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A GOOD APPROXIMATION OF THE INVENTORY LEVEL IN A (Q, r) PERISHABLE INVENTORY SYSTEM (*)

by Huan Neng CHIU ([1])

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Abstract. – *This paper derives a good approach to approximating the expected inventory level per unit time for the continuous review (Q, r) perishable inventory system. Three existing approximation approaches are examined and compared with the proposed approach. Three stockout cases, including the full backorder, the partial backorder, and the full lost sales cases, which customers or material users generally use to respond to a stockout condition are considered. This study reveals the fact that the proposed approximation is simple yet good and suitable for incorporation into the (Q, r) perishable inventory model to determine the best ordering policy. The results from numerical examples and a sensitivity analysis indicate that severe underestimation or overestimation of the expected inventory level per unit time due to the use of an inappropriate approximation approach would result in great distortion in the determination of the best ordering policy.*

Keywords: Perishable inventory, approximation, backorder, lost sales, sensitivity analysis.

Résumé. – *Cet article développe une bonne façon d'approximer le niveau moyen de stock par unité de temps pour le système d'inventaire permanent (Q, r) dans le cas des denrées périssables. Nous examinons trois approches existantes et les comparons avec celle qui est proposée ici. Nous considérons trois cas de ruptures de stock : celui du réapprovisionnement total, celui du réapprovisionnement partiel, et celui où toute demande non satisfaite est entièrement perdue ; ce sont les cas les plus généralement rencontrés. L'étude révèle que l'approximation proposée est simple, et cependant bonne et appropriée à une incorporation dans le modèle (Q, r) pour la détermination de la meilleure politique de réapprovisionnement. Les résultats des exemples numériques et une étude de sensibilité indiquent que d'une sous-estimation ou d'une surestimation sévère du niveau moyen du stock par unité de temps, causée par l'utilisation d'une méthode inappropriée d'approximation, résulterait une grande distorsion dans la détermination de la meilleure politique de réapprovisionnement.*

Mots clés : Stock périssable, approximation, réapprovisionnement, ventes perdues, analyse de sensibilité.

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1. INTRODUCTION

The study of the determination of the optimal or the best ordering policies for perishable or deteriorating inventory systems has received a significant amount of attention in the past three decades. Comprehensive reviews in this area can be found in Nahmias [13] and Raafat [17]. Typically, goods having finite lifetimes are subject to either perishability or decay. A perishable inventory is one in which all the units of one material item remaining in stock will simultaneously lose their utility. The remaining units must be discarded if they have not yet been used (deterministic or random demand) after storage for a fixed period of time. Common examples of perishable inventories are fashion garments, blood, and foodstuffs. On the other hand, a decaying or deteriorating inventory generally has a random lifetime. It can be defined as one in which a fraction of the units of an item remaining in stock loses its utility (e.g., radioactive materials and gasoline) or in which the utility of each unit decreases over time (e.g., fruits and vegetables).

In this paper, the primary focus is placed on the continuous review (Q, r) (order quantity/reorder point) perishable inventory system. Nahmias [11, 12] and most of the other previous studies such as those of Cohen [4], Chazan and Gal [2], and Nandakumar and Morton[14] have concentrated on the periodic review and multi-period lifetime problem with zero lead time. Their considerable efforts have been spent on the development of good approximations of the exact expected outdating (*i.e.*, the expected perished units of an item during a time interval). This is because it is extremely difficult to obtain the optimal expected outdating for a long lifetime item. In fact, this requires solving a multi-dimensional program with corresponding quantities for various ages at the beginning of each period, which involves complex recursive computation. As far as we know, few papers have dealt with the continuous review (Q, r) perishable inventory model, which is known to be an intractable problem. Schmidt and Nahmias [20] commented that the perishable problem appears to be extremely difficult when a positive lead time is introduced. The difficulty is that perishability can only be applied to units on hand, not on order.

Recently, this author [3] developed a simple yet good approximation of the expected outdating for a fixed-life (Q, r) perishable inventory model with a positive lead time. This author used an extremely rough approximation of the expected inventory level per unit time since both the expected outdating of the current order size and the expected shortage quantity per cycle are assumed to be negligible in the calculation of the expected stock level.

