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Errata-Corrige : "Some sporadic groups as Galois groups II"

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ERRATA CORRIGE

«Some Sporadic Groups as Galois Groups II»

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There is an error in Lemma 1 of this paper. As Kai Magaard (Yale University) observed, $\mathcal{C} = (2B, 3A, 10B)$ of $\mathrm{Aut}(McL)$ is not a rational rigid triple. In fact, $l^i(\mathcal{C}) = 0$, as elements $g \in 2B$, $h \in 3A$ with $gh \in 10B$ generate the maximal subgroup $5_+^{1+2}:3:8\cdot 2$.

Furthermore, in the table of the primitive permutation characters of $McL.2$ in [3] the characters corresponding to $3^4:(M_{10} \times 2)$ and $3_+^{1+4}:4\cdot S_5$ are interchanged.

Nevertheless the Theorem of the paper remains correct. One has to replace Lemma 1 *b*) by

LEMMA. For the rational class structure $\mathcal{C} = (4B, 3A, 10B)$ $\mathrm{Aut}(McL)$ one has $l^i(\mathcal{C}) = 1$ and $n(\mathcal{C}) = 3$. Thus $\mathcal{C}$ is rational rigid but not strictly rigid in the sense of Serre [4].

PROOF. An easy computation with the character table of $\mathrm{Aut}(McL)$ shows that $n(\mathcal{C}) = 3$. Let $g \in 4B$, $h \in 3A$ be such that $gh \in 10B$. The table of primitive permutation characters of $\mathrm{Aut}(McL)$ shows that the only maximal subgroups containing elements of the classes $4B$, $3A$, and $10B$ simultaneously are $M_1 = PSU(4,3):2$, $M_2 = 3^4:(M_{10} \times 2)$, and $M_3 = 5_+^{1+2}:3:8\cdot 2$.

Looking at $M_3/O_{2'}(M_3)$ it is obvious that a product of an element of order 4 and an element of order 3 cannot have order 10 in this group. Likewise, since the elements of $3A$ intersect M_2 in a class contained in $O_3(M_2)$ it is clear that a product of elements of order 4 and of elements of order 10 in M_2 cannot be in $3A \cap M_2$.

Hence the only maximal subgroups of $\mathrm{Aut}(McL)$ which can contain

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$\langle g, h\rangle$ are isomorphic to $M_1 = PSU(4,3):2$. The fusion of the classes of M_1 into $Aut(McL)$ is obvious: $3A$, $4C$, and $10A$ of M_1 correspond to the classes of the triple $\mathcal{C}$ and the corresponding normalized structure constant is 2. Observe that M_1 is the group named $PSU(4,3):2_3$ in the ATLAS. Looking at the maximal subgroups of M_1 or at the table of primitive permutation character of this group one sees immediately that any pair of elements x, y with $x \in 3A$, $y \in 4C$, $xy \in 10A$ in M_1 generates M_1. So one concludes that

$$\overline{\sum}(\mathcal{C}) = \{(g_1, g_2, g_3) \in \mathrm{Aut}\,(McL)^3 \mid g_1 \in 4B, g_2 \in 3A, g_3 \in 10B, g_1 g_2 g_3 = 1\}$$

decomposes into 3 regular orbits under the operation of Aut (McL) with two orbits having representatives (g_1, g_2, g_3) with

$$\langle g_1, g_2, g_3\rangle \cong PSU(4,3):2.$$

Hence $l^i(\mathcal{C}) = 1$.

REMARK. Replacing Lemma 1 b) by the Lemma above, i.e. $2B$ by $4B$, the application by W. Feit [1] (Beispiel 3 in Matzat's paper [2]) showing that $3 \cdot McL$ is a Galois group over $Q(t)$ also remains correct.

REFERENCES

[1] W. FEIT, *Some finite groups with nontrivial centers which are Galois groups*, Proceedings of the 1987 Singapore Conference, Berlin, New York, 1989, pp. 87-109.

[2] B. H. MATZAT, *Frattini-Einbettungsprobleme über Hilbertkörpern*, Manuscripta Mathematica, to appear.

[3] H. PAHLINGS, *Some sporadic groups as Galois groups II*, Rend. Sem. Mat. Univ. Padova, 82 (1989), pp. 163-171.

[4] J.-P. SERRE, *Topics in Galois Theory*, Course at Harvard University, Fall 1988, Notes written by Henry Darmom, tentative draft, Cambridge, 1989.

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