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A Duality Approach
for Solving Identification Problems
Related to Integrodifferential Maxwell's Equations.

A. LORENZI - V. PRIYMENKO (*) (**) (***)

SUMMARY - We determine a memory term in Maxwell's equations related to polarized media by means of some additional physical measurement. In the context of duality spaces we prove some existence, uniqueness and stability results

0. Introduction.

In the investigation of quickly changing electromagnetic fields, whose frequencies need not to be small in comparison with the ones characterizing electric and magnetic polarization of the medium, the dependence of vectors D and B on E and H is usually expressed by saying that *the values $D(t)$ and $B(t)$ at time t depend on the corresponding values $E(t)$ and $H(t)$ only*. Yet, in some physical situations [5, chpt. 9] we are forced to assume that the values $D(t)$ and $B(t)$ at time t depend not only on $E(t)$ and $H(t)$, but also on the preceding values $E(\tau)$ and $H(\tau)$ with $\tau \leqslant t$. This situation expresses the fact that the electrical and magnetic polarizations of the medium are affected by the *past history* of the electromagnetic field.

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When dealing with such quickly changing fields we usually assume that vectors D and E are related by an equation of the following type

$$(0.1) \qquad \widehat{D}(\omega, x) = [1 + \widehat{a}(\omega)] \widehat{E}(\omega, x) \qquad (\omega, x) \in \mathbb{R} \times \Omega,$$

where $\frown$ denotes the Fourier transformation with respect to time and

$$(0.2) \qquad \operatorname{Supp} a \subset [0, +\infty)$$

according to the *causality principle* [3].

On the other hand the magnetic conductivity μ, unlike the dielectric one, differs very little from 1, when frequency grows up comparatively quickly. This allows us to assume $\mu = 1$ *everywhere* Hence we get the following constitutive law:

$$(0.3) \qquad \widehat{B}(\omega, x) = \widehat{H}(\omega, x) \qquad (\omega, x) \in \mathbb{R} \times \Omega.$$

In this paper we deal with the following Maxwell's equations, where $'$ denotes differentiation with respect to time, T is a positive number and Ω is a bounded, open and connected set in $\mathbb{R}^3$ with a *smooth* boundary:

$$(0.4) \qquad \operatorname{rot} H(t, x) = D'(t, x) + \delta(t) \otimes f(x) + Y_+(t) F(t, x)$$
$$(t, x) \in (-\infty, T) \times \Omega,$$

$$(0.5) \qquad \operatorname{rot} E(t, x) = -B'(t, x) \quad (t, x) \in (-\infty, T) \times \Omega,$$

$$(0.6) \qquad \operatorname{div} B(t, x) = 0 \quad (t, x) \in (-\infty, T) \times \Omega.$$

Here Y_+, $\delta(t)$ and $\otimes$ denote the Heaviside function, the Dirac measure at $t = 0$ and the tensor product of distributions respectively. We stress here that f *may* denote the Dirac delta (or a derivative of its) with respect to space variables.

From equations (0.1)-(0.6) we easily derive the following differential system for the pair (E, H)

$$(0.7) \qquad \operatorname{rot} H(t, x) = E'(t, x) + \int_0^{+\infty} a(s) E'(t - s, x) \, ds + \delta(t) \otimes f(x) +$$
$$+ Y_+(t) F(t, x) \quad (t, x) \in (-\infty, T) \times \Omega,$$

$$(0.8) \qquad \operatorname{rot} E(t, x) = -H'(t, x) \quad (t, x) \in (-\infty, T) \times \Omega,$$

$$(0.9) \qquad \operatorname{div} H(t, x) = 0 \quad (t, x) \in (-\infty, T) \times \Omega.$$

We assume now to know the *histories* of the electric and magnetic fields in the past (*i.e.* for $t < 0$). Explicitly, this means that we are given the equations

(0.10) $E(t, x) = E_0(x), \quad (t, x) \in (-\infty, 0] \times \Omega,$

(0.11) $H(t, x) = H_0(x), \quad (t, x) \in (-\infty, 0] \times \Omega,$

where the pair (E_0, H_0) satisfies the equations

(0.12) $\operatorname{rot} E_0(x) = \operatorname{rot} H_0(x) = 0 \quad x \in \Omega,$

(0.13) $\operatorname{div} H_0(x) = 0 \quad x \in \Omega.$

REMARK 0.1.   From (0.12)-(0.13) we derive that $H_0 = Dq$, q being *any* (scalar) *harmonic* function in Ω. For the sake of simplicity we shall assume in the sequel $H_0 = 0$.

As far as boundary conditions are concerned, we assume to be able to measure the magnetic field on the boundary $\partial\Omega$ of the body Ω starting from the time $t = 0$. In other words, we have

(0.14) $H(t, x) = \bar{H}(t, x) \quad (t, x) \in (0, T) \times \partial\Omega,$

$\bar{H}: [0, T] \times \Omega \to \mathbb{R}^3$ being a prescribed (smooth) vector function. According to equations (0.9), (0.11), (0.14) and Remark 0.1 function $\bar{H}$ may be supposed to satisfy the additional equations:

(0.15) $\operatorname{div} \bar{H}(t, x) = 0 \quad (t, x) \in (-\infty, T) \times \Omega,$

(0.16) $\bar{H}(0, x) = 0 \quad x \in \Omega.$

Condition (0.16) guarantees the continuity (with respect to time) of vector H at $t = 0$.

We assume now that the kernel $a \in C^1([0, T])$ is itself *unknown*. Thus our problem consists in determining the triplet (E, H, a). To this purpose it is necessary to provide an additional information. Taking the previous results into account, we give the following information involving the magnetic field H

(0.17) $\langle H(t, \cdot), \varphi \rangle = g(t) \quad t \in [0, T].$

Here φ denotes a (smooth) prescribed function, while $\langle \cdot, \cdot \rangle$ stands for a pairing between some function spaces (for details cf. Section 1).

We notice that a problem similar to ours has been dealt with by Wolfersdorf [8]. However, he considers essentially the onedimensional case and assumes that the geometry under consideration is either the

whole space or a slab. On the contrary, we stress that we deal with the threedimensional case, the geometry involved being represented by a general bounded, open, connected set. This forces us to prescribe explicit boundary conditions (cf. equation (0.14)). The novelty of our paper consists just in treating such conditions in the framework of duality spaces.