Though this strong assumption can help simplify computation of the holding cost, distortion in determining a best (Q, r) ordering policy may arise. Brown *et al.* [1] also demonstrated that the penalty associated with ordering is related to not only the lot-size error but also to the holding cost function. Therefore, the derivation of a good approximation of the stock level function to reduce this ordering distortion and cost penalty to a minimum is the focus of this study.

The literature on the (Q, r) inventory system with random demand includes the classical (Q, r) models presented in Hadley and Whitin [6, Sections 4-2 and 4-3]. They discussed the backorder and lost sales cases under the assumption that perishability or decay is not allowed. The optimal policy is that when the inventory position (on hand plus on order stock) reaches the reorder point, r, an order of size Q units is placed. Silver [22] classified inventory management problems into an enormous variety of research schemes. The (Q, r) inventory system with probabilistic lead time demand, stockout, and item shelf-life considerations is of interest for future application. He pointed out that commonly used distributions of lead time demand are the Normal, Gamma, and Poisson distributions. However, there always is a small probability that the lead time demand will be negative when a normal distribution is used for the lead time demand. In this case, a truncated normal distribution is recommended, but this may make the computation difficult. Later, Das [5] introduced a (Q, r) inventory model with time-weighted (time-proportional) backorders. Several (Q, r) inventory models with a mixture of backorders and lost sales were proposed by Posner and Yansouni [16], Montgomery *et al.* [10], Matthews [9], Rosenberg [19], Park [15] and Kin and Park [7]. Almost all the previous research works used β, a fraction of the unsatisfied demand backordered (the remaining fraction 1-β completely lost), to model partial backorders. Recently, Rabinowitz *et al.* [18] modeled a (Q, r) inventory system using a control variable, which limits the maximum number of backorder allowed to accumulate during a cycle. Obviously, these previous research works did not include the underlying perishability assumption in their model formulations.

In general, the cost of a shortage can be assumed to be the time-independent stockout cost ($/unit), the time-proportional shortage cost ($/stockout duration/unit), or the stockout cost per outage. The time-weighted shortage cost is proportional to the duration of a stockout. On the other hand, if the shortage cost is based on an outage, then according to Tersine [23, p. 218], an outage can be defined as one time of the stockout without regard to the number of units out of stock during a replenishment cycle.

In this paper, three stockout cases in which customers or material users can choose to react to a stockout condition are considered. As previously stated, the three stockout cases are the full backorder, the full lost sales, and the partial backorder cases, which influence computation of the expected inventory levels per unit time and, then, the holding costs. Here, we assume that both the backorder and lost sales costs are independent of the duration of the stockout. In addition, other important assumptions are the stochastic demand, fixed item lifetime, β backorder fraction, and no quantity discounts. Therefore, the proposed (Q, r) perishable inventory model is different from Shiue's [21] model and the above mentioned (Q, r) models.

In this paper, we will examine three existing approximations and derive a new one to approximate the expected inventory level per unit time:

(1) An extremely rough approximation, as adopted by this author [3, p. 97, equation (7)] and Hadley and Whitin [6, p. 156, equation (4-1)].

(2) An approximation without considering the stockout duration and the outdate condition, as introduced by Wagner [24, p. 825, equation (14)].

(3) An approximation considering the stockout duration but excluding the outdate condition, as proposed by Kin and Park [7, p. 233, equation (5)].

(4) A good approximation based on our [3, p. 96, equation (4)] approximate expected outdating, as developed in this study.

2. PROBLEM DESCRIPTION

In this paper, only one perishable item (or product) is considered. Each unit of the item has a fixed lifetime equal to m. The inventory level is reviewed continuously and decreased by a satisfaction of demand or by disposal of perished units. An order size of Q is placed when the inventory level reaches the reorder point, r. There is a positive leadtime, L, for each replenishment, and a fixed ordering cost, K, is incurred. All the units of a replenishment order arrive fresh or new. Each unit does not lose or decrease in utility before its useful lifetime ends, but it must be discarded if it has not been used before the expiration date. An outdate cost equal to W per unit is charged. The demand in unit time, d_1, is a nonnegative random variable. Assume that it follows a specific continuous or discrete distribution with density or mass function $f_1(y) = dPr\{d_1 \leq y\}/dy$ and mean D. We also assume that if $N(t)$ is cumulative demand by time t, then $N(t)$ is a stochastic process with stationary, independent increments. This implies that $N(m)$ has density or mass $f_m(z) = dPr\{d_m \leq z\}/dz$ and mean mD. In other words, $N(m+L)$

has density or mass $f_{m+L}(u) = dPr\{d_{m+L} \leq u\}/du$ and mean $(m+L)D$. Units are always depleted according to an FIPO (*i.e.*, First into stock are consumed first) issuing policy.

