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# A HEURISTIC METHOD FOR THE ESTIMATION OF THE PROJECT DURATION IN A STOCHASTIC NETWORK SCHEDULING (*)

by A. C. GARAVELLI ([1]) and P. PONTRANDOLFO ([2])

Abstract. – *In engineering and construction projects, the estimation of the completion date is often a complex task. When the uncertainty concerning project activity durations and the complexity of the project are not negligible, stochastic network techniques are more suitable to gain a reliable appraisal of the completion date. Since in these cases the conventional PERT computation can be inaccurate and the simulation can be too onerous, a heuristic method aimed to obtain fast and reliable results is proposed. An example of application of the heuristics to a real case of project scheduling is provided to show the heuristic implementation and effectiveness.*

Keywords: Project management, scheduling, PERT.

Résumé. – *En Ingenierie et dans les projets de construction, une estimation des temps de projets peut être compliquée. Quand l'incertitude et la complexité composant le projet sont important, des techniques utilisant des réseaux stochastiques peuvent être plus précises. Toutefois, comme les méthodes pratiquées par PERT sont peut précises et des calculs basés sur des simulations sont trop intensifs, nous proposons une méthode heuristique. Un exemple pratique est alors considéré démontrant l'efficacité de notre heuristique.*

Mots clés : Gestion de projet, ordonnancement, PERT.

## 1. INTRODUCTION

In engineering and construction, demand fluctuation, market internationalization, clients' requirements and fast product/process innovation rates, together with the complexity and the risks associated to large projects, are some of the factors which contribute to stir competition [1, 2].

In this context, time plays a particular role. It is considered crucial for both client and contractor: for the former it is a fundamental factor to be evaluated in the contractor's bid, for the latter it represents a factor of competitiveness and a resource in the work plan and execution [3]. Moreover, time is often a constraint, since it usually represents a parameter to be formalized in

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contracts. Thus, appropriate methods and techniques are required to manage time. For example, network techniques, mainly due to their simplicity, are widely used. These techniques, representing a project in terms of a network of activities with stochastic or deterministic durations, also provide a useful modeling framework for project management.

When stochastic activity durations cannot be approximated by deterministic ones, the project evolution is necessarily probabilistic. In this case, the sequence of project events can vary among many possible alternatives, and project performances (as, for example, the completion date), depending on the project evolution, can change [4]. Consequently, when it is necessary to consider the uncertainty of work execution times, the adoption of stochastic networks is necessary, even if it may involve a considerable increase of complexity in project planning and management.

A PERT stochastic network, characterized by activity durations probabilistically distributed, can be solved in many ways, using analytical or approximate methods [5-8]. However, two methods are widely used in practice: conventional PERT analysis [9] and simulation. In the first case, the expected project duration is calculated by the deterministic Critical Path Method (CPM) applied to the mean values of each activity duration. In the second case, the expected project duration is calculated by a statistical analysis of the simulated project durations.

In this paper, a heuristic method based on the PERT technique is proposed to provide a reliable estimation of project duration. This heuristics seeks a trade-off between the results reliability obtained by simulation and the low computational effort required by the conventional PERT. An application to a real case is also provided to show the implementation of the heuristics.

## 2. UNCERTAINTY AND COMPLEXITY IN PROJECT SCHEDULING

In most cases, activity durations cannot be estimated with certainty. The conventional PERT analysis allows to manage activity duration uncertainty. Project scheduling can be supported by this simple method, characterised by few and fast computations. However, conventional PERT analysis is not particularly effective, because it tends to underestimate the whole project duration. This underestimation depends mainly on complex connections among the project activities, each of them characterised by a probabilistic distribution of its duration. In order to explain this inconvenience, an elementary network, made of $n$ parallel activities which connect two nodes

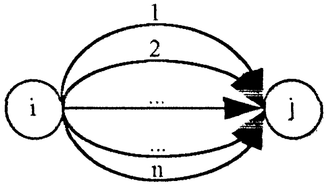



**Figure 1. – An elementary network.**

($i$ and $j$) and are characterized by the same exponential distribution (with parameter $\lambda$) of their duration, is considered (*fig.* 1).

According to conventional PERT analysis, in this simple case a project duration estimate $E(T)$ is given by the maximum value among mean activity durations (all equal to $1/\lambda$), that is $1/\lambda$. However, in this case, project duration is necessarily a random variable, given by $T = \text{Max}\,(x_1, x_2, \ldots, x_n)$, where $x_i$ are known to be exponentially distributed with mean $1/\lambda$. Of course, $E\,(T) > 1/\lambda$. In Figure 2 it is shown how the mean project duration of an elementary network rapidly increases (it doubles for $n = 4$) as the number of parallel activities having the same mean duration ($1/\lambda = 1$) increases.

