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B. ROY
R. SLOWINSKI

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CRITERION OF DISTANCE BETWEEN TECHNICAL PROGRAMMING AND SOCIO-ECONOMIC PRIORITY (*)

by B. Roy (¹) and R. Slowinski (²)

Abstract. — *The problem considered in this paper consists in comparing two partial preorders coming from two quite different points of view. For this purpose, a criterion of distance is proposed. Final form of the criterion is justified by a set of logical and significance conditions. The use of this criterion is illustrated by an example of regional water supply system programming, where the first preorder comes from socio-economic priority and the second one from technical programming.*

Keywords: Decision making, criteria, distance, partial preorders.

Résumé. Critère de distance entre préordres représentant une programmation technique et des priorités socio-économiques. — *Le problème étudié dans cet article concerne la comparaison de deux préordres partiels provenant de deux points de vue instincts. Un critère de distance est proposé dans ce but. La forme finale de ce critère est justifiée par un ensemble comportant d'une part des conditions logiques et, d'autre part, des conditions inhérentes à la signification même de la distance cherchée. L'intérêt de ce critère est illustré sur un cas concret de programmation régionale d'approvisionnement en eau. Dans ce cas, le premier préordre prend en compte des considérations et critères techniques alors que le second traduit des priorités socio-économiques.*

Mots clés : Aide à la décision, critères, distance, préordres partiels.

1. INTRODUCTION

Our interest in the subject of this paper arose from investigation of multicriteria programming of rural water supply systems (*see* Slowinski and Treichel, 1986, 1988).

Construction of rural water supply systems (WSS) shows a tendency to reduce the number of small local installations, supplying one or several farms, in favour of developing bigger installations grouping even several dozen farms, hamlets, villages and food-processing plants. It is due to the fact that rural WSSs have better economic and operational characteristics.

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(¹) LAMSADE, Université de Paris-Dauphine, place du Maréchal De Lattre de Tassigny, 75775 Paris Cedex 16, France.
(²) Technical University of Poznan, Piotrowo 3a, 60-965 Poznan, Poland.

Construction of rural WSS is usually preceded by an analysis of a medium-term decision problem concerning the best use of investment funds and water resources, the most beneficial development of the region and the best improvement of agricultural productivity. This is a complex problem which needs a multicriteria analysis of alternative decisions. This stage of analysis is called WSS programming.

In the decision-aid methodology for dealing with this problem, Slowinski and Treichel (1988) decomposed the programming task into two problems. The first one consists in setting up a priority order in which water users are connected to a new WSS, taking into account economic, agricultural and sociological consequences of the investment. A water user is understood as a topographically compact group of receivers, e. g. a village, a big farm or a food-processing plant. The second problem concerns the selection of a variant of technical construction of the regional WSS evaluated from technical and economic viewpoints.

In the first problem, the users are evaluated using pseudo-criteria (*see* Roy, Vincke, 1984) and the final priority order is a partial preorder. Technical variants considered in the second problem are characterized by location and output of water intakes, lay-out and capacity of main pipeline connections, and by the use of reservoirs, pumps, hydrophores, etc. The variants satisfying users' demands are evaluated using true criteria, *i. e.* traditionally understood criteria which, in contrast to pseudo-criteria, do not involve any thresholds in the comparison. Let us stress that the families of criteria used in both problems are disjoint.

However, the parts of the decomposed decision problem have to be coordinated in the decision-making process. It was observed by Slowinski and Treichel (1988) that due to some precedence constraints, the schedule of connections of users to the WSS construction according to a given technical variant during the investment period is a partial preorder of users. Thus, in order to coordinate both problems of the programming task, the family of criteria used for evaluation of technical variants should be augmented by a criterion expressing the difference between the priority order of users and the order of users following from the technical programming. So, in the second problem, the technical variants are evaluated from the viewpoints of investment and operating costs, reliability and, moreover, from the viewpoint of difference between a partial preorder corresponding to a variant and a partial preorder which is a priority order resulting from the solution of the first problem.

Such a coordinating criterion can be modelled in terms of a measure of distance between two partial preorders coming from different points of view. Construction of this criterion is the subject of the present paper. The construction has, however, nothing in common with the usual techniques of building criteria (*see* Bouyssou, 1990). Although the criterion has been defined in view of the above mentioned application, its definition is general and it can be used to compare any two partial preorders coming from different considerations. In practice, such a comparison arises quite often when one has to deal with a precedence order observing some technical constraints and an order following from a socio-economic preference.

