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GENERALIZED ORDERING POLICIES WITH GENERAL RANDOM MINIMAL REPAIR COSTS AND RANDOM LEAD TIMES (*)

by Shey-Huei SHEU (1) and Ching-Tien LIOU (2)

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Abstract. – *In this article we consider two generalized ordering policies for a complex system with age-dependent minimal repair and general random repair costs. Introducing costs due to ordering, repairs, shortage and holding, we derive the expected cost per unit time in the long run as a criterion of optimality and seek the optimum ordering policies by minimizing that cost. We show that, under certain conditions, there exists a finite and unique optimum policy. Various special cases are discussed. Finally, a numerical example is given.*

Keywords: Ordering, repair, maintenance, reliability.

Résumé. – *Nous considérons dans cet article deux politiques généralisées pour un système complexe avec coûts de réparation minimal dépendant du vieillissement et coûts de réparation généraux aléatoires. Introduisant les coûts dus aux réapprovisionnements, aux réparations, à la pénurie, et à l'immobilisation, nous en déduisons le coût moyen par unité de temps comme critère d'optimalité, et cherchons les politiques de réapprovisionnement optimales par minimisation de ce coût. Nous montrons que, sous certaines conditions, il existe une politique optimale finie. Nous examinons divers cas spéciaux. Nous terminons avec un exemple numérique.*

Mots clés : Réapprovisionnement, répartition, maintenance, fiabilité.

1. INTRODUCTION

It is of great importance to avoid the failure of complex system during actual operation when such an event is costly and/or dangerous. In such situation, one important area of interest in reliability theory is the study of various maintenance policies in order to reduce the operationg cost and the risk of a catastrophe. Many preventive maintenance policies have been proposed and discussed (*see* e.g., [3 to 6]). In particular, a replacement

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policy [7 to 10, 18, 19] is of great interest. In such a replacement policy it is generally assumed that there are an unlimited number of spare units available for replacement. However, it might not be true on some occasions. For instance, it is natural in commercial industries that only one spare unit, which can be delivered by order, is available for replacement. In this case we cannot neglect a random lead time for delivering the spare unit. That is, it is essential and practical to introduce the random lead times. Once we take account of the random lead times, we should consider an ordering policy that determines when to order a spare and when to replace the operating unit after it has begun operating.

Allen and D'Esopo [1, 2] considered an ordering policy in which some failed units are repaired and the others are scrapped with certain probabilities. Wiggins [21] considered an ordering policy in which a spare unit is ordered at a prespecified time t_0 after installation of the operating unit or at failure of the operating unit, whichever occurs first. He obtained the optimal ordering time t_0^* which minimizes the expected cost. He assumed in his model that the lifetimes of the operating units obey the exponential distributions, which implies the trivial ordering policy such as $t_0^* = 0$ or $t_0^* \to \infty$. Several authors [11, 12, 15, 20] have treated ordering policies for a non-repairable unit by assuming the arbitrary lifetime distributions of the operating units and the following assumptions:

(i) The original unit is replaced as soon as the ordered spare arives irrespective of the state of the unit.

(ii) The procured unit is kept in inventory until the original unit fails.

(iii) The procured unit is held in inventory until a predetermined times t_1 (measured from ordering times t_0) if the original unit does not fail till that time.

Recently, Osaki, Kaio, and Yamada [13], and Park and Park [14] have treated ordering policies with minimal repair.

In this paper we consider two generalized ordering policies for a system with age-dependent minimal repairs, general random repair costs and random lead time. The Policy 1 and 2 are described explicitly at the beginning of the next section. The expected cost per unit time in the long run is derived for the Policy 1 and 2. We show that, under certain conditions, there exists a finite and unique optimum policy in both the Policy 1 and 2. As special cases, various results from Barlow and Proschan [5] are obtained as well as many of the results of Osaki [12], Kaio and Osaki [11], Cléroux, Dubuc and Tilquin [10], Boland and Proschan [9], Boland [8], Block, Borges and Savits [7] and Park and Park [14, 15].

In the second section the Policy 1 and 2 are described, and then the expected cost per unit time in the long run is found in both the Policy 1 and 2. Theorem 1 gives a general optimization result for the Policy 1. Theorem 2 gives a general optimization result for the Policy 2. In the third section various special cases are discussed. In the last section a numerical example is given.

2. MODELS AND ANALYSIS

We assume that the unit has a failure time distribution $F(x)$ with finite mean μ and has a density $f(x)$. Then, the failure rate (or the hazard rate) function is $r(x)=f(x)/\bar{F}(x)$ and the cumulative hazard function is $R(x) = \int_0^x r(y)\,dy$, which has a relation $\bar{F}(x)=\exp\{-R(x)\}$ where $\bar{F}(x)=1-F(x)$. It is further assumed that the failure rate function $r(x)$ is continuous, monotone, and remains undisturbed by minimal repair.

We assume that the original unit begins operating at time 0. A system has two types of failures when it fails at age y. Type I failure (minor failure) occurs with probability $q(y)$ and is corrected with minimal repair, and whereas type II failure (catastrophic failure) occurs with probability $p(y)=1-q(y)$ and a unit has to be replaced. If the type II failure occurs before a specified time t_0, then the expedited order is made at the failure time instant. Otherwise, the regular order is made at time t_0. After a replacement the procedure is repeated. We assume all failures are instantly detected and repaired.

Policy 1: We define the following three mutually exclusive and exhaustive states between successive replacements:

(*a*) If the type II failure occurs before t_0, then the unit is shutdown and replaced by the spare as soon as the spare is delivered.

(*b*) If the type II failure occurs between t_0 and the arrival of the regular ordered spare, then the unit is shutdown and replaced by the spare as soon as the spare is delivered.

(*c*) If the type II failure occurs after the arrival of the regular ordered spare, then the unit is replaced by the delivered spare immediately irrespective of the state of the original unit.

Policy 2: We define the following five mutually exclusive and exhaustive states between successive replacements:

(*a*) If the type II failure occurs before t_0, then the unit is shutdown and replaced by the spare as soon as the spare is delivered.

(*b*) If the type II failure occurs between t_0 and the arrival of the regular ordered spare, then the unit is shutdown and replaced by the spare as soon as the spare is delivered.

(*c*) If the type II failure occurs between the arrival of the regular orderded spare and t_0+t_1 where t_1 is measured from ordering time t_0, then the delivered spare is put into inventory and the unit is replaced by that spare at the type II failure time instant.

(*d*) If the regular ordered spare arrives before t_0+t_1 and the type II failure occurs after t_0+t_1, then the delivered spare is put into inventory and the unit is replaced by that spare at the time t_0+t_1.