This effect, shown for a simple elementary network, can be generalized for more complex networks, representing projects with many interrelated activities and nodes. As many simulations have stressed, an expected project duration grows with the network complexity. As stated by Simon [10], a system can be considered complex if it is made of a large quantity of interacting parts. In particular, the complexity of a system is related to the uncertainty concerning event occurrences and is proportional to the number of both network parts and interrelations among these parts. For instance,

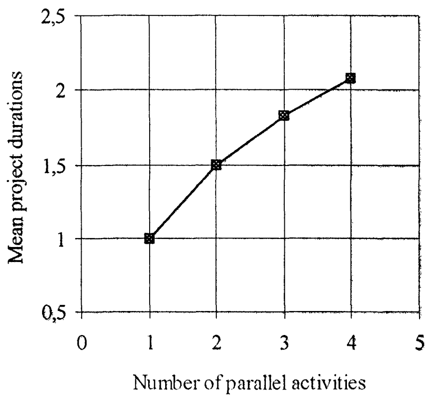



**Figure 2. – Effect of parallel activities on the expected duration of an elementary network.**

some indices that characterize network complexity include the Coefficient of Network Complexity (CNC) and the Index of Relative Complexity IRC, given by [11, 12]:

$$\mathrm{CNC} = \frac{N_a^2}{N_e} \qquad \mathrm{IRC} = \frac{\sum_{i=1}^{p} N_{a_i}}{P}$$

where:

$N_a$ = number of activities;

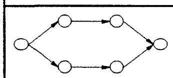
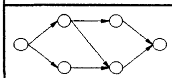
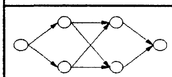
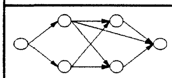
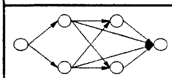
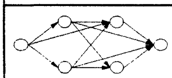
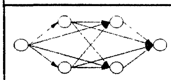
$N_e$ = number of events (equal to the number of nodes);

$N_{ai}$ = number of activities belonging to the path $i$;

$P$ = number of paths joining the start node to the end node.

These indices can be used to appraise the influence of complexity on project duration estimation. In particular, some simulations of different kinds of networks have shown that high values of CNC and low values of IRC are a signal of a mean project duration longer than the estimated one provided by the conventional PERT computation. For instance, these indices have been calculated for the networks shown in Table 1, where each activity duration has an exponential distribution with mean $\lambda = 1$. A comparison between

TABLE 1
*Network complexity indices and project durations.*

| NETWORKS | $T_C$ | $T_S$ | $\varepsilon_C$ | CNC | IRC |
|---|---|---|---|---|---|
|  | 3,00 | 3,91 | 23% | 6,00 | 3,00 |
|  | 3,00 | 4,23 | 29% | 8,16 | 3,00 |
|  | 3,00 | 4,53 | 34% | 10,70 | 3,00 |
|  | 3,00 | 4,60 | 35% | 13,50 | 2,80 |
|  | 3,00 | 4,66 | 36% | 16,70 | 2,66 |
|  | 3,00 | 4,75 | 37% | 20,20 | 2,57 |
|  | 3,00 | 4,82 | 38% | 24,00 | 2,50 |

analytical and simulation approaches has been made for the estimation of the project completion date. The project durations $T_S$ and $T_C$ and the estimation error $\varepsilon_C = \dfrac{|T_C - T_S|}{T_S}$ are provided in Table 1 as well, where $T_S$ and $T_C$ are the expected project durations obtained by simulation and conventional PERT computation, respectively.

## 3. A HEURISTIC ESTIMATION OF THE PROJECT DURATION

The estimation of a project duration can be carried out by conventional PERT computation (*i.e.* by a CPM algorithm applied to mean values for each activity duration) or by simulation. To pursue a trade-off between reliability of results obtained via simulation and low computational effort which characterizes the conventional PERT analysis, a heuristic method is proposed.

The basic principle of our heuristics consists in taking into account the effects that parallel paths of a stochastic network produce on the project duration estimate. The influence of parallel paths is evaluated in order to integrate conventional PERT computations with opportune *mean delay* values, which increase the reliability of PERT duration estimates.