The problem of distance between relations has been considered for at least 30 years. Kemeny (1959, completed by Kemeny and Snell, 1962) proposed an axiomatic characterization of distance of a symmetric difference *(dsd)* between complete orders. The work of Kemeny was continued by Bogart (1973, 1975) who studied the case of transitive relations (specifically, partial orders) and asymmetric relations. Before, Chernyi and Mirkin (1970) considered the case of distance between equivalence relations-still for *dsd.* In fact, all this research derived implicitely from investigation of metric properties of ordered sets. This direction was explored by Barthélémy (1979) who completed the axiomatic characterization of *dsd* in the space of "usual" orderings. Other types of distances were investigated by Cook and Seiford (1978), Armstrong *et al.* (1972), Cook *et al.* (1986), Giakoumakis and Monjardet (1987). Apart from the work of Cook *et al.* (1986 *a* and *b*), this investigation was limited to the comparison of complete orders and preorders.

Let us remark that in all those previous considerations, the authors did not care about the origin of the orders to be compared. We claim, however, that it may influence the axiomatic characterization of the distance. In particular, it is important to distinguish between the case of identical and different points of view (disjoint families of criteria) being at the origin of the two orders.

In the next sections, we shall state two sets of conditions which seem appropriate for derivation of a distance between partial preorders coming from two different points of view. We shall demonstrate that these conditions characterize the distance with two degrees of freedom. Two additional conditions will be proposed in section 5 to have a univocal definition of the criterion. In the final section, a numerical example will illustrate the use of the distance criterion in the context of multicriteria programming of water supply systems.

2. GENERAL DEFINITIONS AND NOTATION

Let A be a finite set of objects and (a_i, a_j) an ordered pair of objects belonging to A. In order to specify a preference (priority) between a_i and a_j, we shall consider three binary relations P, I, R having the following meaning:

$a_i\ P\ a_j$: a_i is preferred to a_j,

$a_i\ I\ a_j$: a_i is indifferent to a_j,

$a_i\ R\ a_j$: a_i is incomparable to a_j.

For each ordered pair (a_i, a_j), one and only one of the following assertions is true: $a_i\ P\ a_j$, $a_j\ P\ a_i$, $a_i\ I\ a_j$, $a_i\ R\ a_j$. For the sake of convenience we shall substitute $a_i\ P^-\ a_j$ for $a_j\ P\ a_i$, where P^- means an inverse preference.

Assuming that P is asymmetric, I is symmetric and reflexive, R is symmetric and $P \cup I$ is transitive, the triple P, I, R defines a partial preorder (*see* Roy, 1985). Let us observe that a particular case of partial preorder P, I, R, where $R = \oslash$, corresponds to what is called complete preorder P, I, *i. e.* to P being a weak order.

Let $Ó_1$ and $Ó_2$ be two partial preorders in set A coming from two different points of view. We want to capture the total divergence between $Ó_1$ and $Ó_2$ which aggregates all elementary divergences defined for the pairs (a_i, a_j) of objects. The elementary divergence appears for the pair (a_i, a_j) if and only if the two objects are differently related in $Ó_1$ and $Ó_2$. The intensity of the elementary divergence, denoted by $d(a_i, a_j)$, depends on the nature of the two relations in question. For instance, the intensity of the elementary divergence between $a_i\ I\ a_j$ and $a_i\ R\ a_j$ can be judged not greater than the one between $a_i\ R\ a_j$ and $a_i\ P\ a_j$. The most intensive elementary divergence appears when $Ó_1$ and $Ó_2$ compare a_i and a_j in a completely contradictory way: $a_i\ P\ a_j$ in $Ó_1$ and $a_i\ P^-\ a_j$ in $Ó_2$. A symbolic definition of $d(a_i, a_j)$ is shown in Table I.

