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BI-DIRECTIONAL NEARNESS IN A NETWORK BY AHP (ANALYTIC HIERARCHY PROCESS) AND ANP (ANALYTIC NETWORK PROCESS) (*)

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Abstract. – *In this paper we study bi-directional nearness in a network based on AHP (Analytic Hierarchy Process) and ANP (Analytic Network Process). Usually we use forward (one-dimensional) direction nearness based on Euclidean distance. Even if the nearest point to i is point j, the nearest point to j is not necessarily point i. So we propose the concept of bi-directional nearness defined by AHP's synthesizing of weights "for" direction and "from" direction. This concept of distance is a relative distance based on the configuration of the set of points located on a plane or network. In order to confirm the usefulness of our study we apply the proposed nearness to solving methods of TSP (Traveling Salesman Problem), where to find an approximate solution of TSP we improved Nearest-Neighbor Method. Some numerical experiments of TSP were carried out. To decide a nearest point we used two kind of nearness, forward direction nearness and bi-directional nearness. As a result, by using bi-directional nearness, we obtained good approximate solution of TSP. Moreover, the relation between AHP and ANP, through an example, is considered.*

Keywords: AHP, ANP, TSP.

1. INTRODUCTION

AHP (Analytic Hierarchy Process) [1, 2] is widely used for decision makers in various fields. Recently AHP is developed to ANP (Analytic Network Process) [3], so that AHP's hierarchy structure is extended to ANP's network structure. In this paper we propose a new concept of distance among a set of points located on a plane or a network based on AHP or ANP. This can be said to be a kind of relative distance depending on the configuration of the set of points. We called it bi-directional nearness.

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A distance, in general, is absolutely defined by Euclidean distance. Usually on a plane we use 2-dimensional Euclidean distance, and ordinary nearness from origin is defined by one-directional distance. However, even if the nearest point of i is point j, the nearest point of j is not necessarily point i among a given set of several points.

So we propose bi-directional nearness based on AHP. It is defined by AHP's synthesizing of weights of bi-directional distance. This new concept of distance is applied to solve Traveling Salesman Problem (TSP), Vehicle Routing Problem or locating problem of various facilities in a city, etc.

In order to confirm the usefulness of our study we apply this concept to TSP. As the simplest heuristic solving procedure of TSP, we have Nearest-Neighbor Method (NNM). However the result of NNM is insufficient because the obtained path has a tendency to include long distance when the final point is connected with the starting point. So, to find an approximate solution of TSP, we also propose Improved Nearest-Neighbor Method (INNM). The concept of INNM is to connect two directions.

In order to compare ordinary nearness and bi-directional nearness, we carried out simulation and some numerical experiments of TSP by INNM. As a result, solutions based on bi-directional nearness give better approximate solutions of TSP.

In Section 2, we describe insufficient points of ordinary nearness, and propose bi-directional nearness based on AHP. In Section 3, we propose solving methods of TSP by bi-directional nearness. In Section 4, the relation between AHP and ANP, through an example, is considered. Finally in Section 5, we conclude our investigation.

2. DEFINITION OF BI-DIRECTIONAL NEARNESS BY AHP

In this section, we describe a problem of ordinary nearness, and propose bi-directional nearness based on AHP. Moreover, constructing procedure of the proposed bi-directional nearness is described in detail through an example.

2.1. A problem of ordinary nearness

Suppose three points i, j, k on a plane and distances among them are given. For example, in Figure 1a, distance from point i to point j is denoted by d_{ij}. Of course, in Figure 1b, $d_{ji} = d_{ij}$. The distance starting at each point of i, j, k is shown in Figures 1a to c.