The notation to be used throughout this paper is defined as follows:

Q = Order quantity.

r = Reorder point.

m = Fixed lifetime of the perishable item.

L = Positive order lead time.

d_L = Demand during lead time with probability function $f_L(x) = d\Pr\{d_L \leq x\}/dx$ and mean LD, where $F_L(x)$ is an L-fold convolution of $f_1(y)$.

C = Replenishment cost per unit.

h = Holding cost per unit per unit time.

K = Fixed ordering cost per order.

W = Outdate cost per unit.

P = Backorder cost per unit.

θ = Lost sales cost per unit.

β = A fraction of the excess (unsatisfied) demand per replenishment cycle can be backordered, and the remaining fraction $1-\beta$ is lost.

ET = Expected cycle length.

EI = Expected inventory level per unit time.

ER = Expected outdate quantity of the current order size Q.

ES = Expected shortage quantity per cycle.

Additionnal notations will be introduced later when needed. Figure 1 shows a (Q, r) perishable inventory model with a mixture of backorders and lost sales.

3. CHIU'S EXPECTED OUTDATING APPROXIMATION

Just as demonstrated by Nahmias [13], who dealt with the periodic review and multi-period lifetime problem with zero order lead time, avoidance of complex computation requires developing a good approximation of the exact expected outdating. The continuous review perishable inventory problem with positive order lead time also involves complex computation, as stated by Schmidt and Nahmias [20] and mentioned before. Thus, this author [3] presented a simple yet good approximation to the expected outdating for the

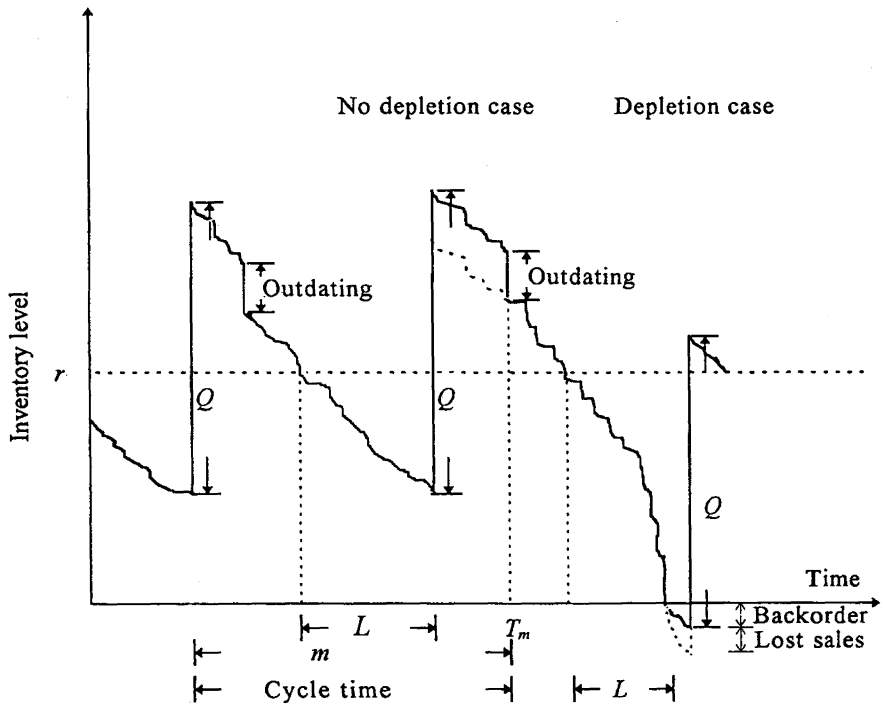

Figure 1.