We assume that the sum of elementary divergences, expressed by $d(a_i, a_j)$ for all pairs (a_i, a_j), reflects well the contradictory character of $Ó_1$ and $Ó_2$. Consequently, the distance $g(Ó_1, Ó_2)$ between preorders $Ó_1$ and $Ó_2$ is defined by:

$$g(Ó_1, Ó_2)= \sum_{\substack{a_i,\, a_j \in A \\ i>j}} d(a_i, a_j). \qquad (1)$$

The problem is then to assign appropriate values to elements δ of Table I. For this purpose, in sections 3 and 4, we shall state two kinds of conditions. Taking them into account, we shall determine in section 5 the unknown values of Table I, and consequently, the criterion defined by formula (1).

TABLE I
Symbolic definition of $d(a_i, a_j)$

$\acute{O}_1$ \ $\acute{O}_2$	$a_i\ P\ a_j$	$a_i\ P^-\ a_j$	$a_i\ I\ a_j$	$a_i\ R\ a_j$
$a_i\ P\ a_j$	0	$\delta(P, P^-)$	$\delta(P, I)$	$\delta(P, R)$
$a_i\ P^-\ a_j$	$\delta(P^-, P)$	0	$\delta(P^-, I)$	$\delta(P^-, R)$
$a_i\ I\ a_j$	$\delta(I, P)$	$\delta(I, P^-)$	0	$\delta(I, R)$
$a_i\ R\ a_j$	$\delta(R, P)$	$\delta(R, P^-)$	$\delta(R, I)$	0

3. LOGICAL CONDITIONS

In order to be a distance, $g(\acute{O}_1, \acute{O}_2)$ has to verify three axioms corresponding to the first three following conditions.

L1. $g(\acute{O}_1, \acute{O}_2)=0$ if and only if $\acute{O}_2$ is identical to $\acute{O}_1$: otherwise $g(\acute{O}_1, \acute{O}_2)>0$.

$\acute{O}_1$ is identical to $\acute{O}_2$ if and only if $d(a_i, a_j)=0$ for all pairs (a_i, a_j). Consequently, condition *L1* is verified if and only if:

$$\delta(q_1, q_2) > 0 \text{ for } q_1 \neq q_2, \qquad q_1, q_2 \in \{P, P^-, I, R\}. \tag{2}$$

L2. $g(\acute{O}_1, \acute{O}_2)=g(\acute{O}_2, \acute{O}_1)$.

This is true if and only if Table I is symmetric:

$$\delta(q_1, q_2) = \delta(q_2, q_1) \text{ for all } q_1, q_2 \in \{P, P^-, I, R\}. \tag{3}$$

L3. $g(\acute{O}_1, \acute{O}_2)+g(\acute{O}_2, \acute{O}_3) \geq g(\acute{O}_1, \acute{O}_3)$ (triangular inequality).

Let us consider the particular case where set A is reduced to two objects a_i and a_j, and assume that relations existing among those objects in preorders $\acute{O}_1$, $\acute{O}_2$, $\acute{O}_3$ are q_1, q_2, q_3, respectively. Condition *L3* for this pair of objects takes then the form:

$$\delta(q_1, q_2) + \delta(q_2, q_3) \geq \delta(q_1, q_3). \tag{4}$$

The next condition is not related to classical axioms of a distance.

L4. For the reason of consistency, it is suitable to have:

$$\delta(P, I) = \delta(P^-, I), \qquad \delta(P, R) = \delta(P^-, R). \tag{5}$$

In order to justifiy the first equality of (5), let us consider two objects a_i, a_j which are compared in the following way:

in $\acute{O}_1$: $a_i\ P\ a_j \Leftrightarrow a_j\ P^-\ a_i$,
in $\acute{O}_2$: $a_i\ I\ a_j \Leftrightarrow a_j\ I\ a_i$.

When calculating the distance between $\acute{O}_1$ and $\acute{O}_2$, we can take into account either the ordered pair (a_i, a_j) or the ordered pair (a_j, a_i). In the first case we have to use $\delta(P, I)$, while in the second, $\delta(P^-, I)$. It is clear that the contribution of both ordered pairs to the distance should be the same. The same justification can be made for the second equality of (5) is we substitute R for I.

4. SIGNIFICANCE CONDITIONS

The purpose of the following conditions is to reflect some subjective requirements which do not follow from pure logical reasons but from practical significance of P, I, R appearing in preorders coming from different points of view. Precisely, the significance concerns the intensity of elementary divergences between $a_i\ P\ a_j$, $a_i\ P^-\ a_j$, $a_i\ I\ a_j$ and $a_i\ R\ a_j$.