(*e*) If the regular ordered spare arrives after t_0+t_1 and the type II failure does not occur before the arrival of the regular ordered spare, then the unit is replaced by the spare as soon as the spare is delivered.

Let L_e denote the random lead time of an expedited order with p.d.f. $k_e(x)$ and finite mean μ_e. Let L_r denote the random lead time of a regular order with p.d.f. $k_r(x)$ and finite mean μ_r. Let us introduce the following five costs: the cost c_e is suffered for each expedited order made up to time t_0, the cost c_r is suffered for each regular order made at time t_0, the cost c_s per unit time is suffered for the shortage, the cost c_h per unit time is suffered for the inventory, the cost of the i-th minimal repair at age y is $g(C(y), c_i(y))$ where $C(y)$ is the age-dependent random part, $c_i(y)$ is the deterministic part which depends on the age and the number of the minimal repair, and g is a positive, nondecreasing and continuous function. Suppose that the random part $C(y)$ at age y have distribution $L_y(x)$, density $l_y(x)$ and finite mean $E[C(y)]$. We assume that $c_e>c_r>0$ and $\mu_r>\mu_e>0$.

Let Y_i^* denote the length of the i-th successive replacement cycle for $i=1, 2, 3, \ldots$ Let R_i^* denote the operational cost over the renewal interval Y_i^*. Thus $\{(Y_i^*, R_i^*)\}$ constitutes a renewal reward process. The pairs (Y_i^*, R_i^*), $i=1, 2, 3, \ldots$ are independent and identically distributed. If $D(t)$ denotes the expected cost of the operating unit over the time interval $[0, t]$, then it is well-known that

$$\lim_{t\to\infty} \frac{D(t)}{t} = \frac{E[R_1^*]}{E[Y_1^*]}. \tag{1}$$

(*see*, e.g., Ross [16, p. 52].) We shall denote the right-hand side of (1) by $B(t_0)$ for the policy 1 and $B(t_0, t_1)$ for the policy 2.

We now give a derivation of the expression for $E[R_1^*]$ and $E[Y_1^*]$ in both the policies. First, however, we must describe in more detail the failure process which governs the cost over the interval $[0, Y_1^*]$.

Consider a non-homogeneous Poisson process $\{N(t), t \geq 0\}$ with intensity $r(t)$ and successive arrival times $S_1, S_2, \ldots$. At time S_n we flip a coin. We designate the outcome by Z_n which takes the value one (head) with probability $p(S_n)$ and the value zero (tail) with probability $q(S_n)$. Let $L(t) = \sum_{n=1}^{N(t)} Z_n$ and $M(t) = N(t) - L(t)$. Then it can be shown that the process $\{L(t), t \geq 0\}$ and $\{M(t), t \geq 0\}$ are independent non-homogeneous Poisson process with respective intensities $p(t)r(t)$ and $q(t)r(t)$. (*see* e.g., Savits [17]). This is similar to the classical decomposition of a Poisson process for constant p. Let Y_1 denote the waiting time until the first type II failure. Then $Y_1 = \inf\{t \geq 0 : L(t) = 1\}$. Note that Y_1 is independent of $\{M(t), t \geq 0\}$. Thus the survival distribution of the time until the first type II failure is given by

$$\overline{F}_p(y) = P(Y_1 > y) = P(L(y) = 0) = \exp\left\{-\int_0^y p(x)\, r(x)\, dx\right\}. \quad (2)$$

We also require the following extended result of Lemma in Block *et al.* [7]. The Lemma is shown by mimicking the proof of Lemma in Block *et al.* [7].

LEMMA 1: *Let* $\{M(t), t \geq 0\}$ *be a non-homogeneous Poisson process with intensity* $q(t)r(t)$ $(t \geq 0)$ *and* $\Lambda(t) = E[M(t)] = \int_0^t q(z)\, r(z)\, dz$. *Denote the successive arrival times by* $S_1, S_2, \ldots$ *Assume that at time* S_i $(i = 1, 2, \ldots)$ *a cost of* $g(C(S_i), c_i(S_i))$ *is incurred. Suppose that* $C(y)$ *at age* y *is a random variables with finite mean* $E[C(y)]$. *If* $A(t)$ *is the total cost incurred over* $[0, t)$, *then*

$$E[A(t)] = \int_0^t h(z)\, q(z)\, r(z)\, dz, \quad (3)$$

where $h(z) = E_{M(z)}[E_{C(z)}[g(C(z), c_{M(z)+1}(z))]]$.

2.1. Analysis of Policy 1

For Policy 1 we have

$$Y_1^* = \begin{cases} Y_1 + L_e, & \text{if } Y_1 < t_0 \\ t_0 + L_r, & \text{if } Y_1 \geq t_0 \end{cases} \quad (4)$$

and

$$R_1^* = \begin{cases} c_e + c_s\, L_e + \sum_{i=1}^{M(Y_1)} g(C(S_i),\, c_i(S_i)), & \text{if} \quad Y_1 < t_0 \\ c_r + c_s(t_0 + L_r - Y_1) + \sum_{i=1}^{M(Y_1)} g(C(S_i),\, c_i(S_i)), & \\ & \text{if} \quad t_0 \le Y_1 < t_0 + L_r \\ c_r + \sum_{i=1}^{M(t_0+L_r)} g(C(S_i),\, c_i(S_i)), & \text{if} \quad Y_1 \ge t_0 + L_r \end{cases} \tag{5}$$

We are now ready to derive the expressions for $E[Y_1^*]$ and $E[R_1^*]$. First note that

$$\begin{aligned} E[Y_1^*] &= \int_0^{t_0} \int_0^{\infty} (y + x)\, k_e(x)\, dx\, dF_p(y) \\ &\quad + \int_{t_0}^{\infty} \int_0^{\infty} (t_0 + x)\, k_r(x)\, dx\, dF_p(y) \\ &= \int_0^{t_0} y\; dF_p(y) + F_p(t_0)\, \mu_e + \overline{F}_p(t_0)(t_0 + \mu_r) \\ &= \int_0^{t_0} \bar{F}_p(y)\, dy + \overline{F}_p(t_0)\, \mu_r + F_p(t_0)\, \mu_e. \end{aligned} \tag{6}$$