The determination of mean delay values is based on the computation of *standard delay*, calculated for elementary networks made by a *start node* $i$, an *end node* $j$ and two paths between them without other nodes in common. Each of the two paths is made by $R_k$ ($k=1, 2$) activities in sequence. Activity durations of each path are characterised by exponential probability distributions and have all the same expected value $1/\lambda = T_k/R_k$, where $T_k$ ($k=1, 2$) is the expected path duration. The longer path is indicated with $P_1$ and $T_1$ is its duration.

The expected project duration $E(T)$ of the elementary network is calculated by simulation. The difference between the expected project duration and $T_1$ gives the desired mean delay value $\Delta T = E(T) - T_1$. In order to generalize the results for various combinations of ($R_1$, $R_2$) and ($T_1$, $T_2$), the delay $\Delta T$ is divided by $T_1$ and the simulations are referred to the ratio $T_2/T_1$. In this way, a *standard delay table* can be defined. In Table 2, to every couple ($R_1$, $R_2$) and ratio $T_1/T_2$, the mean delay values $\Delta T/T_1$ are provided. In this Table a scheme of standard delays is reported, with $R_1 \leq 5$, $R_2 \leq 5$ and a scale interval of $T_2/T_1$ values equal to 0.5.

As an example, consider two elementary networks $A$ and $B$ characterised by the couple ($R_1=2$, $R_2=1$) and, respectively, by ($T_1=3$, $T_2=1$) and

($T_1 = 0.9$, $T_2 = 0.3$), in correspondence of $T_2/T_1 = 0.3$ (for both $A$ and $B$) the value of the standard delay $\Delta T/T_1$ is equal to 0.04. Consequently, $\Delta T_A = 1.2$ and $\Delta T_B = 0.36$. Then, the expected project durations calculated by the heuristics are $E(T_A) = 4.2$ and $E(T_B) = 1.26$, respectively.

These expected durations can be different from the ones provided by the simulation applied to the real projects $A$ and $B$, because the two activities $a$ and $b$ ($R_1 = 2$) of path $P_1$ of projects $A$ and $B$ can have expected durations $T_a = 1/\lambda_a$ and $T_b = 1/\lambda_b$ different from $T_1/T_2$ (for instance, in the project $A$, it could be $T_a = 0.5$ and $T_b = 2.5$ instead of $T_a = T_b = 1.5$).

The simulation of many different cases, however, has shown that this approximation determines an error ($T_H - T_S$) that very seldom reaches the 10% of $T_S$, where $T_S$ and $T_H$ are expected project durations obtained by simulation and heuristics, respectively.

Large and complex projects include in their network representation many elementary networks. In these projects, in fact, there are many nodes (as the previous node $i$) from which more than one activity starts and many nodes (as the previous node $j$) where many activities arrive, with many paths between every couple of nodes $i$ and $j$. Consequently, every elementary network within the whole network determines a mean delay that has to be added to the project duration estimation obtained by the conventional PERT computation. For the heuristic implementation, some general rules are

TABLE 2
*Scheme of a standard delay table.*

| $T_2/T_1$<br>$R_1, R_2$ | 1 | 0.95 | ... | 0.35 | 0.3 | ... |
|---|---|---|---|---|---|---|
| 1, 1 | 0.50 | 0.45 | ... | 0.09 | 0.07 | ... |
| 2, 1 | 0.44 | 0.39 | ... | 0.06 | 0.04 | ... |
| ... | ... | ... | ... | ... | ... | ... |
| 5, 1 | 0.40 | 0.36 | ... | 0.03 | 0.02 | ... |
| 1, 2 | 0.44 | 0.40 | ... | 0.07 | 0.05 | ... |
| ... | ... | ... | ... | ... | ... | ... |
| 5, 2 | 0.32 | 0.28 | ... | 0.02 | 0.01 | ... |
| ... | ... | ... | ... | ... | ... | ... |
| ... | ... | ... | ... | ... | ... | ... |
| 1, 5 | 0.40 | 0.36 | ... | 0.05 | 0.04 | ... |
| ... | ... | ... | ... | ... | ... | ... |
| 5, 5 | 0.24 | 0.21 | ... | 0.01 | 0.005 | ... |

required. These rules have been grouped in the following steps.

1. Determination of the nodes $j$ ($j = 1, 2, \ldots, J$), characterized by 2 or more activity arrivals. The nodes $j$ are numerated according to their precedence ties (from the previous to the following). Independent $j$-nodes can be indifferently ordered.