continuous review (Q, r) perishable inventory system with positive order lead time L. Our approximate expected outdating of the current order size Q is given by

$$ER = \begin{cases} \displaystyle\int_0^{r+Q} (r+Q-u)\, f_{m+L}(u)\,\mathrm{d}u - \int_0^{r} (r-u)\, f_{m+L}(u)\,\mathrm{d}u, \\ \quad \text{if } d_{m+L} \text{ is a nonnegative continuous random variable,} \\ \displaystyle\sum_{u<r+Q} (r+Q-u)\, f_{m+L}(u) - \sum_{u<r} (r-u)\, f_{m+L}(u), \\ \quad \text{if } d_{m+L} \text{ is a nonnegative discrete random variable,} \end{cases} \tag{1}$$

where $f_{m+L}(u)$ is the probability function of the random variable d_{m+L} (*i.e.*, the demand during $m + L$ time units). Equation (1) has been shown to be a fairly acceptable approximation of the exact expected outdating in the situation where the continuous review strategy is used. It should be noted here that equation (1) is analogous to $H(x+y) - H(x)$ presented in

Nahmias [11, p. 1004, equation (2-1)] with x, y, and m replaced by r, Q, and $m + L$, respectively. How equation (1) can approximate the expected outdating effectively has been discussed in more detail elsewhere [3]. In the following three sections, our attention will be focused on the derivations and comparisons of the expected inventory levels per unit time for the three stockout cases.

4. FULL BACKORDER CASE

With full backorders, there is no loss of sales since customers or material users are willing to wait for the arrival of the next order or an outstanding order. The unsatisfied demand is then filled by the arrived order immediately. Four approaches can be used to approximate the expected inventory level per unit time in the (Q, r) perishable inventory system:

(1) *Extremely rough approximation*

This approach assumes that the values of ER and ES are considerably smaller than the current order size, Q. Hence, ER and ES can be neglected, and the expected inventory level per unit time is

$$EI_r = r - DL + Q/2. \tag{2}$$

Equation (2) implies that there are no differences among the three stockout cases. As mentioned earlier, this approach has been adopted by this author as well as by Hadley and Whitin [6]. However, stock level was not correctly accounted for when there was a depletion case (*i.e.*, an out of stock condition).

(2) *Wagner approximation*

In contrast to extremely rough approximation, Wagner [24] considered both the depletion case and the non-depletion case during a lead time. Suppose that ER is much smaller than ES and can be ignored in this approximation. Then, Wagner introduced

$$EI_w = (r - DL + Q/2) + DL\,(ES)/(2Q), \tag{3}$$

where

$$ES = \left\{ \begin{array}{l} \displaystyle\int_r^{\infty} (x - r)\, f_L\,(x)\, \mathrm{d}x, \\ \quad \text{if } d_L \text{ is a nonnegative continuous random variable,} \\ \displaystyle\sum_{x>r} (x - r)\, f_L\,(x), \\ \quad \text{if } \mathrm{d}_L \text{ is a nonnegative discrete random variable,} \end{array} \right\} \tag{4}$$

The function $f_L(x)$ in equation (4) is the probability function of the random variable d_L. In equation (3), $DL(ES)/(2Q)$ is called the correction term of the expected inventory level per unit time. Clearly, this approximation is reduced to the extremely rough approach when ES is ignored. However, this approach does not take into account the duration of the stockout since it assumes that when $d_L > r$, the inventory level becomes zero just before the replenishment arrives. The purpose of this approximation is to make the holding cost formulas uncomplicated.

(3) *Modified Wagner approximation*

EI_w in equation (3), which will be demonstrated in Section 6, is an overestimation of EI due to neglect of the stockout duration and the outdate condition. In this paper, the Kin and Park approximation [7] without the outdate condition is called the modified Wagner approximation. Referring to the derivation of the average carrying inventory in Kin and Park, the expected inventory level per unit time of the modified Wagner model can be expressed by

$$EI_m = (r - DL + Q/2) + DL\left\{\int_r^\infty [(x-r)^2 f_L(x)/x]\,dx\right\}/(2Q). \quad (5)$$

If the right side of the equal sign in equation (5) is multiplied by h, then the result is equivalent to Kin and Park's [7, p. 233, equation (5)] full backorder model with $\beta = 1$. After further manipulation, equation (5) can be rewritten as

$$EI_m = (r - DL + Q/2) + DL(ES/(2Q)) - DL\left\{\int_r^\infty [r(x-r) f_L(x)/x]\,dx\right\}/(2Q). \quad (6)$$

where ES is from equation (4). Note that in equations (5) and (6), the integral notation should be replaced with the summation notation if the lead time demand, d_L, is a discrete random variable. Also, it should be emphasized here that only equations used in the continuous random variable case will be presented later.

