.

Let S be a finite semigroup. Then S is a finite well ordered set of finite groups.

Every semigroup S, with $I(S)$ of the type ω, and with $L(e_i) \cap R(e_k) = D_{ik}$ containing just one element for each e_i, e_k, is isomorphic to the bicyclic semigroup. If S has the classes D_{ik} containing at least one element, then S can be homomorph mapped into bicyclic semigroup (preserving D_{ik}).

The semigroups of required properties, if D_{ik} contain at most one element, can be constructed in a simple manner if $I(S)$ is of the type ω. Firstly, we define in $I(S)$

$$e_i\, e_k = e_k\, e_i = e_k \quad \text{if} \quad e_k \leqslant e_i \ .$$

Then we assign to each $e_i \in I(S)$ two well ordered sets $L^*(e_i)$ and $R^*(e_i)$ orderisomorphic if both are non empty, having the property : Let e be the first element of $I(S)$ with a non empty $L^*(e)$. We begin to define the multiplication between $L^*(e)$ and $I(S)$; for $x \in L^*(e)$, and e_i where $e \leqslant e_i$, let

$$xe_i = e_i\, x = x \ .$$

To each x, we choose a fixed $e_k < e$, this choice is to be orderpreserving (the mapping of $L^*(e)$ into $I(S)$ denote by α), and $e_m x = x$ for $e_k \leq e_m$. Now we define the multiplication in $L^*(e)$:

$$x_1 , x_2 \in L(e) , \quad \alpha x_1 = e_1 < \alpha x_2 = e_2 ,$$

then

$$x_1 x_2 = x_2 x_1 = x_3 \in L^*(e) \text{ with } \alpha x_3 = e_3 \text{ where } \langle e_1 , e_3 \rangle \simeq \langle e , e_2 \rangle .$$

Further to each $e_i < e$, we assign $L(e_i)$ orderisomorphic to $L^*(e)$ (the mapping φ). We continue in the definition of the multiplication between $L^*(e)$ and $L^*(e_i)$, using α and φ. Similarly, for $R^*(e)$, $R^*(e_i)$ (mappings β and ψ), at last between $R^*(e)$ and $L^*(e_i)$, using α, β, φ, ψ.

Remark. - If we have choosen $L^*(e)$ for some e, and in $L^*(e_i)$ would exist some element x which is not an image of an element of $L^*(e)$, this x requires, for each $y \in L^*(e)$, $\alpha y = e_1$, the existence of $y' \in L^*(e)$ where, if $\alpha x = e_k$, then $\alpha y' = e_2$ with $\langle e_1 , e_2 \rangle \simeq \langle e_i , e_k \rangle$. Respecting this, we can construct each semigroup of required property.

This construction can be adapted to the other cases.

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