2. For the first node $j$ ($j=1$), determination of its nodes $i_j$ ($i_j = 1, 2, \ldots, I_j$). Each node $i_j$ is a start node of 2 or more paths that connect that node with the node $j$. The nodes $i_j$ are numerated according to their precedence ties (from the previous to the following). Independent $i_j$-nodes are numerated according to a decreasing order of their distance from node $j$.

3. For every node $j$, $j > 1$, determination of its nodes $i_j$. To this aim, it is necessary, for every node $j$, to consider the nodes $k_j$ individually ($k_j = 1, 2, \ldots, K_j$), where each node $k_j$ is a start node of 2 or more paths arriving at node $j$, and to enumerate them according to their precedence ties (from the previous to the following). Independent $k_j$-nodes are enumerated according to a decreasing order of their distance from the node $j$. Beginning from the last $k_j$-node (*i.e.*, from $k_j = K_j$ to $k_j = 1$), all the paths $P_{nkj}$ ($n = 1, 2, \ldots, N_{ij}$) connecting node $k_j$ with node $j$ are identified. Only if there is at least one path containing activities not previously considered (during the precedent definition of nodes $i_{j'}$, $j' < j$) and/or not all paths have the final activity in common, the node $k_j$ is a $i_j$-type node. Nodes $i_j$ thus defined can be ordered as in the previous step 2. An example of implementation of steps 1 to 3 is reported in Figure 3.

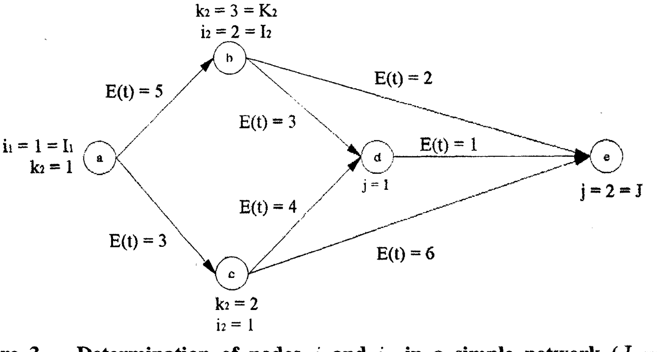



**Figure 3. – Determination of nodes $j$ and $i_j$ in a simple network ($J = 2$).**

4. For every node $i_j$ ($i_j = 1, 2, \ldots, I_j$) of a node $j$ ($j = 1, 2, \ldots, J$), definition of paths $P_{nij}$ ($n = 1, 2, \ldots, N_{ij}$) connecting node $i_j$ with node $j$ and computation of their durations $T_{nij}$ by the conventional PERT. If

two or more paths have initial and/or final activities in common, only the longest path is considered. For each node $i_j$, paths are enumerated according to a decreasing order of their durations. An elementary network between each couple of nodes $(i_j, j)$ is thus defined. For instance, in Figure 3 the following paths are defined: $P_{111}$ and $P_{211}$, connecting (through node $b$ or $c$, respectively) node a with node $d$, with $T_{111}=8$ and $T_{211}=7$; $P_{112}$ and $P_{212}$, connecting (directly or through node $d$, respectively) node $c$ with node $e$, with $T_{112}=6$ and $T_{212}=5$; $P_{122}$ and $P_{222}$, connecting (through node $d$ or directly, respectively) node $b$ with node $e$, with $T_{122}=4$ and $T_{222}=2$.

5. Determination of *mean delays*. This step proceeds from the first to the last node $j$ of the network and, for each node $j$, from the first to the last node $i_j$. When an elementary network $(i_j, j)$ consists of only $N=2$ paths, it is characterised by values of $R_{1ij}$, $R_{2ij}$, $T_{1ij}$ and $T_{2ij}$. The mean delay $\Delta T_{ij}$ is then evaluated by multiplying $T_{1ij}$ times the standard delay $\Delta T_{ij}/T_{ij}$ provided by the standard delay table. When an elementary network contains $N>2$ paths, $N-1$ computations of $\Delta T_{nij}$ $(n=2, 3, N_{ij})$ are required, in correspondence of the $N-1$ couples of paths $(P_{1ij}, P_{nij})$. The sum of all the $\Delta T_{nij}$ provides the mean delay $\Delta T_{ij} = \sum_{n=2}^{N_{ij}} \Delta T_{nij}$. If $i_j=I_j=1$, then $\Delta T_{ij}=\Delta T_j$, otherwise $(I_j>1)$, $\Delta T_{ij} = \sum_{i=1}^{I_j} \Delta T_{ij}$, where $\Delta T_j$, useful for final computations, is the total mean delay associated to node $j$.