(4) *Chiu approximation*

It is a fact that equation (5) is derived under the assumption that ER is considerably smaller than ES and can be neglected. Inevitably, this will

result in an inaccurate value of EI obtained by using the modified Wagner approach. Now, let

L_1 = expected average inventory level during a lead time;

L_2 = expected average inventory level after order arrival until next reorder.

In order to simplify the derivation of the expected inventory level per unit time, it is assumed that when $d_L > r$, the inventory level becomes zero just before the ordered units arrive. Then,

$$L_1 = \int_0^r \frac{1}{2}\left[r + (r - x)\right] f_L(x)\,\mathrm{d}x + \int_r^\infty \frac{1}{2}(r + 0)\, f_L(x)\,\mathrm{d}x. \quad (7)$$

In fact, equation (7) can be further simplified to

$$L_1 = \frac{1}{2}\left[r + \int_0^r (r - x)\, f_L(x)\,\mathrm{d}x\right]. \quad (8)$$

In equation (8), $\int_0^r (r - x)\, f_L(x)\,\mathrm{d}x$ can be easily proved to equal $r - DL + ES$. On the other hand, L_2 can be approximated precisely by considering a rectangle, a triangle, and a parallelogram as shown in Figure 1. Thus,

$$L_2 = r + (Q - DL - ER)/2 + ER(m - \alpha\, ET)/(ET - L), \quad (9)$$

where,

$$ET = (Q - ER)/D, \quad (10)$$

ER is from equation (1), and α denotes the expected number of replenishment cycles that the item lifetime (m) can over; moreover, $0 < (m - \alpha\, ET)/(ET - L) < 1$. In other words, the lifetime of m time units consists of $\lfloor m/ET \rfloor$ replenishment cycles, where $\lfloor v \rfloor$ denotes the greatest integer less than or equal to v. For example, in Figure 1, we set α equal to 1. In addition, T_m represents an outdate point of time dropped in a given cycle, which depends on the actual demand during an $m + L$ time unit interval.

Clearly, equation (8) must be weighted by $DL/(Q - ER)$ (due to $L/ET = DL/(Q - ER)$). Correspondingly, equation (9) should be weighted by $1 - DL/(Q - ER)$. Multiplying the two equations by the two weights, respectively, the expected inventory level per unit time has the following form:

$$\begin{aligned} EI_c = {} & \frac{1}{2}\left[DL/(Q - ER)\right]\left[r + \int_0^r (r - x)\, f_L(x)\,\mathrm{d}x\right] \\ & + \left[1 - DL/(Q - ER)\right] \\ & \times \left[r + (Q - DL - ER)/2 + ER(m - \alpha\, ET)/(ET - L)\right]. \end{aligned} \quad (11)$$

After some manipulations, it is given by

$$EI_c = [r - DL + (Q - ER)/2] + DL\,(ES)/[2\,(Q - ER)] + ER[1 - DL/(Q - ER)]\,[(m - \alpha\,ET)/(ET - L)]. \quad (12)$$

Note that EI_c is reduced to EI_w if ER is ignored (that is, ER is set to zero). Furthermore, EI_c becomes EI_r when both ER and ES are neglected. The last two terms in equation (12) are the correction terms used to make this approximation more effective. In order to reduce the computational effort, it is reasonable to set T_m to the middle point of the time length $(ET - L)$. As a result, $(m - \alpha\,ET)/(ET - L)$ becomes 1/2; thus,

$$EI_c = (r - DL + Q/2) + DL\,(ES - ER)/[2\,(Q - ER)]. \quad (13)$$

From equations (2), (3), and (6), we conclude that

$$EI_r \leq EI_m \leq EI_w. \quad (14)$$

In general, we have $0 \leq ES < Q$. Thus, $(ES - ER)/(Q - ER) \leq ES/Q$ since $ER \geq 0$. Equations (2), (3), and (13) imply that

$$EI_r \leq EI_c \leq EI_w. \quad (15)$$

However, it is difficult to compare EI_c with EI_m. We find that $EI_m \leq EI_c$ when the value of ER is very small in equation (13). This can be seen by comparing equation (13) with equation (6) directly. Now, the total expected average cost per unit time for the full backorder case is given by

$$EAC\,(Q,\, r) = [K + CQ + P\,(ES) + W\,(ER)]/ET + h\,(EI), \quad (16)$$

where EI is one of the above four approximations. It is noted that ET, as given in equation (10), is also a function of the current order size, Q, and the reorder point, r