We recall also that an identification problem related to Maxwell's equations in thredimensional domains (though with a different formulation and in the framework of usual Sobolev function spaces) has been dealt with in [4].

We conclude this section replacing identification problem (0.7)-(0.11), (0.14), (0.17) by an equivalent one involving the pair (H,a) only.

To this purpose we apply then the *rot* operator to equation (0.7) and differentiate equation (0.8) with respect to time. Using the differential identity $\text{rot}^2 = - \varDelta + \text{grad div}$, where grad denotes the space gradient, and taking into account remark 0.1, we easily realize that the magnetic field H solves the initial-boundary value problem

$$(0.18) \quad H''(t, x) - \varDelta H(t, x) + \int_0^{+\infty} a(s) H''(t - s, x)\, ds =$$

$$= \delta(t) \otimes \text{rot} f(x) + Y_+(t) \text{rot} F(t, x) \quad (t, x) \in (- \infty, T) \times \Omega,$$

$$(0.19) \quad \text{div}\, H(t, x) = 0 \quad (t, x) \in (- \infty, T) \times \Omega,$$

$$(0.20) \quad H(t, x) = 0 \quad (t, x) \in (- \infty, 0) \times \Omega,$$

$$(0.21) \quad H(t, x) = \bar{H}(t, x) \quad (t, x) \in (0, T) \times \partial\Omega.$$

To simplify the integral term in (0.18) we recall the continuity of function $t \to H(t, \cdot)$ in $(- \infty, T]$ and use the following distributional formula, where $[H'']$ denotes the second-order *pointwise* derivative:

$$(0.22) \qquad\qquad H'' = \delta(t) \otimes H'(0, \cdot) + [H''].$$

Whence, by an integration by parts, we get the equation

$$(0.23) \quad \int_0^{+\infty} a(s) H''(t - s, x)\, ds =$$

$$= Y_+(t) \left[a(0) H'(t, x) + \int_0^t a'(s) H'(t - s, x)\, ds \right] \quad (t, x) \in (- \infty, T) \times \Omega.$$

Hence equation (0.18) can be rewritten in the following equivalent form

(0.18') $H''(t, x) - \Delta H(t, x) +$

$$+ Y_+(t) \left[a(0) H'(t, x) + \int_0^t a'(s) H'(t - s, x) \, ds \right] =$$

$$= \delta(t) \otimes \operatorname{rot} f(x) + Y_+(t) \operatorname{rot} F(t, x) \quad (t, x) \in (-\infty, T) \times \Omega.$$

We observe now that, once we have determined a solution (H, a) to problem (0.17), (0.18')-(0.21), from (0.7) we immediately get the following equation for the electric field E

(0.24) $D_t \left\{ E(t, x) - E_0(x) + \right.$

$$+ \int_0^{+\infty} a(s)[E(t - s, x) - E_0(x)] \, ds + Y_+(t) f(x) \right\} =$$

$$= \operatorname{rot} H(t, x) - Y_+(t) F(t, x) \quad (t, x) \in (-\infty, T) \times \Omega.$$

From (0.24) and (0.10)-(0.12) we easily derive the equation

(0.25) $E(t, x) - E_0(x) +$

$$+ \int_0^t a(s)[E(t - s, x) - E_0(x)] \, ds = -Y_+(t) f(x) +$$

$$+ Y_+(t) \int_0^t [\operatorname{rot} H(s, x) - F(s, x)] \, ds \quad (t, x) \in (-\infty, T) \times \Omega.$$

Introduce now the kernel $b \in C^1([0, T])$ which solves the linear Volterra integral equation

(0.26) $\displaystyle a(t) + b(t) + \int_0^t a(t - s) \, b(s) \, ds = 0 \quad t \in [0, T].$

From (0.25)-(0.26) we finally get the desired representation formula for E:

$$(0.27) \quad E(t, x) = E_0(x) + Y_+(t)\left\{-\left[1 + \int_0^t b(s)\,ds\right]f(x) + \right.$$

$$\left. + \int_0^t \left[1 + \int_0^{t-s} b(\sigma)\,d\sigma\right]Y_+(s)[\operatorname{rot} H(s, x) - F(s, x)]\,ds\right\}$$

$$(t, x) \in (-\infty, T) \times \Omega\,.$$

Assume now that (E, H) solves problem (0.7), (0.10), (0.18)-(0.21). Apply then the *rot* operator to both members in (0.7) and subtract from it memberwise equation (0.17). Using the equation $\operatorname{rot}^2 H = -\Delta H$ (implied by (0.18)) we derive the following equation

$$(0.28) \quad D_t\left\{\operatorname{rot} E(t, x) + H'(t, x) + \right.$$

$$\left. + \int_0^{+\infty} a(s)[\operatorname{rot} E(t - s, x) + H'(t - s, x)]\,ds\right\} = 0$$

$$(t, x) \in (-\infty, T) \times \Omega\,.$$

From (0.28) we derive that there exists a function $C: \Omega \to \mathbb{R}^3$ such that

$$(0.29) \quad \operatorname{rot} E(t, x) + H'(t, x) + $$

$$+ \int_0^{+\infty} a(s)[\operatorname{rot} E(t - s, x) + H'(t - s, x)]\,ds = C(x)$$

$$(t, x) \in (-\infty, T) \times \Omega\,.$$

Equations (0.10)-(0.13) imply $C = 0$ in Ω and

$$(0.30) \quad \operatorname{rot} E(t, x) + H'(t, x) + $$

$$+ \int_0^t a(s)[\operatorname{rot} E(t - s, x) + H'(t - s, x)]\,ds = 0 \quad (t, x) \in (0, T) \times \Omega\,.$$

Assume then that $t \to \operatorname{rot} E(t, x) + H'(t, x) \in L^1(0, T)$ for a.e. $x \in \Omega$ (cf. also Section 1). From (0.30) we immediately deduce that the pair (E, H) satisfies also equation (0.8).

We have thus shown the equivalence of problems (0.17)-(0.11),

(0.14), (0.17) and (0.17), (0.18')-(0.21) (cf. Section 1 for the functional details).