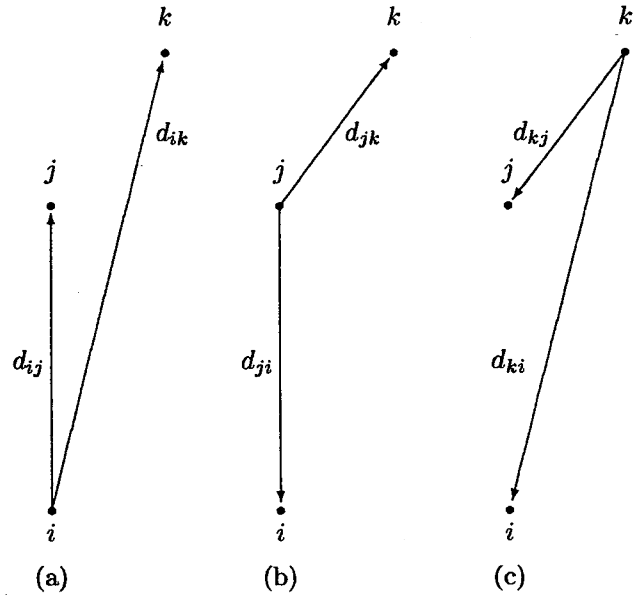

Figure 1. – A problem of ordinary nearness.

In general, nearness distances starting at point i are decided based on forward distance d_{ij} and d_{ik}. In Figure 1a, the nearest point of i is point j because $d_{ij} < d_{ik}$. However, in Figure 1b, the nearest point of j is point k, not point i. Thus ordinary one-directional nearness is seems to be insufficient.

Then by using AHP's synthesizing of weights, the author intends to define bi-directional nearness.

2.2. The concept of bi-directional nearness

The concept of the proposed nearness is based on AHP's synthesizing of weights. AHP model of this concepts, in relation to above example, is shown in Figure 2. The structure of Figure 2 consists of two criteria "from i" and "for i", and two alternatives point j and point k, and this corresponds to calculating of bi-directional distances starting at point i.

Nearness weight w_{ij} and w_{ik} are synthesized by distances weight $w1_{ij}, w1_{ik}, w2_{ji}, w2_{ki}$ and criterion weight v_1, v_2, by the following equations

$$w_{ij} = w1_{ij} \times v_1 + w2_{ji} \times v_2$$

$$w_{ik} = w1_{ik} \times v_1 + w2_{ki} \times v_2 \tag{2.1}$$

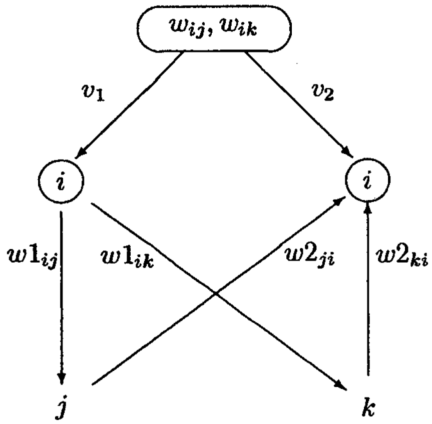

Figure 2. – AHP model of proposed nearness concept.

where $v_1 + v_2 = 1$. In this case, two criteria "from i" and "for i" are equally important, then $v_1 = v_2 = 0.5$.

To determine w_{ij} and w_{ik}, firstly calculate $w1_{ij}$ and $w1_{ik}$, shown in Figure 3. In general, AHP procedure needs pairwise comparisons. However,

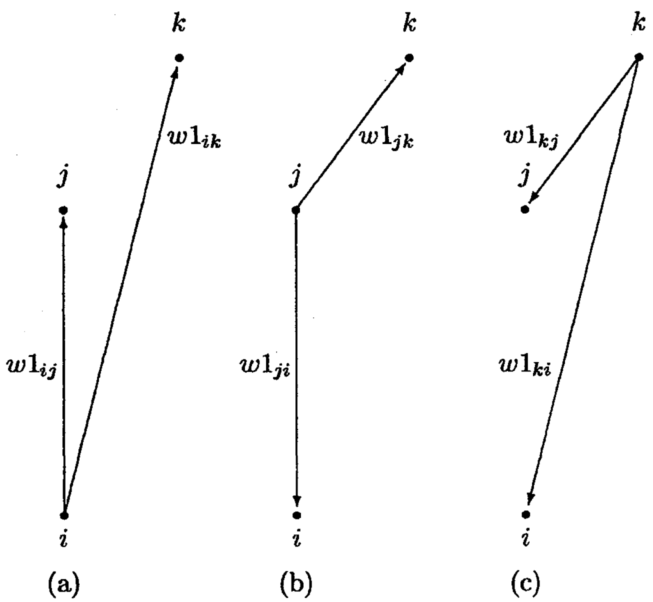

Figure 3. – Normalizing with each "for" distance.