5. PARTIAL BACKORDER AND FULL LOST SALES CASES

In a full lost sales situation, any unsatisfied demand is completely lost, and the customer or material user has presumably filled her or his need from other sources. However, in most practical situations, when the item is out of stock, some customers or material users are patiently waiting for their demand to be satisfied upon initial receipt of the next order while others are impatient and make purchases from other sources to fill their

demand. Under these circumstances, it is reasonable to assume that only a fraction, $\beta\,(0 \leq \beta \leq 1)$, of the shortage quantity is backordered, and that the remaining fraction, $1-\beta$, is lost forever. The derivation of the expected inventory levels per unit time for the two stockout cases is similar to that in the full backorder case. The major difference between the full backorder and the partial backorder cases is that in the partial backorder case, on average, $Q+(1-\beta)\,ES$ units are required in each replenishment cycle, as compared to only Q units in the full backorder case. It should be noted that the quantity of Q (including $\beta\,(ES)$ units backordered) is satisfied while that of $(1-\beta)\,ES$ is lost forever. As a result, the extremely rough approximation remains unchanged, and the other three approximations can easily be derived by simply substituting $Q+(1-\beta)\,ES$ for Q in the relevant equations of the full backorder case. The resultant equations for the partial backorder case are:

$$EI_r = r - DL + Q/2, \tag{17}$$

$$EI_w = \{r-DL+[Q+(1-\beta)\,ES]/2\}+DL\,(ES)/\{2\,[Q+(1-\beta)\,ES]\}, \tag{18}$$

$$\begin{aligned} EI_m = {} & \{r - DL + [Q + (1-\beta)\,ES]/2\} \\ & + DL\,(ES)/\{2\,[Q + (1-\beta)\,ES]\} \\ & - DL\left\{\int_r^{\infty} [r\,(x-r)\,f_L\,(x)/x]\,\mathrm{d}x\right\}/\{2\,[Q + (1-\beta)\,ES]\}, \end{aligned} \tag{19}$$

and

$$\begin{aligned} EI_c = {} & \{r - DL + [Q + (1-\beta)\,ES]/2\} \\ & + DL\,(ES - ER)/\{2\,[Q + (1-\beta)\,ES - ER]\}, \end{aligned} \tag{20}$$

where ER is from equation (1), and ES is from equation (4). At one extreme, $\beta = 0$, the partial backorder case reduces to the full lost sales case. At another extreme, $\beta = 1$, it reduces to the full backorder case. Analogously, the partial backorder and the full lost sales cases have the same properties as expressed in relations (14) and (15). The total expected average cost per unit time for the partial backorder case is given by

$$\begin{aligned} EAC\,(Q,\,r) = {} & [K + CQ + P\,\beta\,(ES) + \theta\,(1-\beta)\,ES + W\,(ER)] \\ & /ET + h\,(EI), \end{aligned} \tag{21}$$

where

$$ET = [Q + (1-\beta)\,ES - ER]/D. \tag{22}$$

6. NUMERICAL EXAMPLES AND SENSITIVITY ANALYSIS

In this section, numerical examples will be given and the results of sensitivity analysis will be presented. Twenty-four test problems which appeared in Chiu [3] were used and are listed in Table 1.

TABLE 1
Cost parameters and relevant data of 24 test problems.

Test problem No.	Cost parameter			
	C	P	K	W
1	5	20	10	5
2	5	20	50	5
3	5	20	100	5
4	5	40	10	5
5	5	40	50	5
6	5	40	100	5
7	15	20	10	5
8	15	20	50	5
9	15	20	100	5
10	15	40	10	5
11	15	40	50	5
12	15	40	100	5
13	5	20	10	15
14	5	20	50	15
15	5	20	100	15
16	5	40	10	15
17	5	40	50	15
18	5	40	100	15
19	15	20	10	15
20	15	20	50	15
21	15	20	100	15
22	15	40	10	15
23	15	40	50	15
24	15	40	100	15

$L = 1$, $h = 1$, $m = 3$, $\theta = P$, and $d_1 \sim$ a Poisson with $D = 10$.