6. Determination of all paths connecting the start node with the end node of the project and estimation of the project duration. This step requires the computation of the path durations, evaluated by adding to each path duration obtained by the conventional PERT the mean delays $\Delta T_j$ associated to the nodes $j$ included in the path. The maximum path duration indicates the expected project duration by the heuristics. For instance, in Figure 3 there are four paths that connect node $a$ to node $e$, passing through nodes $b$, $b$ and $d$, $c$ and $d$, $c$, respectively. The project duration is thus given by the maximum value among: $(T_{abe} + \Delta T_e)$, $(T_{abde} + \Delta T_d + \Delta T_e)$, $(T_{acde} + \Delta T_d + \Delta T_e)$, $(T_{ace} + \Delta T_e)$.

## 4. AN APPLICATION OF THE HEURISTICS TO A REAL PROJECT

In order to show an application of the heuristics, an example of a real project has been considered. The project concerns the construction of an electric power plant, as referred in Albino *et al.* [13]. The whole project has been modeled by a PERT having 28 main activities. To simplify the analysis, the heuristic method has been limited to these activities (their names, for the sake of brevity, have been omitted).

The list of the activities, with the related expected durations and precedence-ties, is reported in Table 3. In particular, two series of mean duration values (Program $A$ and Program $B$) are available, in relation to different resource involvement plans. The project can then be completed according to Program $A$ or $B$ alternatively, as strategic and economic considerations suggest to project management.

TABLE 3

*Project activity durations and precedence ties.*

| Activity | Preceding Activities | A - Mean Activity Duration | B - Mean Activity Duration |
|---|---|---|---|
| 1 | - | 1 | 1 |
| 2 | - | 2 | 2 |
| 3 | - | 2 | 2 |
| 4 | 1 | 3.5 | 2.5 |
| 5 | 1 | 1.25 | 0.75 |
| 6 | 3 | 1.75 | 1.25 |
| 7 | 2-5-6 | 1.5 | 1.25 |
| 8 | 2-5-6 | 29 | 24.75 |
| 9 | 2-5-6 | 33 | 28 |
| 10 | 3 | 30.25 | 28.75 |
| 11 | 3 | 4 | 3 |
| 12 | 4 | 24.25 | 22.25 |
| 13 | 4 | 1.5 | 1.5 |
| 14 | 7 | 1 | 1 |
| 15 | 7 | 1.5 | 1.5 |
| 16 | 11 | 30 | 27 |
| 17 | 13-14 | 22.5 | 18.5 |
| 18 | 13-14 | 4 | 4 |
| 19 | 15 | 2.5 | 2.5 |
| 20 | 15 | 21 | 17.5 |
| 21 | 11 | 33.5 | 28.5 |
| 22 | 18-19 | 15 | 15 |
| 23 | 17-20-22 | 1.5 | 1.5 |
| 24 | 17-20-22 | 2.25 | 2.25 |
| 25 | 12-23 | 3.5 | 3.5 |
| 26 | 8-24-25 | 1.5 | 1.5 |
| 27 | 9-10-16 | 1.5 | 1.5 |
| 28 | 21-27 | 1 | 1 |

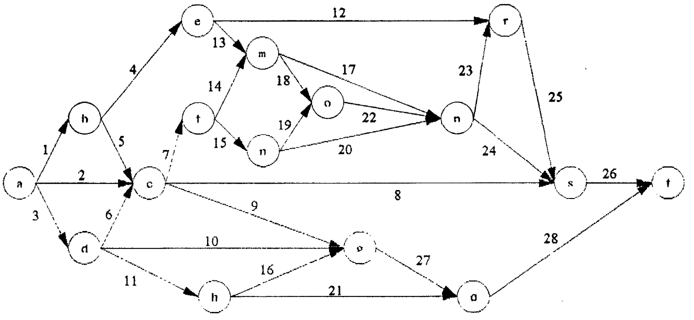


**Figure 4. – Project network representation.**

In Figure 4 the network representation of the project, useful to show the activity sequences necessary to project scheduling, is provided.

In Table 4, all paths connecting node a (project start) with node $r$ (project completion) are provided. In this table, the activity sequence of each path, together with the correspondent total duration $T_{PC}$ calculated by conventional PERT, is shown. According to the conventional approach, then, in correspondence to Programs $A$ and $B$ the expected project duration $T_C$ is equal, respectively, to 40.5 months (Path 1) and 34.5 months (Paths 1, 2).