We conclude this section by reducing our identification problem (0.17), (0.18')-(0.21) to a problem with null boundary data, useful for our abstract formulation (see Section 2). To this purpose we need to introduce the new unknown

$$(0.31) \quad \tilde{H}(t, x) = H(t, x) - Y_+(t)\bar{H}(t, x) \quad (t, x) \in (-\infty, T) \times \Omega.$$

Consequently, according to equation (0.16), our problem transforms into the following

$$(0.32) \quad \tilde{H}''(t, x) - \Delta\tilde{H}(t, x) + Y_+(t)\left\{a(0)[\tilde{H}'(t, x) + \dot{\bar{H}}'(t, x)] + \right.$$

$$\left. + \int_0^t a'(s)[\tilde{H}'(t - s, x) + \bar{H}'(t - s, x)]\,ds\right\} =$$

$$= \delta(t) \otimes [\mathrm{rot}f(x) - \bar{H}'(0, x)] +$$

$$+ Y_+(t)[\mathrm{rot}F(t, x) - \bar{H}''(t, x) + \Delta\bar{H}(t, x)] \quad (t, x) \in (-\infty, T) \times \Omega,$$

$$(0.33) \quad \mathrm{div}\,H(t, x) = 0 \quad (t, x) \in (-\infty, T) \times \Omega,$$

$$(0.34) \quad \tilde{H}(t, x) = 0 \quad (t, x) \in (-\infty, 0) \times \Omega,$$

$$(0.35) \quad \tilde{H}(t, x) = 0 \quad (t, x) \in (0, T) \times \partial\Omega,$$

$$(0.36) \quad \langle\tilde{H}(t, \cdot), \varphi\rangle = g(t) - \langle\bar{H}(t, \cdot), \varphi\rangle \quad t \in [0, T].$$

1. The main result.

In order to state our basic result we need to introduce the following functional spaces related to a bounded, connected, open set of class $C^{1,1}$:

$$(1.1) \quad H = \{u \in L^2(\Omega; \mathbb{R}^3): \mathrm{div}\,u = 0 \text{ in } H^{-1}(\Omega; \mathbb{R}),$$

$$v \cdot u = 0 \quad \text{in } H^{-1/2}(\partial\Omega; \mathbb{R})\},$$

$$(1.2) \quad W = \{u \in H_0^1(\Omega; \mathbb{R}^3): \mathrm{div}\,u = 0 \text{ in } \Omega\}, \quad W^{-1} = W^*.$$

REMARK 1.1. According to the results in Temam[7, chpt. 1], we deduce that W is dense in H and separable, W being a closed subspace in $H_0^1(\Omega; \mathbb{R}^3)$.

REMARK 1.2. Let $-\Delta \in \mathcal{L}(W; W^*)$ be the linear operator defined by the equation

$$(1.3) \qquad \langle -\Delta u, v \rangle = (Du, Dv)_{L^2(\Omega; \mathbb{R}^3)} \qquad \forall u, v \in W.$$

Then, according to Theorem 2.2.3 in Tanabe[6] and well-known regularity results for elliptic equations, we deduce the equations

$$(1.4) \quad W = \mathcal{D}((-\Delta)^{1/2}),$$

$$(1.5) \quad W^2 = \mathcal{D}(-\Delta) = \{u \in H_0^1(\Omega; \mathbb{R}^3) \cap H^2(\Omega; \mathbb{R}^3): \operatorname{div} u = 0$$

$$\text{in } \Omega, \ v \cdot \Delta u = 0 \text{ in } H^{-1/2}(\partial\Omega; \mathbb{R})\}.$$

We can now introduce the following Hilbert spaces

$$(1.6) \quad W^{2m+1} = \{u \in H_0^1(\Omega; \mathbb{R}^3) \cap H^{2m+1}(\Omega; \mathbb{R}^3): \operatorname{div} u = 0$$

$$\text{in } \Omega, \Delta^k u = 0 \text{ on } \partial\Omega, \ k = 0, \dots, m\} \quad m \in \mathbb{N} \backslash \{0\},$$

$$(1.7) \quad W^{2m+2} = \{u \in H_0^1(\Omega; \mathbb{R}^3) \cap H^{2m+2}(\Omega; \mathbb{R}^3): \operatorname{div} u = 0 \text{ in } \Omega,$$

$$\Delta^k u = 0 \text{ on } \partial\Omega, \ k = 0, \dots, m, \ v \cdot \Delta^m u = 0 \text{ in } H^{1/2}(\partial\Omega; \mathbb{R})\}$$

$$m \in \mathbb{N} \backslash \{0\},$$

$$(1.8) \quad W^{-j} = (W^j)^* \quad j \in \mathbb{N} \backslash \{0, 1\}.$$

REMARK 1.3. If Ω is of class $C^{p-1,1}$ for some $p \in \mathbb{N} \backslash \{0, 1\}$, from (1.4)-(1.7) and the regularity results for elliptic operators we deduce the relations

$$(1.9) \qquad \mathcal{D}((-\Delta)^{j/2}) = W^j \quad j = 0, \dots, p.$$

REMARK 1.4. According to Sobolev embedding theorems we get the algebraic and topological inclusions $W^s \hookrightarrow C(\bar{\Omega})$ when $s > 3/2$. Hence $C(\bar{\Omega})^* \hookrightarrow W^{-s}$ when $s > 3/2$.

REMARK 1.5. From definitions (1.6)-(1.7) we easily get $\Delta \in \mathcal{L}(W^j; W^{j-2})$ for any $j = 1, \dots, p$.

Finally, we introduce the following Banach space useful to solve, our identification problem

(1.10) $V_T^{m,\,-r} =$

$$= \{u \in C([0, T]; W^{-r}): D_t^j u \in C([0, T]; W^{-r-j}),\ j = 0, \dots, m\}$$

$$m \in \mathbb{N},\ r \in \mathbb{N} \setminus \{0, 1\}.$$

As far as data are concerned, we assume that they fulfill the following properties for some $r \in \mathbb{N} \setminus \{0, 1\}$ and $T_0 > 0$ (recall that $H_0 = 0$):

(1.11) $F \in C^1([0, T_0]; W^{2-r}),\qquad \bar{H} \in V_{T_0}^{3,\,3-r},$

(1.12) $f, E_0 \in W^{3-r},$

(1.13) $g \in C^3([0, T_0]).$

Finally, we assume that

(1.14) $\phi \in W^r$

and the following inequalities are satisfied

(1.15) $\chi_1 := \langle \operatorname{rot} f - D_t \bar{H}(0, \cdot), \phi \rangle_r \neq 0,$

(1.16) $\chi_2 := \langle \operatorname{rot} F(0, \cdot) - \bar{H}''(0, \cdot), \phi \rangle_r - g''(0) \neq 0,$

where $\langle \cdot, \cdot \rangle_r$ denotes the (canonical) pairing between W^{-r} and W^r.
    Our main result is