S1. The contradiction between P and P^- in two different preorders is not smaller than the sum of contradictions between I and P on the one hand, and I and P^- on the other hand. It means that:

$$\delta(P, P^-) \geq \delta(P, I) + \delta(I, P^-). \tag{6}$$

Let us consider three preorders shown in Figure 1. We have

$$\begin{aligned} g(\acute{O}_1, \acute{O}_2) &= d(a_i, a_j) = \delta(P, I), \\ g(\acute{O}_2, \acute{O}_3) &= d(a_i, a_j) = \delta(I, P^-), \\ g(\acute{O}_1, \acute{O}_3) &= d(a_i, a_j) = \delta(P, P^-). \end{aligned}$$

Conditions $S1$ expresses the idea that the contradiction between $\acute{O}_1$ and $\acute{O}_3$ is at least as big as the sum of contradictions between $\acute{O}_1$ and $\acute{O}_2$ on the one hand, and $\acute{O}_2$ and $\acute{O}_3$ on the other hand.

S2. The contradiction between P and P^- in two different preorders is not smaller than the contradiction between P and R which is, in turn, not smaller than the contradiction between I and R. It means that:

$$\delta(P, P^-) \geq \delta(P, R) \geq \delta(I, R). \tag{7}$$

The condition that $\delta(P, P^-)$ is not smaller than $\delta(P, R)$ and $\delta(I, R)$ expresses the idea that the most intensive elementary divergence arises when two objects are compared in an opposite way in the two preorders.

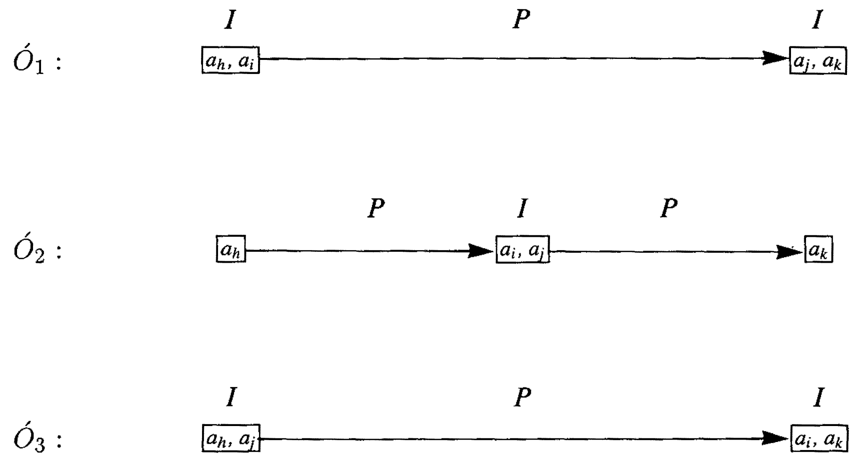
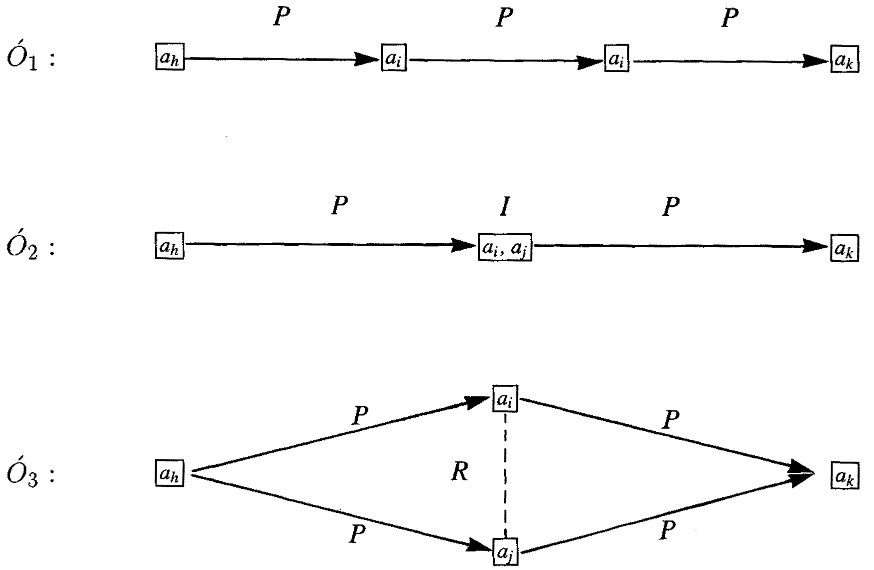


Figure 1.