A simulation lasting about 60 hours on a computer with a 80486DX2/66 processor has shown in this case project mean durations $T_S$, in correspondence to Program $A$ and $B$, equal to 86.8 and 76.7 months, respectively. These values, if compared to the ones provided by the conventional PERT, result more than twice higher, providing an estimation error $\varepsilon_c = \dfrac{|T_C - T_S|}{T_S} > 50\%$.

The effects of duration uncertainties and project complexity on project duration estimates are then considerable. In fact, mainly because of the impossibility of accurately foreseeing the activity durations, with the assumption of their probability distributions, and of the great number of project activities and precedence-ties, especially in terms of parallel paths

TABLE 4

*Network paths and related duration estimations by the conventional PERT.*

| Path No. | Path activities | $T_{PC}$ A | $T_{PC}$ B |
|---|---|---|---|
| 1 | 3-11-21-28 | **40.5** | **34.5** |
| 2 | 3-11-16-27-28 | 38.5 | **34.5** |
| 3 | 3-10-27-28 | 34.75 | 33.25 |
| 4 | 3-6-9-27-28 | 39.25 | 33.75 |
| 5 | 3-6-8-26 | 34.25 | 29.5 |
| 6 | 3-6-7-15-20-24-26 | 31.5 | 27.25 |
| 7 | 3-6-7-15-20-23-25-26 | 34.25 | 30 |
| 8 | 3-6-7-15-19-22-24-26 | 28 | 27.25 |
| 9 | 3-6-7-15-19-22-23-25-26 | 30.75 | 30 |
| 10 | 3-6-7-14-18-22-24-26 | 29 | 28.25 |
| 11 | 3-6-7-14-18-22-23-25-26 | 31.75 | 31 |
| 12 | 3-6-7-14-17-24-26 | 32.5 | 27.75 |
| 13 | 3-6-7-14-17-23-25-26 | 35.25 | 30.5 |
| 14 | 2-9-27-28 | 37.5 | 32.5 |
| 15 | 2-8-26 | 32.5 | 28.25 |
| 16 | 2-7-15-20-24-26 | 29.75 | 26 |
| 17 | 2-7-15-20-23-25-26 | 32.5 | 28.75 |
| 18 | 2-7-15-19-22-24-26 | 26.25 | 26 |
| 19 | 2-7-15-19-22-23-25-26 | 29 | 28.75 |

| Path No. | Path activities | $T_{PC}$ A | $T_{PC}$ B |
|---|---|---|---|
| 20 | 2-7-14-18-22-24-26 | 27.25 | 27 |
| 21 | 2-7-14-18-22-23-25-26 | 30 | 29.75 |
| 22 | 2-7-14-17-24-26 | 30.75 | 26.5 |
| 23 | 2-7-14-17-23-25-26 | 33.5 | 29.25 |
| 24 | 1-5-9-27-28 | 37.75 | 32.25 |
| 25 | 1-5-8-26 | 32.75 | 28 |
| 26 | 1-5-7-15-20-24-26 | 30 | 25.75 |
| 27 | 1-5-7-15-20-23-25-26 | 32.75 | 28.5 |
| 28 | 1-5-7-15-19-22-24-26 | 26.5 | 25.75 |
| 29 | 1-5-7-15-19-22-23-25-26 | 29.25 | 28.5 |
| 30 | 1-5-7-14-18-22-24-26 | 27.5 | 26.75 |
| 31 | 1-5-7-14-18-22-23-25-26 | 30.25 | 29.5 |
| 32 | 1-5-7-14-17-24-26 | 31 | 26.25 |
| 33 | 1-5-7-14-17-23-25-26 | 33.75 | 29 |
| 34 | 1-4-13-18-22-24-26 | 28.75 | 27.75 |
| 35 | 1-4-13-18-22-23-25-26 | 31.5 | 30.5 |
| 36 | 1-4-13-17-24-26 | 32.25 | 27.25 |
| 37 | 1-4-13-17-23-25-26 | 35 | 30 |
| 38 | 1-4-12-25-26 | 33.75 | 30.75 |

among nodes, project execution times cannot be effectively estimated by the conventional PERT.

Instead of the simulation approach, which is time consuming, the heuristics described in Paragraph 3 can be used to estimate the duration of the project considered.

As indicated in Table 5, the project network is characterized by nine $j$-type nodes. Each node $j$ is characterized by one or two $i$-type nodes. In Table 5 the different paths connecting each node $j$ to every node $i_j$ are also indicated, together with the activities involved, the mean path durations given by the conventional PERT computation and the delay values $\Delta T_j$ obtained by the heuristics.