in this case, pairwise comparisons are not needed. Instead we have only to normalize as shown below.

$$w1_{ij} = d_{ij}/(d_{ij} + d_{ik})$$

$$w1_{ik} = d_{ik}/(d_{ij} + d_{ik}) \tag{2.2}$$

where $w1_{ij} + w1_{ik} = 1$ (Fig. 3a).

Next we calculate $w2_{ji}$ and $w2_{ki}$ by the normalizing $w1_{ji}$ and $w1_{ki}$, and thus we have

$$w2_{ji} = w1_{ji}/(w1_{ji} + w1_{ki})$$

$$w2_{ki} = w1_{ki}/(w1_{ji} + w1_{ki}) \tag{2.3}$$

where $w2_{ji} + w2_{ki} = 1$ (Fig. 4).

Thus we find bi-directional nearness weight w_{ij} and w_{ik} by equation (2.1).

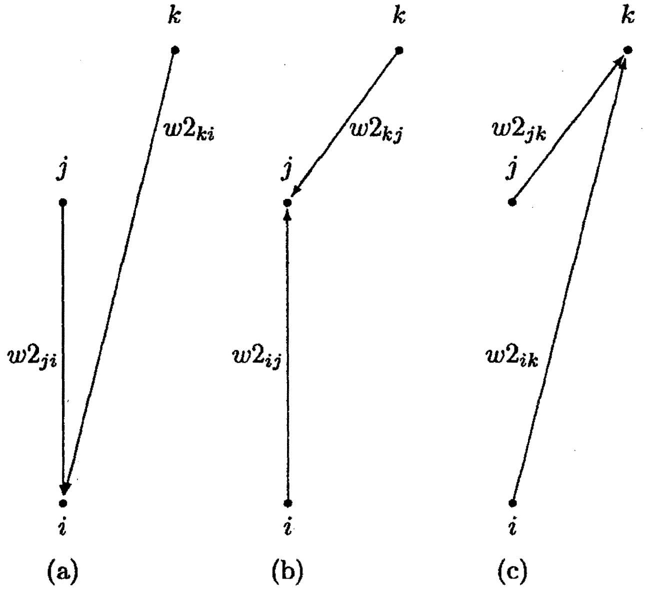

Figure 4. – Normalizing with each "from" distance.

2.3. Procedure to obtain the proposed bi-directional nearness

AHP model of the proposed bi-directional nearness is shown in Figure 5. A number of points n on a plane and distance d_{ij} between point i and point j ($i = 0 \sim n-1$, $j = 0 \sim n-1$) are given.

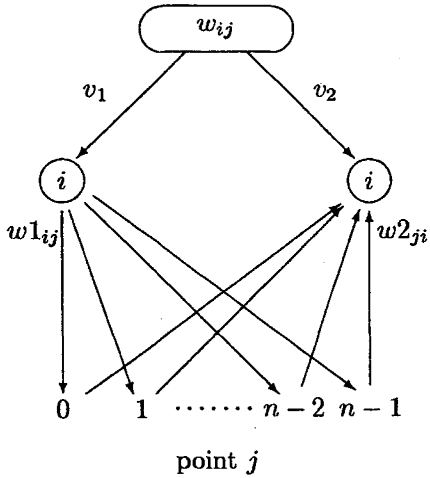

Figure 5. – AHP model of the proposed bi-directional nearness.