Among three elementary divergences considered in (7), $\delta(I, R)$ seems to be the less intensive (see *fig.* 2). Indeed, as R reflects an impossibility



$$g\,(\acute{O}_1,\, \acute{O}_3) = d(a_i,\ a_j) = \delta(P,\ R)$$

$$g\,(\acute{O}_2, \acute{O}_3) = d(a_i,\ a_j) = \delta(I,\ R)$$

Figure 2.

of finding any convincing reason for choosing one of relations P, I, P^-, some authors have a tendency to identify I with R, *i. e.* to reduce $\delta(I, R)$ to zero. In contrast to the first tendency, the distance derived from the axiomatic basis of Cook *et al.* (1986 *a*) leads to $\delta(I, R)=\delta(P, R)$. The latter proposal does not seem less appropriate than the former, so we admit that $\delta(P, R) \geq \delta(I, R)$.

Let us observe than the distance of a symmetric difference imposes $\delta(I, R) = \delta(P, P^-) > \delta(P, R)$. For this reason, *dsd* is incompatible with condition *S2*, thus inappropriate for the problem under study.

5. FINAL RESULT

5.1. Domain of variation of $\delta(q_1, q_2)$

There are 12 strictly positive variables $\delta(q_1, q_2)$, $q_1, q_2 \in \{P, P^-, I, R\}$ in Table I (*see L1*). According to *L2* and *L4*, it is sufficient to give a value to 4 of them in order to determine the value of 8 others. These 4 variables are, for example, $\delta(P, I)$, $\delta(P, P^-)$, $\delta(I, R)$, $\delta(P, R)$.

It is not restrictive to let $\delta(P, I)=2$. According to *S1*, $\delta(P, P^-)=4$. For the sake of convenience, let $\delta(I, R)=x$ and $\delta(P, R)=y$ (*see* Table II). Due to *S2*, we have

$$4 \geq y \geq x. \tag{8}$$

If we apply *L3* (precisely, (4)) to the triangle $(a_i\ P\ a_j)\,(a_i\ R\ a_j)\,(a_i\ P^-\ a_j)$, we get (see *fig.* 3):

$$y + y \geq 4 \quad \text{or simply, } y \geq 2. \tag{9}$$

If we consider now the triangle $(a_i\ P\ a_j)\ (a_i\ I\ a_j)\ (a_i\ R\ a_j)$, we obtain (see *fig.* 3):

$$2 + x \geq y. \tag{10}$$

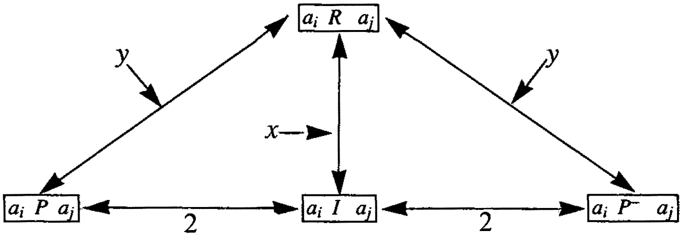


Figure 3.

After aggregating (2), (8), (9), (10), we finally obtain:

$$0 < \max\{2,\ x\} \leq y \leq \min\{4,\ 2 + x\} \tag{11}$$

which defines the domain of variation of the two unknowns x and y (see *fig.* 4).

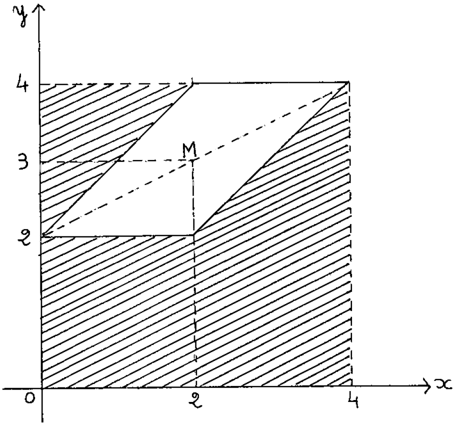


Figure 4.