Using the Lemma 1 and the independence of Y_1 and $\{M(t),\, t \ge 0\}$. We can write

$$\begin{aligned} E[R_1^*] &= P(Y_1 < t_0)\left(c_e + c_s \int_0^{\infty} x k_e(x)\, dx\right) \\ &\quad + \int_0^{t_0} E\left[\sum_{i=1}^{M(y)} g(C(S_i),\, c_i(S_i))\right] dF_p(y) + c_r\, P(Y_1 \ge t_0) \\ &\quad + \int_0^{\infty} \left\{ c_s \int_{t_0}^{t_0+x} (t_0 + x - y)\, dF_p(y) \right. \\ &\quad + \int_{t_0}^{t_0+x} E\left[\sum_{i=1}^{M(y)} g(C(S_i),\, c_i(S_i))\right] dF_p(y) \end{aligned}$$

$$+ \int_{t_0+x}^{\infty} E\left[\sum_{i=1}^{M(t_0+x)} g(C(S_i), c_i(S_i))\right] dF_p(y)\Bigg\} k_r(x)\,dx,$$

$$= F_p(t_0)\,c_e + F_p(t_0)\,c_s\,\mu_e$$

$$+ \int_0^{t_0}\int_0^y h(z)\,q(z)\,r(z)\,dz\,dF_p(y) + \overline{F}_p(t_0)\,c_r$$

$$+ \int_0^{\infty}\Bigg\{c_s\left(x\,\overline{F}_p(t_0) - \int_{t_0}^{t_0+x}\overline{F}_p(y)\,dy\right)$$

$$+ \int_{t_0}^{t_0+x}\int_0^y h(z)\,q(z)\,r(z)dz\;dF_p(y)$$

$$+ \overline{F}_p(t_0+x)\int_0^{t_0+x} h(z)\,q(z)\,r(z)dz\Bigg\} k_r(x)\,dx,$$

which on simplification is equal to

$$F_p(t_0)\,c_e + \overline{F}_p(t_0)\;c_r + c_s\Bigg(F_p(t_0)\mu_e$$

$$+ \overline{F}_p(t_0)\mu_r - \int_0^{\infty}\int_{t_0}^{t_0+x}\overline{F}_p\;(y)\,dy\,k_r(x)\,dx\Bigg)$$

$$+ \int_0^{\infty}\left[\int_0^{t_0+x}\overline{F}_p(y)\,h(y)\,q(y)\,r(y)\,dy\right] k_r(x)\,dx, \qquad (7)$$

where $\overline{F}_p(y) = \exp\left\{-\int_0^y p(x)\,r(x)\,dx\right\}$.

For the infinite-horizon case we want to find a t_0^* which minimise $B(t_0)$, the expected cost per unit in the longrun. Recall that

$$B(t_0) = \Bigg\{F_p(t_0)\,c_e + \overline{F}_p(t_0)\,c_r + c_s\Bigg(F_p(t_0)\,\mu_e$$

$$+ \overline{F}_p\;(t_0)\,\mu_r - \int_0^{\infty}\int_{t_0}^{t_0+x}\overline{F}_p(y)\;dy\,k_r(x)\,dx\Bigg)$$

$$+ \int_0^{\infty}\left[\int_0^{t_0+x}\overline{F}_p(y)\,h(y)\;q(y)\,r(y)\,dy\right] k_r(x)\,dx\Bigg\}$$

$$/\left\{\int_0^{t_0}\overline{F}_p(y)\,dy + \overline{F}_p(t_0)\,\mu_r + F_p(t_0)\,\mu_e\right\}. \qquad (8)$$

We obtain the following two special expected costs:

$$B(0) = \left\{ c_r + c_s \int_0^\infty \int_0^x F_p(y)\,dy\,k_r(x)\,dx + \int_0^\infty \int_0^x \overline{F}_p(y)\,h(y)\,q(y)\,r(y)\,dy\,k_r(x)\,dx \right\} / \mu_r, \tag{9}$$

in which it is ordered at the same time as the installation of a working unit, and

$$B(\infty) = \left\{ c_e + c_s\,\mu_e + \int_0^\infty \overline{F}_p(y)\,h(y)\,q(y)\,r(y)\,dy \right\} / \left\{ \mu_e + \int_0^\infty \overline{F}_p(y)\,dy \right\}, \tag{10}$$

in which it is ordered just after the type II failure of a working item.

Define the numerator of the derivative of the right-hand side in (8) divided by $\overline{F}_p(t_0)$ as $w(t_0)$;

$$\begin{aligned} w(t_0) = & \left\{ (c_e - c_r)\,p(t_0)\,r(t_0) + c_s \left[(\mu_e - \mu_r)\,p(t_0)\,r(t_0) + \int_0^\infty F_p(x|t_0)\,k_r(x)\,dx \right] \right. \\ & \left. + \int_0^\infty (1 - F_p(x|t_0))\,h(t_0 + x)\,q(t_0 + x)\,r(t_0 + x)\,k_r(x)\,dx \right\} \\ & \times \left\{ \int_0^{t_0} \overline{F}_p(y)\,dy + F_p(t_0)\,\mu_e + \overline{F}_p(t_0)\,\mu_r \right\} \\ & - \left\{ F_p(t_0)\,c_e + \overline{F}_p(t_0)\,c_r + c_s \left(F_p(t_0)\,\mu_e + \overline{F}_p(t_0)\,\mu_r - \int_0^\infty \int_{t_0}^{t_0+x} \overline{F}_p(y)\,dy\,k_r(x)\,dx \right) \right. \\ & \left. + \int_0^\infty \int_{t_0}^{t_0+x} \overline{F}_p(y)\,h(y)\,q(y)\,r(y)\,dy\,k_r(x)\,dx \right\} \\ & \times \{ (\mu_e - \mu_r)\,p(t_0)\,r(t_0) + 1 \}, \end{aligned} \tag{11}$$

where

$$F_p(x|t_0) = \frac{F_p(t_0 + x) - F_p(t_0)}{\overline{F}_p(t_0)}.$$

LEMMA 2: *Assume F has a density f, with $F(0^-)=0$. If $r(t)$ and $p(t)$ are increasing in t, then $F_p(x|t)$ is increasing in t for $x \geq 0$.*

Proof: For $t_1 < t_2$, $r(t_1) \leq r(t_2)$ and $p(t_1) \leq p(t_2)$

$$\text{imply} \int_0^x p(t_1 + y)\, r(t_1 + y)\, dy \leq \int_0^x p(t_2 + y)\, r(t_2 + y)\, dy.$$

That is,

$$\exp\left\{-\int_{t_2}^{t_2+x} p(y)\, r(y)\, dy\right\} \leq \exp\left\{-\int_{t_1}^{t_1+x} p(y)\, r(y)\, dy\right\}$$

which implies $F_p(x|t_2) \geq F_p(x|t_1)$ using the identity

$$\overline{F}_p(t) = \exp\left\{-\int_0^t p(y)\, r(y)\, dy\right\}.$$

Q.E.D.