The constructing procedure of the proposed bi-directional nearness is summarized as follows:

1) Calculate $w1_{ij}$:

$$w1_{ij} = d_{ij}/d_{i\cdot} \tag{2.4}$$
$$\text{where } d_{i\cdot} = \sum_{j=0}^{n-1} d_{ij}.$$

2) Calculate $w2_{ji}$:

$$w2_{ji} = w1_{ji}/d_{\cdot i} = (d_{ji}/d_{j\cdot})/d_{\cdot i} \tag{2.5}$$
$$\text{where } d_{\cdot i} = \sum_{j=0}^{n-1} w1_{ji} = \sum_{j=0}^{n-1} (d_{ji}/d_{j\cdot}).$$

3) Synthesis w_{ij}:

Assuming weights of criteria, from point i and for point i, are equally important, we have $v_1 = v_2 = 0.5$ and

$$w_{ij} = w1_{ij} \times v_1 + w2_{ji} \times v_2 = (d_{ij}/d_{i\cdot} + (d_{ji}/d_{j\cdot})/d_{\cdot i}) \times 0.5. \qquad (2.6)$$

4) Decide the order of nearness of point i based on w_{ij}.

2.4. An example of the proposed bi-directional nearness

We explain the constructing procedure of the proposed bi-directional nearness in detail through an example. Suppose, for example, six points, 0 to 5, on a plane and distances among them $d_{ij}(i = 0 \sim 5,\ j = 0 \sim 5)$ are given in Table 1.

TABLE 1
Given distance d_{ij} for an example.

$i\backslash j$	0	1	2	3	4	5
0	0.0	56.0	35.0	2.0	51.0	60.0
1	56.0	0.0	21.0	57.0	78.0	70.0
2	35.0	21.0	0.0	36.0	68.0	68.0
3	2.0	57.0	36.0	0.0	51.0	61.0
4	51.0	78.0	68.0	51.0	0.0	13.0
5	60.0	70.0	68.0	61.0	13.0	0.0

In general, the order of nearness of each point in Table 1 is decided based on one-directional nearness, and its result is shown in Table 2. For point 0, in Table 2, point 0 itself is order 0 and point 3 is order 1, the nearest point, and point 2 is the second nearest and so on.

Based in Table 2, we have bi-directional nearness by using the proposed procedure.

Firstly calculate $w1_{ij}$ with respect to criterion "from i". To calculate $w1_{ij}$, by normalizing with sum of distance from point i equal to 1. The result is shown in Table 3.

TABLE 2
The nearness by ordinary method.

Point\Order	0	1	2	3	4	5
0	0	3	2	4	1	5
1	1	2	0	3	5	4
2	2	1	0	3	4	5
3	3	0	2	4	1	5
4	4	5	3	0	2	1
5	5	4	0	3	2	1

TABLE 3
Result of $w1_{ij}$.

$i\backslash j$	0	1	2	3	4	5	sum
0	0.000	0.275	0.172	0.010	0.250	0.294	1.000
1	0.199	0.000	0.074	0.202	0.277	0.248	1.000
2	0.154	0.092	0.000	0.158	0.298	0.298	1.000
3	0.010	0.275	0.174	0.000	0.246	0.295	1.000
4	0.195	0.299	0.261	0.195	0.000	0.050	1.000
5	0.221	0.257	0.250	0.224	0.048	0.000	1.000

Next, calculate $w2_{ij}$, based on $w1_{ij}$, by normalizing with sum of distance for point i equal to 1. The result is shown in Table 4.

Finally, based on $w1_{ij}$ and $w2_{ij}$, we synthesize w_{ij}. The weight of criterion, v_1 ("from i") and v_2 ("for i"), in this example, are equally important, so both are 0.5. The weights w_{ij} are calculated by equation (2.7).

$$w_{ij} = w1_{ij} \times 0.5 + w2_{ji} \times 0.5. \tag{2.7}$$

The result of w_{ij} is shown in Table 5.

TABLE 4
Result of $w2_{ij}$.

$i\backslash j$	0	1	2	3	4	5
0	0.000	0.229	0.184	0.012	0.223	0.248
1	0.255	0.000	0.080	0.256	0.247	0.209
2	0.197	0.077	0.000	0.200	0.267	0.252
3	0.012	0.230	0.187	0.000	0.220	0.249
4	0.251	0.249	0.280	0.248	0.000	0.042
5	0.284	0.215	0.269	0.284	0.043	0.000
sum	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 5
Result of w_{ij}.