    THEOREM 1.1. *Let Ω be an open, bounded, connected set in $\mathbb{R}^3$ of class $C^{r-3,1}$ with $r \geq 5$ and let ϕ be a function satisfying (1.14). Assume that data $(F, f, E_0, \bar{H}, g)$ satisfy properties (0.12), (0.15), (0.16), (1.11)-(1.13), (1.15)-(1.16).*
    Then there exists $T \in (0, T_0]$ such that problem (0.7-11), (0.14) (0.17) admits a unique solution $(E, H, a) \in C^1([0, T]; W^{3-r}) \times \times V_T^{3,3-r} \times C^1([0, T])$ continuously depending on data in the norms pointed out. Moreover the map Data $\to$ Solution is uniformly continuous on bounded sets in $C^1([0, T]; W^{2-r}) \times W^{3-r} \times W^{3-r} \times V_T^{3,3-r} \times \times C^1([0, T])$ consisting of vector functions $(F, f, E_0, \bar{H}, g)$ such that

(1.17) $|\langle \operatorname{rot} f - D_t \bar{H}(0, \cdot), \phi \rangle_r| \geq m,$

(1.18) $|\langle \operatorname{rot} F(0, \cdot) - \bar{H}''(0, \cdot), \phi \rangle_r - g''(0)| \geq m,$

for some positive (fixed) constant m.

REMARK 1.6. In the case where $f = \delta(x_0) z$ for some $x_0 \in \bar{\Omega}$ and some vector $z \in C(\bar{\Omega})^3$, according to our assumption $r \leq 5$, we derive that $\delta(x_0) z \in W^{3-r}$. Moreover, taking advantage of the equation

(1.19) $\langle \operatorname{rot}[\delta(x_0)z], \phi \rangle_r = z(x_0) \cdot \operatorname{rot} \phi(x_0)$

we can rewrite (1.17) in the form

(1.20) $|z(x_0) \cdot \operatorname{rot} \phi(x_0) - \langle D_t \bar{H}(0, \cdot), \phi \rangle_r| \geq m$.

2. An abstract setting.

Let H and W be two real Hilbert spaces such that $W \hookrightarrow H \hookrightarrow W^*$ densely, W^* denoting the dual space to W. Assume that $a: W \times W \to \mathbb{R}$ is a symmetric, positive, W-elliptic and bounded bilinear form satisfying the following estimates, where $\|\cdot\|_1$ denotes the norm in W:

(2.1) $\alpha \|v\|_1^2 \leq a(v, v)$ for any $v \in W$ and some $\alpha \in \mathbb{R}_+$;

(2.2) $|a(v, u)| \leq M \|v\|_1 \|u\|_1$ for any $v, u \in W$ and some $M \in \mathbb{R}_+$.

Then there exists an operator $A \in \mathcal{L}(W; W^*)$ such that

(2.3) $a(v, u) = \langle v, Au \rangle_1$ $\forall v, u \in W$.

Here $\langle \cdot, \cdot \rangle_1$ denotes the pairing between W and W^*.

We recall that $A: \mathcal{D}(A) \subset H \to H$ is a closed linear self-adjoint, (unbounded) positive operator.

We define now the spaces W^s ($s = 0, \dots, N$) by the equations

(2.4) $W^s = \mathcal{D}(A^{s/2})$ $(W^0 = H; W^1 = W)$

and we endow them with their graph-norms denoted by $\|\cdot\|_s$. We recall [1, chpt. 8, Theor. 3.13] that

    i) W^s *are Hilbert spaces* $(0 \leq s \leq N)$;

    ii) $W^q \hookrightarrow W^s$ *densely* $(0 \leq s \leq q \leq N)$;

    iii) $A^{s/2} \in \mathcal{L}(W^q; W^{q-s})$ $(0 \leq s \leq q \leq N)$;

    iv) $A^{1/2}$ *and* A *are invertible and* $A^{-1/2} \in \mathcal{L}(H; W)$ *and* $A^{-1} \in \mathcal{L}(W^*; W)$.

Then we define the dual spaces

(2.5) $W^{-s} = (W^s)^*$ $(0 \leq s \leq N)$

and endow them with their usual norms. Moreover $\langle \cdot, \cdot \rangle_s$ denotes the pairing between W^{-s} and W^s ($s = 1, \ldots, N$).

Finally, we define, by duality, operators $A^{s/2}$ on W^{-q} ($q = 1, \ldots, N$). This implies $A^{s/2} = (A^{s/2})^* \in \mathcal{L}(W^{s-q}; W^{-q})$ ($1 \leqslant s < q \leqslant N$). Moreover we assume

(2.6) $A^{-m} \in \mathcal{L}(W^j; W^{2m+j})$ ($j = 0, 1; m = 1, \ldots, [N/2]$).

REMARK 2.1. Using equations $A^{-s/2} = A^{-m}$ (if $s = 2m$, $m \in \mathbb{N} \setminus \{0\}$) and $A^{-s/2} = A^{-m}A^{1/2} = A^{1/2}A^{-m}$ (if $s = 2m - 1$, $m \in \mathbb{N} \setminus \{0\}$), from (2.6) we deduce the properties

(2.7) $A^{-s/2} \in \mathcal{L}(W^j; W^{s+j})$ ($j = 0, 1; s = 1, \ldots, N$).

REMARK 2.2. In our concrete case, where $A = -\Delta$ (cf. Section 1), properties (2.6) are implied by regularity results for elliptic boundary value problems, requiring the suitable smoothness of the boundary $\partial\Omega$.

Then we introduce the following Banach spaces $V_T^{m, -r}$, basic to our investigations:

(2.8) $V_T^{m,-r} = \{u \in C([0, T]; W^{-r}): u^{(j)} \in C([0, T]; W^{-r-j}), 0 \leqslant j \leqslant m\}$

normed by

(2.9) $\|u\|_{m, r, T} = \sum_{j=0}^{m} \|u^{(j)}\|_{C([0, T]; W^{-r-j})}$.