TABLE 5
*Heuristic data and results.*

| Node j | Node $i_j$ | Paths $P_{nij}$ | Path i-j activities | $T_{ij}$ (PERT A) | $\Delta T_j$ A | $T_{ij}$ (PERT B) | $\Delta T_j$ B |
|---|---|---|---|---|---|---|---|
| | | $P_{111}$ | 3-6 | 3.75 | | 3.25 | |
| c (1) | a | $P_{211}$ | 1.5 | 2.25 | 0.97 | 2.00 | 0.91 |
| | | $P_{311}$ | 2 | 2 | | 1.75 | |
| | | $P_{112A}$ ($P_{212B}$) | 6-9 | 34.75 | | 29.25 | |
| g (2) | d | $P_{212A}$ ($P_{112B}$) | 11-16 | 34 | 23.9 | 30.00 | 22.4 |
| | | $P_{312}$ | 10 | 30.25 | | 28.75 | |
| q (3) | h | $P_{113}$ | 21 | 33.5 | 13.1 | 28.5 | 12.3 |
| | | $P_{213}$ | 16-27 | 31.5 | | 28.5 | |
| m (4) | b | $P_{114}$ | 4-13 | 5 | 0.93 | 4 | 0.88 |
| | | $P_{214}$ | 5-7-14 | 3.75 | | 3 | |
| o (5) | f | $P_{115}$ | 14-18 | 5 | 1.18 | 5 | 1.18 |
| | | $P_{215}$ | 15-19 | 4 | | 4 | |
| | m (1) | $P_{116A}$ ($P_{216B}$) | 17 | 22.5 | | 18.5 | |
| p (6) | | $P_{216A}$ ($P_{116B}$) | 18-22 | 19 | 13.9 | 19 | 15.4 |
| | n (2) | $P_{126}$ | 20 | 21 | | 17.5 | |
| | | $P_{226}$ | 19-22 | 17.5 | | 17.5 | |
| | | $P_{117A}$ | 13-17-23 | 25.5 | | | |
| r (7) | e | $P_{217A}$ ($P_{117B}$) | 12 | 24.25 | 9.77 | 22.25 | 8.57 |
| | | ($P_{217B}$) | 13-18-22-23 | | | 22 | |
| | | $P_{118A}$ | 7-14-17-23-25 | 30 | | | |
| | c (1) | $P_{218}$ | 8 | 29 | | 24.75 | |
| s (8) | | ($P_{118B}$) | 7-14-18-22-23-25 | | 11.5 | 26.25 | 9.5 |
| | p (2) | $P_{128}$ | 23-25 | 5 | | 5 | |
| | | $P_{228}$ | 24 | 2.25 | | 2.25 | |
| | | $P_{119}$ | 9-27-28 | 35.5 | | 30.5 | |
| t (9) | c | $P_{219A}$ | 7-14-17-23-25-26 | 31.5 | 7.45 | | 6.1 |
| | | ($P_{219B}$) | 7-14-18-22-23-25-26 | | | 27.75 | |

In Table 6, heuristic path durations $T_{PH}$, calculated by adding to the correspondent conventional PERT values $T_{PC}$ (provided by Table 4) all the $\Delta T_j$ associated to the nodes $j$ included in each path, are reported. According to the heuristics, then, the expected project duration $T_H$ results equal to 84.7 and 73.5 months for the Program $A$ and $B$, respectively.