$i\backslash j$	0	1	2	3	4	5
0	0.000	0.265	0.184	0.011	0.251	0.289
1	0.214	0.000	0.076	0.216	0.263	0.232
2	0.169	0.086	0.000	0.172	0.289	0.283
3	0.011	0.266	0.187	0.000	0.247	0.289
4	0.209	0.273	0.264	0.208	0.000	0.046
5	0.234	0.233	0.251	0.236	0.045	0.000

Based in Table 5, we can decide the order of nearness of this example. The result is shown in Table 6.

In Table 6, figures different from Table 2 are underlined.

Which nearness, by Table 2 or by Table 6, is better?

Comparisons of these two kinds of nearness are explained in the next section.

TABLE 6
Result of bi-directional nearness by AHP.

Point\Order	0	1	2	3	4	5
0	0	3	2	4	1	5
1	1	2	0	3	5	4
2	2	1	0	3	<u>5</u>	<u>4</u>
3	3	0	2	4	1	5
4	4	5	3	0	2	1
5	5	4	<u>1</u>	<u>0</u>	<u>3</u>	<u>2</u>

3. APPLICATION AND EVALUATION OF BI-DIRECTIONAL NEARNESS

In this section, we apply the proposed bi-directional nearness to Traveling Salesman Problem (TSP) to verify its usefulness. The simplest heuristic solving procedure of TSP is Nearest-Neighbor Method (NNM). NNM is based on ordinary nearness table such as presented in Table 2.

We explain this by the example shown in the previous section. The exact shortest path and the shortest distance of TSP are:

$$\begin{aligned} \text{Shortest path}: &\ 0 \to 2 \to 1 \to 5 \to 4 \to 3 \to 0 \\ \text{Shortest distance}: &\ 192.0. \end{aligned}$$

However by NNM, starting at point 0 and using Table 2, we obtain the result presented below.

$$\begin{aligned} \text{NNM path}: &\ 0 \to 3 \to 2 \to 1 \to 5 \to 4 \to 0 \\ \text{NNM distance}: &\ 193.0. \end{aligned}$$

The result of NNM is insufficient because the path obtained has a tendency to include long distance when we connect the final point with the starting point. So the author proposes improved NNM (INNM) and demonstrates it here with the following example.

3.1. Proposed heuristic solving procedure of TSP

Here we propose Improved Nearest-Neighbor Method (INNM). The concept of INNM is to connect two paths of different directions. INNM procedure is explained based in Table 2 data.

Firstly we construct the path of TSP starting at point 0 like Figure 6. In Figure 6a, we connect the nearest point 3 ($d_{03} < d_{0i}, i = 1, 2, 4, 5$, where d_{ij} is the distance of point i and j) and the second nearest point 2 ($d_{02} < d_{0i}, i = 1, 4, 5$) with the starting point 0. Next, in Figure 6b, we connect point 3 with its nearest point of the rest and next, connect point 2 with its nearest point of the rest. Finally, in Figure 6c, we connect point 4 and point 1 with the last point 5. Then we have an approximate TSP path, and if fortunate, it is the exact shortest path.

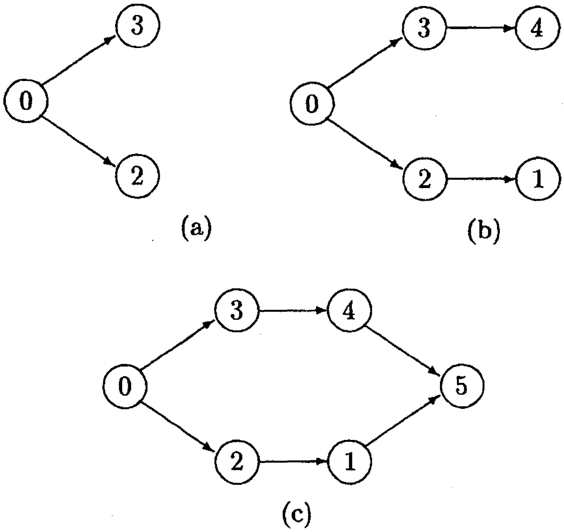

Figure 6. – INNM starting at point 0.