We consider now the following abstract Cauchy problem, r being a fixed integer in $[2, N]$: *determine a pair of functions* $u \in C((-\infty, T); X) \cap$ $\cap V_T^{3, 3-r}$, $h \in C([0, T])$ *and a real number* β *such that*

(2.10) $u''(t) + Au(t) + Y_+(t)\{\beta[u'(t) + p(t)] + h * (u' + p)(t)\} =$

$$= \delta(t) \otimes u_1 + Y_+(t) f(t) \quad t \in (-\infty, T),$$

(2.11) $u(t) = 0 \quad t \in (-\infty, 0)$,

(2.12) $\langle u(t), \varphi \rangle_r = g(t) \quad t \in [0, T]$,

where $*$ denotes convolution and

(2.13) $\varphi \in W^r$.

As far as data p, f, u_1, g are concerned, we assume that they enjoy the

following properties for some given $T_0 > 0$:

(2.14) $$p, f \in C^1([0, T_0]; W^{1-r}),$$

(2.15) $$u_1 \in W^{2-r},$$

(2.16) $$g \in C^3([0, T_0]).$$

Using standard arguments (for details cf., *e.g.* [1, chpt. 16, sect. 4]), we easily realize that problem (2.10)-(2.12) is equivalent to the following: *determine a pair of functions* $u \in V_T^{3,\,3-r}$, $h \in C([0, T])$ *and a real number* β *such that*

(2.17) $$u''(t) + Au(t) + \beta[u'(t) + p(t)] + h * (u' + p)(t) = f(t)$$

$$t \in (0, T),$$

(2.18) $$u(0) = 0,$$

(2.19) $$u'(0) = u_1,$$

(2.20) $$\langle u(t), \varphi \rangle_r = g(t). \quad t \in [0, T].$$

We observe now that β can be *a priori* determined in terms of data, if the following condition is satisfied:

(2.21) $$\chi_1 := \langle u_1 + p(0), \varphi \rangle_r \neq 0.$$

In fact, if we compute at $t = 0$ the duality products of both members in (2.17) multiplied by φ and use equations (2.18)-(2.20), we easily get the following equation for β:

(2.22) $$g''(0) + \beta \langle u_1 + p(0), \varphi \rangle_r = \langle f(0), \varphi \rangle_r.$$

From (2.21) and (2.22) we derive the required value for β:

(2.23) $$\beta = \chi_1^{-1} \{ \langle f(0), \varphi \rangle_r - g''(0) \}.$$

We rewrite equation (2.17) in the following form

(2.17') $$u''(t) + Au(t) + B(u' + p, h) = f(t) \quad t \in (0, T).$$

Operator B is assigned by the formula

(2.24) $$B(w, h)(t) = \beta w(t) + h * w(t)$$

β being defined by (2.23).

It is immediate to verify that B satisfies the properties:

(2.25) $B \in C(V_T^{0,\,j-r} \times C([0, T]; V_T^{0,\,j-r})$ $j = 0, 1, 2,$

(2.26) $B(w, h)(0) = w(0)$ $\forall w \in V_T^{0,\,j-r}, \ \forall h \in C([0, T]), j = 0, 1, 2,$

(2.27) $D_t B(w, h)(t) = h(t)\, w(0) + B(w', h)(t)$

$$\forall t \in [0, T], \ \forall w \in V_T^{1,\,2-r}, \ \forall h \in C([0, T]),$$

(2.28) $\|B(w_2, h_2) - B(w_1, h_1)\|_{0,\,1-r,\,t}^2 \leqslant$

$$\leqslant m_1 \left(\sum_{j=1}^{2} \|h_j\|_{0,\,t}, \ T \right) \|w_2 - w_1\|_{0,\,1-r,\,t}^2 +$$

$$+ m_2 \left(\sum_{j=1}^{2} \|w_j\|_{0,\,1-r,\,t}, \ T \right) \int_0^t \|h_2 - h_1\|_{0,\,s}^2 \, ds$$

$$\forall t \in [0, T], \ \ \forall w_1, w_2 \in V_T^{0,\,1-r}, \ \ \forall h_1, h_2 \in C([0, T]),$$

where

(2.29) $m_1(s) = 3\beta^2 + 3Ts^2,$

(2.30) $m_2(s) = 6[\|p\|_{0,\,T}^2 + s^2],$

(2.31) $\|h\|_{0,\,t} = \|h\|_{C([0,\,t])}$ $t \in (0, T].$

Our basic abstract result is the following

THEOREM 2.1. *Let operator A satisfy properties (2.1)-(2.3), (2.6). Let* $(p, f, u_1, g) \in C^1([0, T_0]; W^{1-r}) \times C^1([0, T_0]; W^{1-r}) \times W^{2-r} \times C^3([0, T_0])$ *be a quadruplet satisfying conditions (2.21) and*

(2.32) $\chi_2 := \langle f(0), \varphi \rangle_r - g''(0) \neq 0.$

Then there exists $T \in (0, T_0]$ such that problem (2.17')-(2.20) admits a unique solution $(u, h) \in V_T^{3,\,3-r} \times C([0, T])$ depending continuously on data with respect to the norms pointed out. Moreover, the map $(p, f, u_1, g) \to (u, h)$ is Lipschitz continuous on the bounded sets in $C^1([0, T]; W^{1-r}) \times C^1([0, T_0]; W^{1-r}) \times W^{2-r} \times C^3([0, T_0])$, whose ele-

ments satisfy the bounds

(2.33) $|\langle u_1 + p(0), \varphi \rangle_r| \geqslant m$,

(2.34) $|\langle f(0), \varphi \rangle_r - g''(0)| \geqslant m$,

for some (fixed) positive constant m.

3. An equivalence result for the abstract problem.

Assume $u \in V_T^{3, \, 3-r}$ solves problem (2.17′)-(2.20). Introduce then the function $v \in V_T^{2, \, 2-r}$ defined by

(3.1) $v(t) = u'(t) \quad t \in [0, T]$.

Differentiating equation (2.17′) and using properties (2.26)-(2.27), we easily deduce that the pair (v, h) satisfies the identification problem: *determine a pair* $(v, h) \in V_T^{2, \, 2-r} \times C([0, T])$ *such that*

(3.2) $v''(t) + Av(t) + \beta h(t)[u_1 + p(0)] + B(v' + p', h)(t) = f'(t)$

$$t \in (0, T),$$

(3.3) $v(0) = u_1$,

(3.4) $v'(0) = f(0) - \beta[u_1 + p(0)]$,

(3.5) $\langle v(t), \varphi \rangle_r = g'(t) \quad t \in [0, T]$.

