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Séminaire Delange-Pisot-Poitou. Théorie des nombres, tome 20, nº 2 (1978-1979), exp. nº 33, p. 1

<http://www.numdam.org/item?id=SDPP_1978-1979__20_2_A10_0>

Séminaire DELANGE-PISOT-POITOU
(Théorie des nombres)
20e année, 1978/79, n° 33, 1 p.

11 juin 1979

A COUNTREXAMPLE IN THE GALOIS MODULE STRUCTURE OF WILD EXTENSIONS OF THE RATIONAL FIELD

by Stephen M. J. WILSON (*)

Abstract

Let K be a finite galois extension of $\mathbb{Q}$ with group $\mathcal{G}$ and let $\mathbb{Z}_K$ be the integers in K. Choose a maximal order $\mathfrak{M}$ containing $\mathbb{Z}\mathcal{G}$. It was conjectured by J. MARTINET, and proved in 1974 by A. FRÖHLICH that if $K/\mathbb{Q}$ is tamely ramified then $[\mathbb{Z}_K \otimes_{\mathbb{Z}\mathcal{G}} \mathfrak{M}] = [\mathfrak{M}]$ in $\mathcal{G}_0(\mathfrak{M})$. In 1975, J. COUGNARD showed that

$$[\mathbb{Z}_K \otimes_{\mathbb{Z}\mathcal{G}} \mathfrak{M}/\text{torsion}] \neq [\mathfrak{M}]$$

in general, but he has subsequently shown that $[\mathbb{Z}_K \otimes_{\mathbb{Z}\mathcal{G}} \mathfrak{M}] = [\mathfrak{M}]$ in many cases.

In this talk an example (with $\mathcal{G} = C_{23} \times D_{18}$) was presented such that both

$$[\mathbb{Z}_K \otimes_{\mathbb{Z}\mathcal{G}} \mathfrak{M}] - [\mathfrak{M}] \quad \text{and} \quad [\mathbb{Z}_K \otimes_{\mathbb{Z}\mathcal{G}} \mathfrak{M}/\text{torsion}] - [\mathfrak{M}]$$

are dependant on the choice of $\mathfrak{M}$ (This statement makes sense as the groups $\mathcal{G}_0(\mathfrak{M})$ for different $\mathfrak{M}$ are canonically isomorphic). In particular, in this example, there exists an $\mathfrak{M}$ such that the two elements are non-zero.

(The full details will shortly be submitted for publication under same title.)

(*) Texte reçu le 6 juillet 1979.
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