However when TSP path starts at point 1, as shown in Figure 7, this procedure makes a closing path. In Figure 7a, point 3 is the nearest point of both point 2 and point 0. In this case, we consider the second nearest point of the rest for point 2 and point 0, and we have point 4. In Figure 7b we compare the distances $d_{24} + d_{03}$ with $d_{23} + d_{04}$ and select the shorter one. (In general, the second nearest point of 2 and 0 do not coincide. For example, 2 and 0 has the second nearest point 4 and 5, respectively, then we compare $d_{24} + d_{03}$ with $d_{23} + d_{05}$ and select the shorter one.) Thus, in Figure 7c, we connect point 2 with point 4 and connect point 0 with point 3. Then we have an approximate path. The result is shown in Figure 7d.

The procedure of INNM is summarized as follows:

1) choose the starting point;

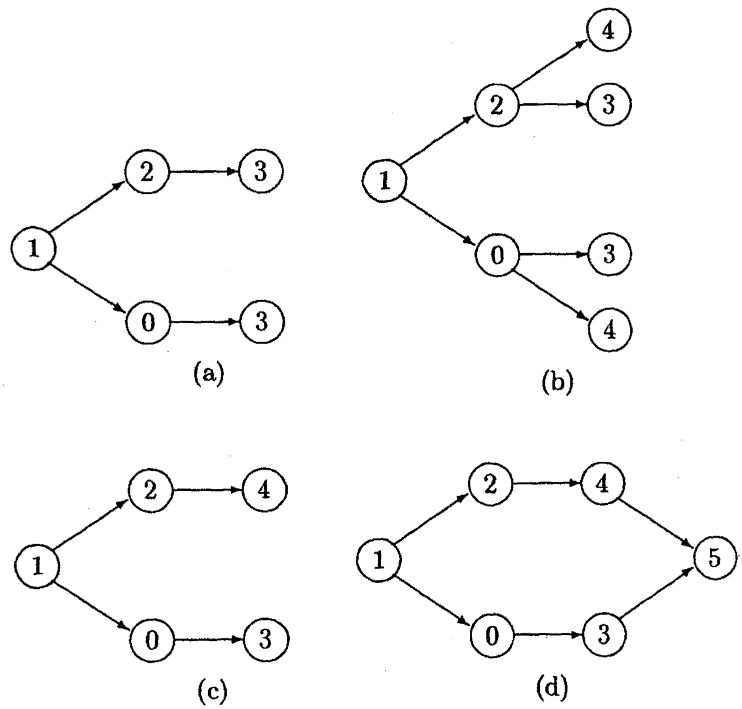

Figure 7. – INNM starting at point 1.

2) connect the nearest point and the second nearest point with the starting point;

3) next, connect the nearest point of the rest with each connected points, without closing the path;

4) continue above process until all points are connected;

5) select the shortest of paths derived from all starting points.

Based on two kinds of nearness, we calculate path and distance of TSP for each starting point by INNM. The result is shown in Table 7.

In Table 7, as underlined, we have 4 incorrect cases of 6 by ordinary nearness and 2 incorrect cases of 6 by bi-directional nearness.

3.2. Evaluation by INNM using random number

To evaluate two kinds of nearness, simulation of TSP by INNM was carried out.

TABLE 7
Result of an example by INNM.

	The shortest distance	
Starting point	By ordinary nearness	By bi-directional nearness
0	192.0	192.0
1	221.0	211.0
2	192.0	192.0
3	193.0	193.0
4	211.0	192.0
5	221.0	192.0

The procedures of simulation using random number by personal computer are as follows:

1) n random points are scattered in a square of area 1 for each $n = 5 \sim 10$;

2) we have 1000 cases for each $n = 5 \sim 10$;

3) for each case calculate the path and distance of TSP by INNM based on ordinary nearness and bi-directional nearness, and further true shortest path and distance by surveying all possible paths;

4) count the number of incorrect cases not attaining the true shortest path, and calculate maximum error and mean error;

where error calculate (calculated distance – shortest distance)/shortest distance.