Now we discuss the optimum ordering policy which minimizes $B(t_0)$.

THEOREM 1: *(1) Suppose that either (a) the functions $r(t_0)$, $p(t_0)$ and*

$$[(c_e - c_r) + c_s(\mu_e - \mu_r)]\, p(t_0)\, r(t_0) + c_s F_p(x|t_0) + (1 - F_p(x|t_0))\, h(t_0 + x)\, q(t_0 + x)\, r(t_0 + x)$$

are continuous and strictly increasing in t_0 for each $x \geq 0$, or (b) the functions $r(t_0)$, $p(t_0)$, and $(1 - F_p(x|t_0))\, h(t_0 + x)\, q(t_0 + x)$ are continuous and strictly increasing in t_0 for each $x \geq 0$ and $c_e + c_s \mu_e > c_r + c_s \mu_r$.

(i) *If* $w(0) < 0$ and $w(\infty) > 0$, *then there exists a finite and unique optimum ordering time t_0^* $(0 < t_0^* < \infty)$ satisfing $w(t_0^*) = 0$ and the expected cost is*

$$B(t_0^*) = \Bigg\{(c_e - c_r)\, p(t_0^*)\, r(t_0^*) + c_s\left[(\mu_e - \mu_r)\, p(t_0^*)\, r(t_0^*) + \int_0^\infty F_p(x|t_0^*)\, k_r(x)\, dx\right] + \int_0^\infty (1 - F_p(x|t_0^*))\, h(t_0^* + x)\, q(t_0^* + x)\, r(t_0^* + x)\, k_r(x)\, dx\Bigg\} \Big/ \{(\mu_e - \mu_r)\, p(t_0^*)\, r(t_0^*) + 1\}. \tag{12}$$

(ii) *If* $w(\infty) \leq 0$, *then the optimum ordering time is* $t_0^* \to \infty$, i.e. *order for a spare is made at the same time instant as type II failure of the original unit, and the expected cost is given by* (10).

(iii) *If* $w(0) \geq 0$, *then the optimum ordering time is* $t_0^* = 0$, i.e. *order for a spare is made at the same time instant as the beginning of the original unit, and the expected cost is given by* (9).

Proof: By differentiating $B(t_0)$ with respect to t_0 and setting it equal to zero, we have the equation $w(t_0) = 0$. Further, we have

$$\begin{aligned} w'(t_0) = & \left\{ \int_0^\infty \frac{d}{dt_0} \left[((c_e - c_r) + c_s(\mu_e - \mu_r))\, p(t_0)\, r(t_0) \right.\right. \\ & + c_s F_p(x|t_0)\, h(t_0 + x) \\ & \left.\left. \times q(t_0 + x)\, r(t_0 + x) \right] k_r(x)\, dx \right\} \\ & \times \left\{ \int_0^{t_0} \overline{F}_p(y)\, dy + F_p(t_0)\, \mu_e + \overline{F}_p(t_0)\, \mu_r \right\} \\ & - \left\{ F_p(t_0)\, c_e + \overline{F}_p(t_0)\, c_r + c_s \left(F_p(t_0)\, \mu_e + \overline{F}_p(t_0)\, \mu_r \right.\right. \\ & \left. - \int_0^\infty \int_{t_0}^{t_0+x} \overline{F}_p(y)\, dy\, k_r(x)\, dx \right) \\ & \left. + \int_0^\infty \int_{t_0}^{t_0+x} \overline{F}_p(y)\, h(y)\, q(y)\, r(y)\, dy\, k_r(x)\, dx) \right\} \\ & \times \left\{ \mu_e - \mu_r)\, (p'(t_0)\, r(t_0) + p(t_0)\, r'(t_0)) \right\}. \end{aligned} \tag{13}$$

First, we treat the case (1). If the conditions of (1) in the Theorem 1 are satisfied, using the Lemma 2, then we have that $w'(t_0) > 0$, i.e. $w(t_0)$ is strictly increasing.

If $w(0) < 0$ and $w(\infty) > 0$, then there exists a finite and unique t_0^* $(0 < t_0^* < \infty)$ which minimizes the expected cost $B(t_0)$ as a finite and unique solution to $w(t_0) = 0$, since $w(t_0)$ is strictly increasing and continuous. Substituting the relation of $w(t_0^*) = 0$ into $B(t_0^*)$ in (8) yields (12).

If $w(\infty) \leq 0$, then for any non-negative t_0, $w(t_0) \leq 0$ and thus $B(t_0)$ is a strictly decreasing function, thus the optimum ordering time is $t_0^* \to \infty$.

If $w(0) \geq 0$, then for any non-negative t_0, $w(t_0) \geq 0$ and thus $B(t_0)$ is a strictly increasing function. Thus, the optimum ordering time is $t_0^* = 0$.

Q.E.D.

2.2. Analysis of Policy 2

For Policy 2 we have

$$Y_1^* = \begin{cases} Y_1 + L_e, & \text{if} \quad Y_1 < t_0 \\ t_0 + L_r, & \text{if} \quad t_0 \leq Y_1 < t_0 + L_r \\ Y_1, & \text{if} \quad t_0 + L_r \leq Y_1 < t_0 + t_1 \\ t_0 + t_1, & \text{if} \quad t_0 + L_r < t_0 + t_1 \leq Y_1 \\ t_0 + L_r, & \text{if} \quad t_0 + t_1 \leq t_0 + L_r \leq Y_1 \end{cases} \tag{14}$$

and

$$R_1^* = \begin{cases} c_e + c_s L_e + \sum_{i=1}^{M(Y_1)} g(C(S_i), c_i(S_i)), & \text{if } Y_1 < t_0, \\ c_r + c_s(t_0 + L_r - Y_1) + \sum_{i=1}^{M(Y_1)} g(C(S_i), c_i(S_i)), & \text{if } t_0 \leq Y_1 < t_0 + L_r, \\ c_r + c_h(Y_1 - t_0 - L_r) + \sum_{i=1}^{M(Y_1)} g(C(S_i), c_i(S_i)), & \text{if } t_0 + L_r \leq Y_1 < t_0 + t_1, \\ c_r + c_h(t_1 - L_r) + \sum_{i=1}^{M(t_0+t_1)} g(C(S_i), c_i(S_i)), & \text{if } t_0 + L_r < t_0 + t_1 \leq Y_1, \\ c_r + \sum_{i=1}^{M(t_0+L_r)} g(C(S_i), c_i(S_i)), & \text{if } t_0 + t_1 \leq t_0 + L_r \leq Y_1. \end{cases} \tag{15}$$