Hence, from (2.18)-(2.20), (3.3)-(3.5) we deduce that our data have to satisfy the following consistency conditions

(3.6) $g(0) = 0, \quad \langle u_1, \varphi \rangle_r = g'(0)$.

Conversely, assume that $(v, h) \in V_T^{2, \, 2-r} \times C([0, T])$ solves (3.2)-(3.5). Then, the function

(3.7) $u(t) = \displaystyle\int_0^t v(s) \, ds \quad t \in [0, T]$

is easily seen to be a solution to problem (2.17′)-(2.20) satisfying the properties $u \in C([0, T]; W^{2-r})$, $u' \in V_T^{2, \, 2-r}$. Moreover, from equation (2.27′) we derive that $Au \in C([0, T]; W^{1-r})$: this implies $u \in C([0, T]; W^{3-r})$. Since $u' \in V_T^{2, \, 2-r}$, we deduce $u \in V_T^{3, \, 3-r}$.

To check that u satisfies also equation (2.20), we note that, accord-

ing to (3.1), equation (3.5) can be rewritten in the form

(3.8) $D_t \{\langle u(t), \varphi \rangle_r - g(t)\} = 0 \quad t \in [0, T].$

Such an equation is equivalent to the following

(3.9) $\langle u(t), \varphi \rangle_r - g(t) = \langle u(0), \varphi \rangle_r - g(0) \quad t \in [0, T].$

Finally, from (3.6)-(3.7) we deduce that u satisfies (2.20).

Assume now, again, that $(v, h) \in V_T^{2, 2-r} \times C([0, T])$ solves (3.2)-(3.5) and take the duality products of both members in (3.2) multiplied by φ. Under assumption (2.32) we deduce the following equation for h:

(3.10) $h(t) =$

$$= \chi_2^{-1} \{\langle f'(t), \varphi \rangle_r - g^{(3)}(t) - \langle Av(t), \varphi \rangle_r - \langle B(v' + p', h)(t), \varphi \rangle_r\}$$

$$t \in [0, T].$$

Conversely, assume that $(v, h) \in V_T^{2, 2-r} \times C([0, T])$ solves problem (3.2)-(3.4), (3.10). Observe then that, owing to (3.2), (3.10) is equivalent to the equation

(3.11) $D_t^2 \{\langle v(t), \varphi \rangle_r - g'(t)\} = 0 \quad t \in [0, T].$

In fact, using consistency conditions (3.6) and initial condition (3.4), from (3.11) it is immediate to derive equation (3.5).

In conclusion, we have proved the equivalence of problems (2.17′)-(2.20) and (3.2)-(3.4), (3.10).

THEOREM 3.1. *Let operator A satisfy properties* (2.1)-(2.3), (2.6). *Assume further that* $(f, u_1, g) \in C^1([0, T]; W^{1-r}) \times W^{2-r} \times C^3([0, T])$ *satisfies conditions* (2.21) *and* (2.32). *Let* $(u, h) \in V_T^{3, 3-r} \times C([0, T])$ *be a solution to problem* (2.17′)-(2.20). *Then the pair* $(v, h) \in V_T^{2, 2-r} \times C([0, T])$, *$v$ being defined by formula* (3.1), *solves problem* (3.2)-(3.4), (3.10). *Vice versa, if* (v, h) *has such a property, then the pair* (u, h), *u being defined by* (3.8) *belongs to* $V_T^{3, 3-r} \times C([0, T])$ *and solves problem* (2.17′)-(2.20).

4. Solving the direct problem (3.2)-(3.4).

First we observe that problem (3.2)-(3.4) can be rewritten in the form

(4.1) $v''(t) + Av(t) + D(v, h)(t) = \bar{f}(t) \quad t \in (0, T),$

(4.2) $v(0) = v_0,$

(4.3) $v'(0) = v_1.$

We have set

(4.4) $D(v, h)(t) = h(t)[u_1 + p(0)] + B(v' + p', h)(t)$ $t \in (0, T)$,

(4.5) $\bar{f}(t) = f'(t)$ $t \in [0, T]$,

(4.6) $v_0 = u_1$,

(4.7) $v_1 = f(0) - \beta[u_1 + p(0)]$.

According to properties (2.25)-(2.28) we derive that D satisfies the estimate

(4.8) $\|D(v_2, h_2) - D(v_1, h_1)\|_{0, 1-r,t}^2 \leqslant$

$$\leqslant m_1\left(\sum_{j=1}^{2} \|h_j\|_{0, t}, T\right)\left[\|v_2 - v_1\|_{1, 2-r,t}^2 + m_3\left(\sum_{j=1}^{2} \|v_j\|_{1, 2-r,t}, T\right)\|h_2 - h_1\|_{0, t}^2\right]$$

$$\forall t \in [0, T], \quad \forall v_1, v_2 \in V_T^{1, 2-r}, \quad \forall h_1, h_2 \in C([0, T]),$$

where

(4.9) $$m_3(s, T) = 2\|u_1 + p(0)\|^2 + 2Tm_2(s, T).$$

We are now in a position to prove

THEOREM 4.1. *For any integer* $r \in \mathbb{N} \setminus \{0\}$ *and any quadruplet* $(h, \bar{f}, v_0, v_1) \in C([0, T]) \times V_T^{0, 1-r} \times W^{2-r} \times W^{1-r}$ *problem* (4.1)-(4.3) *admits a unique solution* $v \in V_T^{2, 2-r}$ *satisfying the estimate*

(4.10) $\|v\|_{1, 2-r,t}^2 \leqslant$

$$\leqslant m_4(\|h\|_{0, T}, T)\left\{\|v_0\|_{2-r}^2 + \|v_1\|_{1-r}^2 + \int_0^t [\|\bar{f}(s)\|_{1-r}^2 \|h\|_{0, s}^2] \, ds\right\}$$

$$\forall t \in [0, T],$$

m_4 *being a continuous and nondecreasing function of its arguments.*
    Moreover, if $v, v^* \in V_T^{2, 2-r}$ *are solutions to problems* (4.1)-(4.3) *corresponding to data* (h, f, v_0, v_1), $(h^*, f^*, v_0^*, v_1^*) \in C([0, T]) \times$

$\times V_T^{0,\,1-r} \times W^{2-r} \times W^{1-r}$, *then* $v - v^*$ *satisfies the estimate*