TABLE 6

*Path durations by heuristic computations.*

| Path | nodes j included | $\Sigma\Delta T_i$ A | $T_{PH}$ A | $\Sigma\Delta T_i$ B | $T_{PH}$ B |
|---|---|---|---|---|---|
| 1 | q-t | 20.5 | 61 | 18.4 | 52.9 |
| 2 | g-q-t | 44.5 | 83 | 30 | 64.5 |
| 3 | g-q-t | 44.5 | 79.2 | 30 | 63.3 |
| 4 | c-g-q-t | 45.4 | **84.7** | 30.9 | 64.7 |
| 5 | c-s-t | 19.9 | 54.2 | 16.5 | 46 |
| 6 | c-p-s-t | 33.8 | 65.3 | 31.9 | 59.2 |
| 7 | c-p-r-s-t | 43.6 | 77.8 | 40.5 | 70.5 |
| 8 | c-o-p-s-t | 35 | 63 | 33.1 | 60.3 |
| 9 | c-o-p-r-s-t | 44.8 | 75.5 | 41.7 | 71.7 |
| 10 | c-m-o-p-s-t | 35.9 | 64.9 | 34 | 62.2 |
| 11 | c-m-o-p-r-s-t | 45.7 | 77.5 | 42.5 | **73.5** |
| 12 | c-m-p-s-t | 34.7 | 67.3 | 32.8 | 60.5 |
| 13 | c-m-p-r-s-t | 44.5 | 79.8 | 41.4 | 71.9 |
| 14 | c-g-q-t | 45.4 | 82.9 | 30.9 | 63.4 |
| 15 | c-s-t | 19.9 | 52.4 | 16.5 | 44.8 |
| 16 | c-p-s-t | 33.8 | 63.6 | 31.9 | 57.9 |
| 17 | c-p-r-s-t | 43.6 | 76.1 | 40.5 | 69.2 |
| 18 | c-o-p-s-t | 35 | 61.3 | 33.1 | 60.1 |
| 19 | c-o-p-r-s-t | 44.8 | 73.8 | 41.7 | 71.4 |
| 20 | c-m-o-p-s-t | 35.9 | 63.2 | 34 | 60.5 |
| 21 | c-m-o-p-r-s-t | 45.7 | 75.7 | 42.5 | 71.8 |
| 22 | c-m-p-s-t | 34.7 | 65.5 | 32.8 | 65 |
| 23 | c-m-p-r-s-t | 44.5 | 78 | 41.4 | 69.4 |
| 24 | c-g-q-t | 45.4 | 83.2 | 30.9 | 56.7 |
| 25 | c-s-t | 19.9 | 52.7 | 16.5 | 45 |
| 26 | c-p-s-t | 33.8 | 63.8 | 31.9 | 58.7 |
| 27 | c-p-r-s-t | 43.6 | 76.3 | 40.5 | 70 |
| 28 | c-o-p-s-t | 35 | 61.5 | 33.1 | 59.3 |
| 29 | c-o-p-r-s-t | 44.8 | 74 | 41.7 | 70.7 |
| 30 | c-m-o-p-s-t | 35.9 | 63.4 | 34 | 61.7 |
| 31 | c-m-o-p-r-s-t | 45.7 | 75 | 42.5 | 73 |
| 32 | c-m-p-s-t | 34.7 | 65.7 | 32.8 | 60 |
| 33 | c-m-p-r-s-t | 44.5 | 78.3 | 41.4 | 71.4 |
| 34 | m-o-p-s-t | 35 | 63.7 | 33.1 | 63.8 |
| 35 | m-o-p-r-s-t | 44.7 | 76.2 | 41.6 | 41.6 |
| 36 | m-p-s-t | 33.8 | 66 | 31.9 | 31.9 |
| 37 | m-p-r-s-t | 43.5 | 78.5 | 40.4 | 40.5 |
| 38 | r-s-t | 28.7 | 62.5 | 24.2 | 24.2 |

The comparison between the expected project duration values provided by the simulation approach ($T_S$) and by the heuristics ($T_H$) shows the following error $\varepsilon_H$ of the heuristic estimation:

$$\varepsilon_H = \frac{|T_H - T_S|}{T_S} = \frac{|84.7 - 86.8|}{86.8} = 2.4\% \qquad \text{(Program A)}$$

$$\varepsilon_H = \frac{|T_H - T_S|}{T_S} = \frac{|73.5 - 76.7|}{76.7} = 4.2\% \qquad \text{(Program B)}$$

These error values, especially if compared with those obtained by the conventional PERT computation ($\varepsilon_C > 50\%$), show the reliability of the heuristic results in the case examined.

## 5. CONCLUSIONS

Project duration is usually estimated, using the PERT approach, by simulation or conventional PERT computation. Even if widely adopted, however, both these methods present some disadvantages, that appear more evident as project complexity grows: while the conventional PERT analysis provides an expected project duration often considerably underestimated, the simulation approach requires long simulation runs, even if provides a very accurate time estimation.

In the paper, a heuristic method has been proposed to estimate the project duration. The main objective of the heuristics is to provide a reliable estimation of the project duration by a low computational effort. This objective is mainly pursued by considering the conventional PERT computation combined with the determination of the *delay* that parallel paths cause in a stochastic network scheduling.

A test based on a real project has pointed out the effectiveness of the heuristics in terms of accuracy and computational effort. Further investigations on the heuristic implementation can be addressed, for example, on the generalization of the probability distributions of activity durations.

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