For the purpose of illustration, Test Problem 1 in Table 1 was chosen. Then, the relevant equations of the proposed approach (including equations (1), (4), and (20)-(22)) were applied for $\beta = 1$, 0.5, and 0, respectively. After solving this test problem with Gino [8], a summary of the final solution was a given in Table 2.

It can be seen from Table 2 that the values of EI_c are 11.4899, 11.4080, and 11.0981 for $\beta = 1$, 0.5, and 0, respectively. We may conclude here that the expected inventory level per unit time decreases as the fraction β

TABLE 2
Summary of results using the proposed approximation and Test Problem 1 for $\beta = 1$, 0.5 and 0.

Final solution	$\beta = 1$ (Full backorder)	$\beta = 0.5$ (Partial backorder)	$\beta = 0$ (Full lost sales)
Q	13.8417	13.9178	13.6224
r	14.5414	14.3792	14.1564
EI_c	11.4899	11.4080	11.0981
ER	0.0571	0.0549	0.0431
ES	0.1331	0.1471	0.1693
ET	1.3785	1.3936	1.3749
$EAC(Q, r)$	71.0898	70.8247	70.5319

decreases. A more detailed analysis to verify this conclusion was further conducted in this study.

Table 3 presents the solution values of EI_c using the proposed approximation and Test Problem 1 for various values of m. The results indicate that for each fraction of β, the expected inventory level per unit time converges to a fixed value when the lifetime, m, increases to a large value (this value of m is five in this example). The longer the lifetime of a perishable item has, the greater is the tendency that the perishability assumption being released. This implies that $ER \rightarrow 0$ as the lifetime, m, increases to a sufficently large value, and that equation (20) then approaches equation (18). Consequently, the (Q, r) perishable inventory model reduces to the (Q, r) no-outdating model in the extremely long lifetime situation.

TABLE 3
Solution values of EI using the proposed approximation and Test Problem 1 for various values of m.

m	$\beta = 1$	$\beta = 0.5$	$\beta = 0$
2	8.7777	8.5570	8.4990
3	11.4899	11.4080	11.0981
4	12.5202	12.3676	12.1288
5	12.5803	12.4032	12.1890
6	12.5803	12.4032	12.1890
7	12.5803	12.4032	12.1890

A question arises about whether careless approximation of the expected inventory level per unit time has a significant impact on determination of the ordering policy (Q, r). Table 4 presents a summary of the results of sensitivity analysis in which 24 test problems, given in Table 1, were used.

Each average percentage in Table 4 is the result of, first, subtracting the policy parameter (e.g., Q) which was obtained using the proposed

TABLE 4
Average percentages for ordering policy deviations.

Approximation approach	$\beta = 1$		$\beta = 0.7$		$\beta = 0.4$		$\beta = 0$	
	Q	r	Q	r	Q	r	Q	r
Extremely rough	– 0.10%	– 0.53%	+ 0.10%	– 0.84%	+ 0.29%	– 0.92%	+ 0.54%	– 1.82%
Modified Wagner	– 0.14%	– 0.49%	– 0.24%	– 0.37%	+ 0.82%	– 1.20%	– 0.52%	– 0.40%
Wagner	– 0.03%	– 0.52%	– 0.35%	– 0.18%	– 0.16%	– 0.03%	– 0.07%	– 0.19%

approximation from the policy parameter which was obtained using one of the other three approximations, and then dividing this value by the proposed policy parameter. A positive average percentage shows the extent to which the policy parameter obtained by using an approximation approach has been overestimated while a negative one means that the policy parameter obtained has been underestimated. Some important conclusions drawn from Table 4 are as follows:

(1) For each fraction of β, the reorder points obtained by using the extremely rough, modified Wagner, and Wagner approaches are consistently underestimated.

(2) In the full lost sales case ($\beta = 0$), deviations on the order quantity Q are greater than those in the full backorder case ($\beta = 1$). This may be because order quantity, Q, does not include the backordered quantity of ES in the full lost sales case.

(3) Most of the policy parameters obtained by using the Wagner approach have much smaller deviations than do those obtained using the extremely rough and modified Wagner approximations. Presumedly, the main reason is that the solution value of ER is very small (one example is shown in Table 2). Thus, equation (18) is almost identical to equation (20). Nevertheless, all policy parameters determined by using the Wagner approach are underestimated.