(4.11) $\|v - v^*\|_{1,\,2-r,t}^2 \leqslant$

$$\leqslant m_5(\|v\|_{1,\,2-r,T} + \|v^*\|_{1,\,2-r,T}, \|h\|_{T,\,s} + \|h^*\|_{T,\,s}, T) \cdot$$

$$\cdot \left\{ \|v_0 - v_0^*\|_{2-r}^2 + \|v_1 - v_1^*\|_{1-r}^2 + \int_0^t [\|\bar{f} - \bar{f}^*\|_{0,\,1-r,s}^2 + \|h - h^*\|_{0,\,s}^2 \right\} ds$$

$$\forall t \in (0, T],$$

m_5 *being a continuous and nondecreasing function of its arguments.*

PROOF. Assume that $(h, \bar{f}, v_0, v_1) \in C([0, T]) \times V_T^{0,\,1} \times W^2 \times W$. Then the solution v to problem (4.1)-(4.3) belongs [2] to $V_T^{2,\,0}$. Consequently, accordingly to (2.7), we get $A^{1-r}v'(t) \in W^{2r-1}$, $A^{(1-r)/2}v'(t) \in W^r$, $A^{(1-r)/2}v''(t) \in W^{r-1}$ and $A^{(2-r)/2}v(t) \in W^r$ $\forall t \in [0, T]$. Multiplying scalarly both members of equation (4.1) by $2A^{1-r}v'(t)$, we easily deduce the following chain of inequalities, $\|\cdot\|_{\alpha,\beta}$ denoting the norm in $\mathcal{L}(W^\alpha; W^\beta)$:

(4.12) $\dfrac{d}{dt} \{ \|A^{(1-r)/2}v'(t)\|_0^2 + \|A^{(2-r)/2}v(t)\|_0^2 \} =$

$$= \frac{d}{dt} \{ \langle v'(t), A^{1-r}v'(t) \rangle + \langle Av(t), A^{1-r}v'(t) \rangle \} = 2\langle v''(t) + Av(t), A^{1-r}v'(t) \rangle =$$

$$= 2\langle \bar{f}(t) - D(v, h), A^{1-r}v'(t) \rangle = 2\langle A^{(1-r)/2}\bar{f}(t), A^{(1-r)/2}v'(t) \rangle -$$

$$- 2\langle A^{(1-r)/2}D(v, h), A^{(1-r)/2}v'(t) \rangle \leqslant \|A^{(1-r)/2}\bar{f}(t)\|_0^2 +$$

$$+ \|A^{(1-r)/2}D(v, h)(t)\|_0^2 + 2\|A^{(1-r)/2}v'(t)\|_0^2 \leqslant \|A^{(1-r)/2}\|_{1-r,0}^2 \cdot$$

$$\cdot \{ \|\bar{f}(t)\|_{1-r}^2 + \|D(v, h)(t)\|_{1-r}^2 + 2\|v'(t)\|_{1-r}^2 \} \leqslant$$

$$\leqslant \|A^{(1-r)/2}\|_{1-r,0}^2 \{ \|\bar{f}(t)\|_{1-r}^2 + 3\|D(0, 0)(t)\|_{1-r}^2 +$$

$$+ 3\|D(0, h)(t) - D(0, 0)(t)\|_{1-r}^2 + 3\|D(v, h)(t) - D(0, h)(t)\|_{1-r}^2 +$$

$$+ 2\|v'(t)\|_{1-r}^2 \} \leqslant \|A^{(1-r)/2}\|_{1-r,0}^2 \{ \|\bar{f}(t)\|_{1-r}^2 + 3\|D(0, 0)(t)\|_{1-r}^2 +$$

$$+ 3m_3(0, T)\|h\|_{0,\,t}^2 + [3m_1(\|h\|_{0,\,t}, T) + 2]\|v\|_{1,2-r,t}^2 \} \quad \forall t \in (0, T].$$

Observe now that the identity $w = A^{\sigma/2}(A^{-\sigma/2}w)$, valid for any $w \in H$,

easily implies the estimate

(4.13) $$\|w\|_{-\sigma} \le \|A^{\sigma/2}\|_{\sigma,0}\|A^{-\sigma/2}w\|_0 \quad \forall w \in H.$$

Whence and from (2.9), (4.2)-(4.3) and (4.12) we derive the inequality

(4.14) $$\|v(t)\|_{2-r}^2 + \|v'(t)\|_{1-r}^2 \le C(r)\left\{\|v_0\|_{2-r}^2 + \|v_1\|_{1-r}^2 + \int_0^t [\|\bar{f}(s)\|_{1-r}^2 \right.$$
$$+ 3\|D(0,0)(s)\|_{1-r}^2 + 3m_3(0,T)\|h\|_{0,s}^2]\,ds +$$
$$\left. + \int_0^t [3m_1(\|h\|_{0,s},T) + 2]\|v\|_{1,2-r,s}^2\,ds\right\} \quad \forall t \in (0,T].$$

Set now

(4.15) $$N(v)(t) = \|v\|_{1,2-r,t}^2 \quad \forall t \in (0,T].$$

Then from (4.14) we easily deduce the integral inequality

$$N(v)(t) \le C(r)\left\{\|v_0\|_{2-r}^2 + \|v_1\|_{1-r}^2 + \int_0^t [\|\bar{f}(s)\|_{1-r}^2 \right.$$
$$+ 3\|D(0,0)(s)\|_{1-r}^2 + 3m_3(0,T)\|h\|_{0,s}^2]\,ds +$$
$$\left. + \int_0^t [3m_1(\|h\|_{0,s},T) + 2]N(v)(s)\,ds\right\} \quad \forall t \in (0,T].$$

From Gronwall's inequality we easily derive estimate (4.10) under the previous regularity assumption on our data. Since $V_T^{0,1} \times W^2 \times W$ is dense in $V_T^{0,1-r} \times W^{2-r} \times W^{1-r}$, a density argument implies that problem (4.1)-(4.4) admits a unique solution $v \in V_T^{2,2-r}$ satisfying estimate (4.10).