We are now ready to derive the expressions for $E[Y_1^*]$ and $E[R_1^*]$. First note that we can write

$$\begin{aligned} E[Y_1^*] = & \int_0^\infty \int_0^{t_0} (y + x)\, dF_p(y)\, k_e(x)\, dx \\ & + \int_0^\infty \int_{t_0}^{t_0+x} (t_0 + x)\, dF_p(y)\, k_r(x)\, dx \\ & + \int_0^{t_1} \int_{t_0+x}^{t_0+t_1} y\, dF_p(y)\, k_r(x)\, dx \end{aligned}$$

$$+ \int_0^{t_1} \int_{t_0+t_1}^{\infty} (t_0 + t_1)\, dF_p(y)\, k_r(x)\, dx$$
$$+ \int_{t_1}^{\infty} \int_{t_0+x}^{\infty} (t_0 + x)\, dF_p(y)\, k_r(x)\, dx,$$

which on simplification is equal to

$$F_p(t_0)\,\mu_e + \overline{F}_p(t_0)\,\mu_r + \int_0^{t_0} \overline{F}_p(y)\, dy$$
$$+ \int_0^{t_1} \int_{t_0+x}^{t_0+t_1} \overline{F}_p(y)\, dy\, k_r(x)\, dx.$$

Using the Lemma 1 and the independence of Y_1 and $\{M(t),\ t \geq 0\}$, we can write

$$E[R_1^*] = P(Y_1 < t_0)\left(c_e + c_s \int_0^{\infty} x\, k_e(x)\, dx\right)$$
$$+ \int_0^{t_0} E\left[\sum_{i=1}^{M(y)} g(C(S_i),\, c_i(S_i))\right] dF_p(y)$$
$$+ c_r \int_0^{\infty} P(t_0 \leq Y_1 < t_0 + x)\, k_r(x)\, dx$$
$$+ \int_0^{\infty} \left\{ c_s \int_{t_0}^{t_0+x} (t_0 + x - y)\, dF_p(y)\right.$$
$$\left. + \int_{t_0}^{t_0+x} E\left[\sum_{i=1}^{M(y)} g(C(S_i),\, c_i(S_i))\right] dF_p(y)\right\} k_r(x)\, dx$$
$$+ c_r \int_0^{t_1} P(t_0 + x \leq Y_1 < t_0 + t_1)\, k_r(x)\, dx$$
$$+ \int_0^{t_1} \left\{ c_h \int_{t_0+x}^{t_0+t_1} (y - t_0 - x)\, dF_p(y)\right.$$
$$\left. + \int_{t_0+x}^{t_0+t_1} E\left[\sum_{i=1}^{M(y)} g(C(S_i),\, c_i(S_i))\right] dF_p(y)\right\} k_r(x)\, dx$$
$$+ c_r \int_0^{t_1} P(t_0 + t_1 \leq Y_1)\, k_r(x)\, dx$$

$$+ \overline{F}_p (t_0 + t_1) c_h \int_0^{t_1} (t_1 - x) k_r (x) dx$$

$$+ \int_0^{t_1} \overline{F}_p (t_0 + t_1) E \left[\sum_{i=1}^{M(t_0+t_1)} g (C (S_i), c_i (S_i)) \right] k_r (x) dx$$

$$+ c_r \int_{t_1}^{\infty} P (Y_1 \geq t_0 + x) k_r (x) dx$$

$$+ \int_{t_1}^{\infty} \int_{t_0+x}^{\infty} E \left[\sum_{i=1}^{M(t_0+t_1)} g (C (S_i), c_i (S_i)) \right] dF_p (y) k_r (x) dx$$

$$= F_p (t_0) c_e + F_p (t_0) c_s \mu_e + \int_0^{t_0} \int_0^y h (z) q (z) r (z) dz dF_p (y)$$

$$+ c_r \int_0^{\infty} P (t_0 \leq Y_1 < t_0 + x) k_r (x) dx$$

$$+ \int_0^{\infty} \left[c_s \left(x \overline{F}_p (t_0) - \int_{t_0}^{t_0+x} \overline{F}_p (y) dy \right) \right.$$

$$\left. + \int_{t_0}^{t_0+x} \int_0^y h (z) q (z) r (z) dz dF_p (y) \right] k_r (x) dx$$

$$+ c_r \int_0^{t_1} P (t_0 + x \leq Y_1 < t_0 + t_1) k_r (x) dx$$

$$+ \int_0^{t_1} \left[c_h \left(-(t_1 - x) \overline{F}_p (t_0 + t_1) + \int_{t_0+x}^{t_0+t_1} \overline{F}_p (y) dy \right) \right.$$

$$\left. + \int_{t_0+x}^{t_0+t_1} \int_0^y h (z) q (z) r (z) dz dF_p (y) \right] k_r (x) dx$$

$$+ c_r \int_0^{t_1} P (Y_1 \geq t_0 + t_1) k_r (x) dx$$

$$+ \overline{F}_p (t_0 + t_1) c_h \int_0^{t_1} (t_1 - x) k_r (x) dx$$

$$+ \int_0^{t_1} \overline{F}_p (t_0 + t_1) \int_0^{t_0+t_1} h (z) q (z) r (z) dz k_r (x) dx$$

$$+ c_r \int_{t_1}^{\infty} \overline{F}_p (t_0 + x) k_r (x) dx$$

$$+ \int_{t_1}^{\infty} \overline{F}_p (t_0 + x) \int_0^{t_0+x} h (z) q (z) r (z) dz k_r (x) dx,$$

which on simplification is equal to

$$
\begin{aligned}
F_p(t_0)\,c_e + \overline{F}_p(t_0)\,c_r + c_s \Bigg(& F_p(t_0)\,\mu_e + \overline{F}_p(t_0)\,\mu_r \\
& - \int_0^{\infty} \int_{t_0}^{t_0+x} \overline{F}_p(y)\,dy\,k_r(x)\,dx \Bigg) \\
& + c_h \int_0^{t_1} \int_{t_0+x}^{t_0+t_1} \overline{F}_p(y)\,dy\,k_r(x)\,dx \\
& + \int_0^{t_1} \int_0^{t_0+t_1} \overline{F}_p(y) h(y)\,q(y)\,r(y)\,dy\,k_r(x)\,dx \\
& + \int_{t_1}^{\infty} \int_0^{t_0+x} \overline{F}_p(y) h(y)\,q(y)\,r(y)\,dy\,k_r(x)\,dx. \qquad (17)
\end{aligned}
$$