The result of simulation by ordinary one-directional nearness is shown in Table 8 and by bi-directional nearness is shown in Table 9.

In Tables 8 and 9, for $n = 10$, we have 460 incorrect cases of 1 000 by using ordinary one-directional nearness and have 315 incorrect cases of 1 000 by using bi-directional nearness.

From these tables, we can conclude the proposed bi-directional nearness gives better results.

TABLE 8
Result by INNM using ordinary one-directional nearness.

n	Incorrect cases	Max. error (%)	Mean error (%)
5	3	1.04	0.51
6	170	20.7	3.78
7	175	17.6	3.44
8	329	17.8	3.47
9	354	19.3	3.44
10	460	21.2	3.90

TABLE 9
Result by INNM using bi-directional nearness.

n	Incorrect cases	Max. error (%)	Mean error (%)
5	1	0.28	0.28
6	91	9.82	2.49
7	88	10.6	2.29
8	221	11.9	2.60
9	225	12.1	2.35
10	315	16.6	2.49

3.3. Evaluation by INNM using traveling Salesman Problem Library

To evaluate the usefulness of ordinary one-directional nearness and the proposed bi-directional nearness, calculation by INNM using Traveling Salesman Problem LIBrary (TSPLIB) given by 2-dimensional Euclidean distance are carried out. These TSPLIB files are downloaded by the Internet.

We calculate 12 cases of TSPLIB by workstation and compare calculated tour length with the minimum tour length. The results are shown in Table 10.

From Table 10, as a result, we have 20% or less of error by bi-directional nearness. Therefore the proposed bi-directional nearness is better than one-directional nearness, except in 3 cases of 12.

TABLE 10
Results of TSPLIB by INNM.

File	Nodes	The minimum tour length	Calculate length (Error%) by one-directional nearness	Calculate length (Error%) by bi-directional nearness
eil101.tsp	101	629	755(20.0)	727(15.6)
eil51.tsp	51	426	499(17.1)	476(11.3)
eil76.tsp	76	538	592(10.0)	604(12.3)
kroA100.tsp	100	21,282	24,469(15.0)	25,367(19.2)
kroC100.tsp	100	20,749	22,743(9.6)	22,165(6.8)
kroD100.tsp	100	21,294	25,353(19.1)	24,622(15.6)
lin105.tsp	105	14,379	16,081(11.8)	15,459(7.5)
pcb442.tsp	442	50,779	57,546(13.3)	60,918(20.0)
pr2329.tsp	2,329	378,032	455,628(20.5)	450,538(19.2)
pr76.tsp	76	108,159	134,617(24.5)	129,791(20.0)
rd100.tsp	100	7,910	9,016(14.0)	8,957(13.2)
st70.tsp	70	675	780(15.6)	758(12.3)

4. ANP ANALYSIS

Finally the relation between AHP and ANP, through an example, was considered. For example, as was mentioned in Section 2.4, the simplest ANP model of the proposed bi-directional nearness is shown in Figure 8.

This model consists of two clusters. In Figure 8, cluster 1 consists of two criteria and cluster 2 consists of six alternatives.

For any point i, we have supermatrix (4.1).

$$\mathbf{W}_i = \begin{bmatrix} 0 & W_{12} \\ W_{21} & 0 \end{bmatrix}. \tag{4.1}$$

In this supermatrix, if cluster i is independent of cluster j, then submatrix W_{ij} becomes zero matrix, otherwise columns of W_{ij} are the weights of cluster i with respect to cluster j. The details of equation (4.1) is shown in equation (4.2).