Finally, we observe that stability estimate (4.11) can be deduced likewise using the equations

(4.17) $$(v - v^*)''(t) + A(v - v^*)(t) + D(v,h)(t) - D(v^*,h^*)(t) =$$
$$= \bar{f}(t) - \bar{f}^*(t) \quad t \in (0,T),$$

(4.18) $$(v - v^*)(0) = v_0 - v_0^*,$$

(4.19) $$(v - v^*)'(0) = v_1 - v_1^*. \quad \blacksquare$$

5. Proof of Theorem 2.1.

In this section first we prove

THEOREM 5.1. *Let operator A satisfy properties (2.1)-(2.3), (2.6).
Let $(p, f, u_1, g) \in C^1([0, T_0]; W^{1-r}) \times C^1([0, T_0]; W^{1-r}) \times W^{2-r} \times \\
\times C^3([0, T_0])$ be a quadruplet satisfying conditions (2.21), (2.32). Then
there exists $T \in (0, T_0]$ such that problem (3.2)-(3.5) admits a unique
solution $(v, h) \in V_T^{2, 2-r} \times C([0, T])$ depending continuously on the
data with respect to the norms pointed out. Moreover, the map the
$(p, f, u_1, g) \to (u, h)$ is Lipschitz continuous on the bounded sets in
$C^1([0, T]; W^{1-r}) \times C^1([0, T_0]; W^{1-r}) \times W^{2-r} \times C^3([0, T_0])$, whose ele-
ments satisfy bounds (2.33)-(2.34) for some (fixed) positive constant m.*

PROOF. From our assumptions on (p, f, u_1, g) we easily deduce
that the triplet $(\bar{f}, v_0, v_1)$ defined by equations (4.5)-(4.7) belongs to
$V_T^{0, 1-r} \times W^{2-r} \times W^{1-r}$.

Associate then with any $h \in C([0, T]$ the unique solution

$$(5.1) \qquad\qquad v = M(h) \in V_T^{2, 2-r}$$

to problem (3.2)-(3.4). According to (4.10) and (4.11), we deduce that
$M \in C(C([0, T]); V_T^{1, 2-r})$ and satisfies the estimate

$$(5.2) \quad \|M(h_2) - M(h_1)\|_{1, 2-r, t}^2 \leqslant m_6 \left(\sum_{j=1}^{2} \|h_j\|_{0, T}, T \right) \int_0^t \|h_2 - h_1\|_{0, s}^2 \, ds$$

$$\forall t \in (0, T], \forall h_1, h_2 \in C([0, T]).$$

As usual, function m_6 depends continuously and nondecreasingly on its
arguments.

We observe now that identification problem (3.2)-(3.4), (3.12) is
equivalent to the following: *determine a function $h \in C([0, T])$ such
that*

$$(5.3) \quad h(t) = \chi_2^{-1} \{ \langle f'(t), \varphi \rangle_r - g^{(3)}(t) - \langle AM(h)(t), \varphi \rangle_r +$$

$$- \langle B(D_t M(h) + p', h), \varphi \rangle_r \} := Q(h)(t) \quad \forall t \in [0, T].$$

We prove now that Q admits a unique fixed point in $C([0, T])$. To this
purpose we show that for a large enough $\rho > 0$ operator Q maps the
closed ball $S(\rho)$ with center at $h = 0$ and radius ρ into itself. Moreover,
Q, restricted to $S(\rho)$, is a contraction. We observe that such properties
are implied by the following estimates, where we make use of the uni-

form boundedness of $M(h)$, implied by (4.10) and of the embedding $W^{1-r} \hookrightarrow W^{-r}$, $C_j(r, \rho, T)$ ($j = 3, 6, 7$) denoting positive functions which remain bounded as $T \to 0$:

(5.4) $\quad \|AM(h_2) - AM(h_1)\|_{0,\,-r,\,t}^2 \leqslant C_2(r)\|M(h_2) - M(h_1)\|_{0,\,2-r,t}^2 \leqslant$

$$\leqslant C_3(r, \rho, T) \int_0^t \|h_2 - h_1\|_{0,\,s}^2 \, ds \quad \forall t \in (0, T], \forall h_1, h_2 \in S(\rho),$$

(5.5) $\quad \|B(D_t M(h_2), h_2) - B(D_t M(h_1), h_1)\|_{0,\,-r,\,t}^2 \leqslant$

$$\leqslant C_4(r)\|B(D_t M(h_2), h_2) - B(D_t M(h_1), h_1)\|_{0,\,1-r,t}^2 \leqslant$$

$$\leqslant C_5(r, \rho) \left\{ \|M(h_2) - M(h_1)\|_{0,\,2-r,t}^2 + \int_0^t \|h_2 - h_1\|_{0,\,s}^2 \, ds \right\} \leqslant$$

$$\leqslant C_6(r, \rho, T) \int_0^t \|h_2 - h_1\|_{0,\,s}^2 \, ds \quad \forall t \in [0, T], \ \forall h_1, h_2 \in S(\rho).$$

In fact, they imply

(5.6) $\quad \|Q(h_2) - Q(h_1)\|_{0,\,t}^2 \leqslant C_8(r, \rho, T) \int_0^t \|h_2 - h_1\|_{0,\,s}^2 \, ds$

$$\forall t \in [0, T], \ \forall h_1, h_2 \in S(\rho).$$

Performing standard computations as, *e.g.*, in [2], we prove the existence and uniqueness of the solution to equation (5.2). The continuous dependence on the data, as in the statement of the theorem, can be proved likewise. ∎

PROOF OF THEOREM 2.1.   It is an immediate consequence of theorems 3.1, 5.1 and representation (3.7).   ∎

6. Proof of Theorem 1.1.

Observe that in our concrete case $\beta = a(0)$ and conditions (2.21) and (2.32) coincide with (1.17) and (1.18) respectively (owing to (0.16)). Hence, according to abstract theorem 2.1 and section 1, we deduce that there exists $T \in (0, T_0]$ such that our identification problem (0.32)-(0.36) admits a unique solution $(H, a) \in V_T^{3,\,3-r} \times$

$\times C^1([0, T])$ continuously depending on data $(F, f, \bar{H}, g)$ with respect to the norms related to the spaces in (1.11)-(1.13).

Using representation formula (0.27), it is easy to check that the electric_ field $E \in C^1([0, T]; W^{3-r})$ continuously depends on data $(F, f, \bar{H}, g)$ with respect to the norms related to the spaces in (1.11)-(1.13).

Finally, we conclude that the map *data* → *solution* is uniformly continuous on bounded sets in $C^1([0, T]; W^{2-r}) \times W^{3-r} \times W^{3-r} \times$ $\times V_T^{3, 3-r} \times C^1([0, T])$ consisting of vector functions $(F, f, E_0\bar{H}, g)$ satisfying conditions (1.17)-(1.18). ∎

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