For the infinite-horizon case we want to find t_1^* which minimizes $B(t_0^*, t_1)$ for a fixed value of t_0^* given by a solution to $w(t_0) = 0$ in (11). Recall that

$$
\begin{aligned}
B(t_0, t_1) = \Bigg\{ & F_p(t_0)\,c_e + \overline{F}_p(t_0)\,c_r + c_s \Bigg(F_p(t_0)\,\mu_e \\
& + \overline{F}_p(t_0)\,\mu_r - \int_0^{\infty} \int_0^{t_0+x} \overline{F}_p(y)\,dy\,k_r(x)\,dx \Bigg) \\
& + c_h \int_0^{t_1} \int_{t_0+x}^{t_0+t_1} \overline{F}_p(y)\,dy\,k_r(x)\,dx \\
& + \int_0^{t_1} \int_0^{t_0+t_1} \overline{F}_p(y)\,h(y)\,q(y)\,r(y)\,dy\,k_r(x)\,dx \\
& + \int_{t_1}^{\infty} \int_{t_0}^{t_0+x} \overline{F}_p(y)\,h(y)\,q(y)\,r(y)\,dy\,k_r(x)\,dx \Bigg\} \\
& \Bigg\{ F_p(t_0)\,\mu_e + \overline{F}_p(t_0)\,\mu_r + \int_0^{t_0} \overline{F}_p(y)\,dy \\
& + \int_0^{t_1} \int_{t_0+x}^{t_0+t_1} \overline{F}_p(y)\,dy\,k_r(x)\,dx \Bigg\}.
\end{aligned}
$$

We see that $dB(t_0^*, t_1)/dt_1 = 0$ if and only if

$$\begin{aligned} J(t_0^*, t_1) = {} & h(t_0^*, t_1)\, q(t_0^*, t_1)\, r(t_0^*, t_1) \\ & \times \left[F_p(t_0^*)\, \mu_e + \bar{F}_p(t_0^*)\, \mu_r + \int_0^{t_0^*} \overline{F}_p(y)\, dy \right. \\ & \left. + \int_0^{t_1} \int_{t_0^*+x}^{t_0^*+t_1} \overline{F}_p(y)\, dy\, k_r(x)\, dx \right] \\ & + c_h \left(F_p(t_0^*)\, \mu_e + \overline{F}_p(t_0^*)\, \mu_r + \int_0^{t_0^*} \overline{F}_p(y)\, dy \right) \\ & - \left\{ F_p(t_0^*)\, c_e + \overline{F}_p(t_0^*)\, c_r + c_s \left(F_p(t_0^*)\, \mu_e \right. \right. \\ & \left. + \overline{F}_p(t_0^*)\, \mu_r - \int_0^{\infty} \int_{t_0^*}^{t_0^*+x} \overline{F}_p(y)\, dy\, k_r(x)\, dx \right) \\ & + \int_0^{t_1} \int_0^{t_0^*+t_1} \overline{F}_p(y)\, h(y)\, q(y)\, r(y)\, dy\, k_r(x)\, dx \\ & \left. + \int_{t_1}^{\infty} \int_0^{t_0^*+x} \overline{F}_p(y)\, h(y)\, q(y)\, r(y)\, dy\, k_r(x)\, dx \right\} = 0. \end{aligned} \tag{19}$$

Next, we discuss optimum ordering policies, which minimize $B(t_0^*, t_1)$ for a fixed value of t_0^* given by a solution to $w(t_0) = 0$ in (11).

THEOREM 2: *Under the assumptions of (1) in Theorem 1 and*

$$h(t_0^* + t_1)\, q(t_0^* + t_1)\, r(t_0^* + t_1)$$

is continuous and strictly increasing in t_1, *we have*

(i) *if* $J(t_0^*, 0) \geq 0$, *then* $t_1^* = 0$,

(ii) *if* $J(t_0^*, 0) < 0$ *and* $J(t_0^*, \infty) > 0$, *there exists a unique* t_1^* *such that* $J(t_0^*, t_1^*) = 0$ *and the expected cost is*

$$B(t_0^*, t_1^*) = c_h + h(t_0^* + t_1^*)\, q(t_0^* + t_1^*)\, r(t_0^* + t_1^*), \tag{20}$$

(iii) *if* $J(t_0^*, \infty) < 0$, *then* $t_1^* \to \infty$.

Proof: Differentiating $J(t_0^*, t_1)$ with respect to t_1 yields

$$J'(t_0^*, t_1) = \left[\frac{d}{dt_1}(h(t_0^*+t_1)\, q(t_0^*+t_1)\, r(t_0^*+t_1))\right] \times \left[F_p(t_0^*)\, \mu_e + \overline{F}_p(t_0^*)\, \mu_r + \int_0^{t_0^*} \overline{F}_p(y)\, dy + \int_0^{t_1} \int_{t_0^*+x}^{t_0^*+t_1} \overline{F}_p(y)\, dy\, k_r(x)\, dx\right] > 0.$$

Therefore, $J(t_0^*, t_1^*)$ is a strictly increasing function of t_1.

Hence

(i) if $J(t_0^*, 0) \geq 0$, then $t_1^* = 0$,

(ii) if $J(t_0^*, 0) < 0$ and $J(t_0^*, \infty) > 0$, there exists a unique t_1^* such that $J(t_0^*, t_1^*) = 0$.

Substituting $J(t_0^*, t_1^*) = 0$ into $B(t_0^*, t_1^*)$ in (18) yields (20),

(iii) if $J(t_0^*, \infty) < 0$, then $t_1^* \to \infty$.

Q.E.D.

3. SPECIAL CASES OF THE POLICY 2

Case 1 ($p(y)=1$, $c_e=c_r=c_0$, $\mu_e=\mu_r=m$, $k_e(x)=k_r(x)=k(x)$): This is the case considered by Park and Park [15]. In this case, if we put $p(y)=1$, $q(y)=0$, $\mu_e=\mu_r=m$, $c_e=c_r=c_0$, $k_e(x)=k_r(x)=k(x)$ in (18), then we have the expression for the expected cost per unit time as

$$\overline{B}(t_0, t_1) = \left\{c_0 + c_s \int_0^{\infty} \int_{t_0}^{t_0+x} F(y)\, dy\, k(x)\, dx + c_h \int_0^{t_1} \int_{t_0+x}^{t_0+t_1} \overline{F}(y)\, dy\, k(x)\, dx\right\} \Big/ \left\{m + \int_0^{t_0} \overline{F}(y)\, dy + \int_0^{t_1} \int_{t_0+x}^{t_0+t_1} \overline{F}(y)\, dy\, k(x)\, dx\right\}, \quad (21)$$

which agrees with equation (5) in Park and Park [15] and can be optimized for t_0 and t_1 as Park and Park [15].