Table 5 presents the sums of 24 solution values of EI. Figure 2 gives the associated graph which shows the relative values of EI for the four approximations and three stockout cases. Here, we conclude that the expected inventory level per unit time decreases with the decrease of the fraction, β. It is also evident that Relations (14) and (15) are consistent with the results shown in Table 5 or Figure 2. Furthermore, just as expected, the values of EI obtained by using the modified Wagner approximation are smaller than

those obtained using the proposed approximation since the solution values of ER are very small in this analysis.

TABLE 5
Total solution values of EI (24 test problems).

Approximation approach	$\beta = 1$	$\beta = 0.7$	$\beta = 0.4$	$\beta = 0$
Extremely rough	299.4603	293.4229	283.7080	268.7160
Modified Wagner	299.5945	295.4538	290.4995	283.3413
Chiu	301.2300	297.8750	293.2553	286.8325
Wagner	302.4031	298.7430	294.8651	287.9539

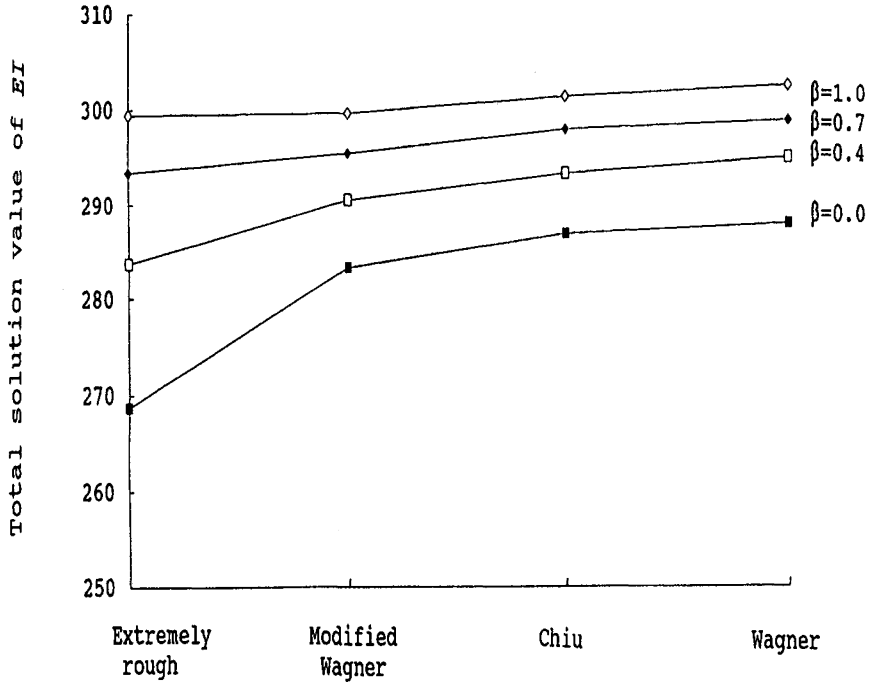

Figure 2.

7. CONCLUSIONS

This paper has presented a good approach to approximation of the expected inventory level per unit time for the (Q, r) perishable inventory system. Three stockout cases which customers or material users may adopt in response

to a stockout condition have been considered. The study has compared the proposed approximation approach with three existing approaches which have been used in a situation where persishability or decay is not allowed. Obviously, the proposed approximation is much simpler than the modified Wagner approach. This can be observed by comparing equation (20) with equation (19). It is not complicated, as compared with the Wagner approximation. Therefore, equation (20) is a practical formula, suitable for incorporation into the (Q, r) perishable inventory model, which can be formulated in the form of equations (21), (22), (1), and (4). The best ordering policy (Q, r) can, thereby, be obtained correctly, and distortion in determining Q and r can be reduced to a minimum.

In addition, results from numerical examples and a sensitivity analysis indicate that the solution values of EI are underestimated when the extremely rough approach and the modified Wagner approach are used. This result often causes deviations in the policy parameters. More importantly, severe underestimation of EI due to the use of the extremely rough approach will result in great distortion when determining the best ordering policy. It is worth noting here, as pointed out by Brown *et al.* [1, p. 607], that the importance of accurately estimating the holding cost function is readily apparent for decision makers whose firms operate in an environment of diseconomies of scale (e.g., perishability, decay, or deterioration).

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