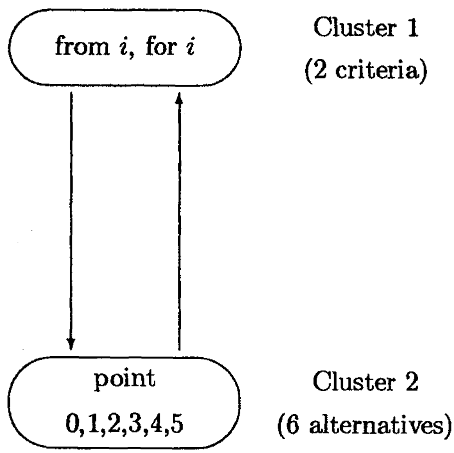

Figure 8. – ANP model of the proposed bi-directional nearness.

$$
\boldsymbol{W}_i = \left[\begin{array}{c||cc|cccccc}
 & \text{from } i & \text{for } i & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline\hline
\text{from } i & 0 & 0 & w12_{10} & w12_{11} & w12_{12} & w12_{13} & w12_{14} & w12_{15} \\
\text{for } i & 0 & 0 & w12_{20} & w12_{21} & w12_{22} & w12_{23} & w12_{24} & w12_{25} \\
\hline
0 & w21_{10} & w21_{20} & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & w21_{11} & w21_{21} & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & w21_{12} & w21_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & w21_{13} & w21_{23} & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & w21_{14} & w21_{24} & 0 & 0 & 0 & 0 & 0 & 0 \\
5 & w21_{15} & w21_{25} & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right] \tag{4.2}
$$

In this case, for any point i, from Table 3 the values of $w21_{1j}$ are equal to $w1_{ij}$ and from Table 4 the values of $w21_{2j}$ are equal to $w2_{ij}$. And the values of $w12_{1j}$ and $w12_{2j}$ are equal to 0.5. This is because in Figure 9, any point j evaluates two criteria, from i and for i, so the weights of criteria are equally important.

Then for point 0, we have supermatrix (4.3).

$$
W_0 = \left[\begin{array}{cc|cccccc}
0.0 & 0.0 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 \\
0.0 & 0.0 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 \\
\hline
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.275 & 0.255 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.172 & 0.197 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.010 & 0.012 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.250 & 0.251 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.294 & 0.284 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0
\end{array}\right] \tag{4.3}
$$

Limiting supermatrix for point 0 converges W_0 to 3. The result is shown in equation (4.4).

$$
W_0^3 = \left[\begin{array}{cc|cccccc}
0.0 & 0.0 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 \\
0.0 & 0.0 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 & 0.500 \\
\hline
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.265 & 0.265 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.184 & 0.184 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.011 & 0.011 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.251 & 0.251 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
0.289 & 0.289 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0
\end{array}\right] \tag{4.4}
$$

As a result, for point 0, the weights of ANP, in the lefthand column of equation (4.4), coincide with the result of AHP, $i = 0$ in Table 5. Then we have same weights by AHP and ANP.

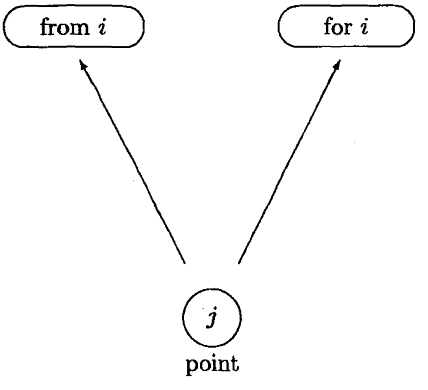

Figure 9. – Evaluate the criteria.

5. CONCLUSION

In this paper the concept of bi-directional nearness based on AHP was proposed and applying the proposed nearness to TSP. To find shorter path of TSP, INNM, heuristic procedure of TSP, was also proposed. To evaluate the usefulness of the proposed nearness by INNM, simulation using random number and calculation for some example of TSPLIB were carried out. As a result, we confirmed our proposed bi-directional nearness is better than ordinary one-directional nearness. Moreover, through an example, we obtained the same weights by ANP and AHP.

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REFERENCES

1. T.L. Saaty, A Scaling Method for Priorities in Hierarchical Structures. *J. Math. Psych.* **15** (1977) 234-281.
2. T.L. Saaty, *The Analytic Hierarchy Process*. McGraw Hill (1980).
3. T.L. Saaty, *The Analytic Network Process*. RWS Publications (1996).