Case 2 ($p(y)=0$, $c_e=c_r=c_0$, $g(C(y), c_i(y))=c$, $k_e(x)=k_r(x)=k(x)$): This is the case considered by Park and Park [14]. In this case, we assume all failures are type I failure (*i. e.* $p(y)=0$, $q(y)=1$). If we put $p(y)=0$, $q(y)=1$,

$c_e = c_r = c_0$, $g(C(y), c_i(y)) = c$, $k_e(x) = k_r(x) = k(x)$ in (18), then we get the following result as Park and Park [14] obtained,

$$B(t_0, t_1) = \left\{ c_0 + c \int_0^{t_1} R(t_0 + t_1)\, k(x)\, dx + c \int_{t_1}^{\infty} R(t_0 + x)\, k(x)\, dx + c_h \int_0^{t_1} (t_1 - x)\, k(x)\, dx \right\} \Big/ \left\{ t_0 + t_1 + \int_{t_1}^{\infty} \int_x^{\infty} k(y)\, dy\, dx \right\}. \tag{22}$$

Case 3 ($t_1 = 0$): In this case, Policy 2 reduces to Policy 1. If we put $t_1 = 0$ in (18), then we get the expression (8).

Case 4 ($t_1 = 0$, $p(y) = 1$, $k_e(x) = k_r(x) = k(x)$ and $k(x)$ is degenerated at L): Osaki [12] considered this case. If we put $t_1 = 0$, $p(y) = 1$ and $k_e(x) = k_r(x) = k(x)$ is degenerated at L in (18), then we get the following result as Osaki [12] obtained,

$$B(t_0) = \frac{c_e F(t_0) + c_r \overline{F}(t_0) + c_s \int_{t_0}^{t_0 + L} F(y)\, dy}{\int_0^{t_0} \overline{F}(y)\, dy + L} \tag{23}$$

Case 5 ($t_1 = 0$, $p(y) = 1$, $k_e(x)$ is degenerated at L_e, $k_r(x)$ is degenerated at L_r): Kaio and Osaki [11] considered this case.

Case 6 ($t_1 = 0$, $g(C(y), c_i(y)) = c_i(y)$, $k_e(x)$ and $k_r(x)$ are degenerated at 0): This is the case considered by Block *et al.* [7].

Case 7 ($t_1 = 0$, $p(y) = 0$, $g(C(y), c_i(y)) = c(y)$, $k_e(x)$ and $k_r(x)$ are degenerated at 0): This is the case considered by Boland [8].

Case 8 ($t_1 = 0$, $p(y) = 0$, $g(C(y), c_i(y)) = c_i$, $k_e(x)$ and $k_r(x)$ are degenerated at 0): Boland and Proschan [9] investigated this case. In particular they considered the cost structure $c_i = a + ic$.

Case 9 ($t_1 = 0$, $p(y) = p$, $g(C(y), c_i(y)) = C$, $k_e(x)$ and $k_r(x)$ are degenerated at 0): This is the case considered by Cléroux *et al.* [10].

Case 10 ($t_1 = 0$, $p(y) = 1$, $k_e(x)$ and $k_r(x)$ are degenerated at 0): This is the classical age replacement considered by Barlow and Proschan [5].

Case 11 ($t_1 = 0$, $p(y) = 0$, $g(C(y), c_i(y)) = c$, $k_e(x)$ and $k_r(x)$ are degenerated at 0): The problem reduces to the classical periodic replacement with minimal repair at failure. Barlow and Hunter [4] considered this case.

Case 12 ($g(C(y), c_i(y)) = C(y) + c_i(y)$): Here the cost of the minimal repair is the age-dependent random part $C(y)$ plus the deterministic part $c_i(y)$ and so $h(y) = E[C(y)] + E[c_{M(y)+1}(y)]$ in (18).

4. A NUMERICAL EXAMPLE

In the numerical analysis here we shall consider the system with the Weibull distribution which is one of the most common in reliability studies. The p.d.f. of the Weibull distribution with shape parameter β and scale parameter θ is given by

$$f(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} \exp\left[-\left(\frac{t}{\theta}\right)^{\beta}\right], \qquad t > 0, \quad \beta, \theta > 0, \tag{26}$$

and the parameter of the distribution will be chosen $\beta = 2$.

The p.d.f. of the random lead time of an expedited order is given by

$$k_e(x) = \frac{1}{\mu_e} \exp\left[-\frac{x}{\mu_e}\right], \qquad x > 0, \quad \mu_e > 0. \tag{27}$$

The p.d.f. of the random lead time of a regular order is given by

$$k_r(x) = \frac{1}{\mu_r} \exp\left[-\frac{x}{\mu_r}\right], \qquad x > 0, \quad \mu_r > 0. \tag{28}$$

Suppose that $g(C(y), c_i(y)) = C(y) + c_i(y)$. Here we discuss a policy where, at failure, one replaces the system or repairs it depending on the random cost C of repair. Let c_∞ be the constant cost. A replacement (type II failure) upon at age y takes place if $C > \delta(y)\, c_\infty$, if $C \leq \delta(y)\, c_\infty$, then on proceeds a minimal repair (type I failure). The parameter $\delta(y)$ can be interpreted as a fraction of the constant cost c_∞ at age y, and $0 \leq \delta(y) \leq 1$. Here we consider the following parametric form of the repair cost limit function $\delta(y) = \delta e^{-ay}$ with $0 \leq \delta \leq 1$ and $a \geq 0$. Suppose that the random repair cost C has a normal distribution $L(.)$ and density $l(.)$, with mean μ and standard deviation σ (the probability of a negative cost is negligible). If an operating system fails at age y, it is either replaced with a new system with probability

$$p(y) = 1 - \int_0^{\delta(y)\, c_\infty} l(x)\, dx \tag{29}$$

or it undergoes minimal repair with probability

$$q(y) = \int_0^{\delta(y)\, c_\infty} l(x)\, dx \tag{30}$$

Then the random part $C(y)$ of minimal repair at age y has density $l_y(x) = l(x)/q(y)$ for $0 \leq x \leq \delta(y)\, c_\infty$ and

$$\begin{aligned} h(y) &= E_{M(y)}\left[E_{C(y)}\left[g\left(C(y),\, c_{M(y)+1}(y)\right)\right]\right] \\ &= E_{M(y)}\left[E_{C(y)}\left[C(y) + c_{M(y)+1}(y)\right]\right] \\ &= \int_0^{\delta(y)\, c_\infty} x\, l_y(x)\, dx + E_{M(y)}\left[c_{M(y)+1}(y)\right] \\ &= \frac{1}{q(y)} \int_0^{\delta(y)\, c_\infty} x\, l(x)\, dx + E_{M(y)}\left[c_{M(y)+1}(y)\right], \end{aligned} \tag{31}$$

where $\{M(y),\, y \geq 0\}$ is a non-homogeneous Poisson process with intensity $q(y)\, r(y)$.

Furthermore, when $c_i(y) = c_3 + c_4 y + c_5 i\, (c_3 \geq 0,\ c_4 \geq 0,\ c_5 \geq 0)$, then

$$\begin{aligned} h(y) = \frac{1}{q(y)} \int_0^{\delta(y)\, c_\infty} x\, l(x)\, dx + c_3 + c_4\, y \\ + c_5 \left(\int_0^y q(x)\, r(x)\, dx + 1\right). \end{aligned} \tag{32}$$

In the numerical analysis we first compute the optimum solution t_0^* which minimizes $B(t_0)$ in (8) for the policy 1. Then, we seek an optimum solution t_1^* which minimizes $B(t_0^*,\, t_1)$ in (18) for a fixed value t_0^* given by a solution to (8). The parameters δ, a and c_h were varied to take the different values in order to see thier influence on the optimal solution. The results are given in Tables 1 and 2. It should be noted that we can check the minimum cost per unit time $B(t_0^*,\, t_1^*)$ in Tables 1 and 2 is correct by putting t_0^* and t_1^* in the expression (20).

In this paper, the repair cost limit function $\delta(y) = \delta e^{-ay}$ is chosen for the purpose of easy computation. From the numerical results, we can derive the following remarks:

Optimal ordering policies with general random repair costs, for the Weibull distribution.

$\beta = 2$, $\theta = 1{,}012$, $\mu_e = 100$, $\mu_r = 120$, $c_r = 1{,}100$, $c_e = 1{,}300$, $c_\infty = 1{,}100$, $c_s = 5$, $C \sim N(700,\ 200^2)$.

TABLEAU 1
$c_i(y) = 0$

δ	a	$q(y)$	c_h	t_0^*	$B(t_0^*)$	t_1^*	$B(t_0^*, t_1^*)$
1	0	0.977	0.10	1,172	1.7278	65	1.7275
10/11	0	0.933	0.10	1,203	1.7077	108	1.7065
9/11	0	0.841	0.10	1,263	1.6787	226	1.6722
8/11	0	0.691	0.10	1,354	1.6549	536	1.6265
7/11	0	0.500	0.10	1,457	1.6563	1,381	1.5974
5/11	0	0.158	1.00	1,566	1.7300	4,273	1.7154
3/11	0	0.001	1.75	1,562	1.7866	2,050	1.7863

TABLEAU 2
$c_i(y) = 0.2y, \ c_i(y) = 0.2y + 2i$

				$c_i(y) = 0.2y$			
δ	a	$q(y)$	c_h	t_0^*	$B(t_0^*)$	t_1^*	$B(t_0^*, t_1^*)$
1	0	0.977	0.10	967	1.9076	85	1.9062
1	0.00015	*	0.10	1,035	1.8642	140	1.8607
1	0.00070	*	∞	1,433	1.6816	0	1.6816
10/11	0	0.933	0.10	990	1.8826	106	1.8802
10/11	0.00010	*	0.10	1,053	1.8430	174	1.8370
10/11	0.00060	*	∞	1,436	1.6866	0	1.6866
9/11	0	0.841	0.10	1,039	1.8407	158	1.8352
9/11	0.00010	*	0.10	1,133	1.7925	365	1.7706
9/11	0.00050	*	∞	1,440	1.6947	0	1.6947
8/11	0	0.691	0.10	1,121	1.7911	288	1.7740
8/11	0.00005	*	0.10	1,176	1.7692	464	1.7367
8/11	0.00036	*	∞	1,439	1.7063	0	1.7063
7/11	0	0.500	0.10	1,235	1.7540	588	1.7111
7/11	0.00005	*	0.10	1,293	1.7408	1,120	1.6769
7/11	0.00024	*	∞	1,442	1.7214	0	1.7214
5/11	0	0.158	0.10	1,463	1.7576	1,155	1.7446
3/11	0	0.001	1.75	1,546	1.7902	96	1.7902

				$c_i(y) = 0.2y + 2i$			
δ	a	$q(y)$	c_h	t_0^*	$B(t_0^*)$	t_1^*	$B(t_0^*, t_1^*)$
1	0	0.977	0.10	964	1.9109	89	1.9095
1	0.00015	*	0.10	1,032	1.8674	139	1.8639
1	0.00070	*	∞	1,433	1.6826	0	1.6826
10/11	0	0.933	0.10	987	1.8880	106	1.8834
10/11	0.00010	*	0.10	1,051	1.8460	172	1.8402
10/11	0.00060	*	∞	1,436	1.6876	0	1.6876
9/11	0	0.841	0.10	1,036	1.8436	158	1.8380
9/11	0.00010	*	0.10	1,131	1.7951	361	1.7735
9/11	0.00050	*	∞	1,440	1.6956	0	1.6956
8/11	0	0.691	0.10	1,118	1.7934	287	1.7762
8/11	0.00005	*	0.10	1,174	1.7713	460	1.7390
8/11	0.00036	*	∞	1,439	1.7073	0	1.7073
7/11	0	0.500	0.10	1,233	1.7555	584	1.7127
7/11	0.00005	*	0.10	1,291	1.7421	1,109	1.6783
7/11	0.00024	*	∞	1,442	1.7221	0	1.7221
5/11	0	0.158	0.10	1,462	1.7580	1,149	1.7450
3/11	0	0.001	1.75	1,546	1.7902	94	1.7902

*: age-dependent.

(i) from Table 1, some improvement can be made in the minimum cost per unit time for the Policy 1 and 2 if one allows for minimal repair at failure;

(ii) from Table 2, the minimum cost per unit time for the Policy 1 and 2 will be reduced when the probability of minimal repairing is age-dependent;

(iii) from Table 2, when holding cost c_h is small, Policy 2 is better than Policy 1 and when holding cost is sufficiently large Policy 2 has optimal solution $t_1^* = 0$ and $B(t_0^*, t_1^*) = B(t_0^*)$;

(iv) it can be seen that the present models are a generalization on previously known policies.

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