Taking into account definition (1), Table II and formula (11), it can be seen that for any partial preorders $\acute{O}_1$, $\acute{O}_2$:

$$0 \leq g(\acute{O}_1,\ \acute{O}_2) \leq 2n(n-1)$$

where n is the cardinality of set A. Moreover, $g(\acute{O}_1,\ \acute{O}_2) = 0$ if and only if $\acute{O}_1$ and $\acute{O}_2$ are identical (*see L1*), and $g(\acute{O}_1,\ \acute{O}_2) = 2n(n-1)$ if and only if $\acute{O}_1$ and $\acute{O}_2$ are two exactly inverse complete orders.

TABLE II

Definition of $d(a_i, a_j)$ taking into account L1, L2, L3, L4, S1, S2. ((x, y) have to verify (11)).

$\acute{O}_1$ \ $\acute{O}_2$	a_i P a_j	a_j P a_i	a_i I a_j	a_i R a_j
a_i P a_j	0	4	2	y
a_j P a_i	4	0	2	y
a_i I a_j	2	2	0	x
a_i R a_j	y	y	x	0

Let us observe that:

– in the case of complete preorders, the distance criterion defined above is reduced to the classical symmetric difference;

– the distance obtained by Cook *et al.* (1986 *a*) corresponds to the one defined above for $x=y=2$.

5.2. Solution finally adopted

One may wish, as it is the case of our application presented in section 1, not to keep intervals in the definition of the distance criterion. To give a value to x and y, one has to introduce some additional restrictions, perhaps more questionable than the previous ones.

Let us remark that according to (11), x may be greater or lesser than 2. We find no convincing argument neither for $\delta(I, R) > \delta(I, P)$ nor for $\delta(I, R) < \delta(I, P)$. Moreover, having

$$a_i\ I\ a_j \text{ in } \acute{O}_1,\ a_i\ R\ a_j \text{ in } \acute{O}_2 \text{ and } a_i\ P\ a_j \text{ in } \acute{O}_3,$$

we consider that $\acute{O}_1$ contradicts $\acute{O}_2$ and $\acute{O}_3$ to the same extend. This leads to adopt

$$\delta(I, R) = \delta(I, P) = x = 2. \tag{12}$$

Taking into account (11) and (12), we get

$$2 \le y \le 4.$$

In order to fix the position of y within the interval [2, 4], we can compare the differences $\delta(P, P^-) - \delta(P, R)$ and $\delta(P, R) - \delta(P, I)$. We find, however, no argument which would justify that one of these differences is greater than another. Thus, it seems natural to consider them as equal:

$$\delta(P, P^-) - \delta(P, R) = \delta(P, R) - \delta(P, I).$$

It follows that

$$\delta(P, R) = y = 3.$$

Let us notice that the values finally selected: $x=2$ and $y=3$ correspond to the central point M of the feasible domain defined by (11) (see *fig*. 4).

6. NUMERICAL EXAMPLE

Let us come back to the WSS programming problem presented in the introduction. We shall consider an example of 10 water users (villages or big farms). Our aim is to use this example in order to present a part of the methodology for WSS programming which is concerned with the use of the proposed distance measure. In particular, we want to show the context in which the distance measure plays a coordinating role between two problems of the programming task. The complete methodology is presented in another paper by Roy *et al.* (1991).

The first problem of the programming task consists in finding a priority order of users taking into account the following criteria:

c_1: water defficiency in the user's area,

c_2: farm production potential,

c_3: function and standing of the user,

c_4: structure of settlements in the user's area,

c_5: water demands,

c_6: share of water supply installations in overall investments concerning the user,

c_7: possibility of connecting the user to another existing WSS.

Evaluation of the users by the above criteria is shown in Table III.

TABLE III
Evaluation of water users.

Criterion / Water user	c_1	c_2	c_3	c_4	c_5	c_6	c_7
a_1	1	1.9	0	0.055	151	0	0
a_2	0	2	6	0.066	320	2	0
a_3	1	2.125	0	0.06	164	2	0
a_4	2.5	2.25	12	0.278	873	5	0
a_5	3	2.05	4	0.1	147	5	0
a_6	0.5	2.25	12	0.433	916	5	0
a_7	4.5	2.2	6	0.167	568	0	0
a_8	1.5	2.05	20	0.187	570	5	5
a_9	2	2.2	6	0.167	440	2	2
a_{10}	1	2.25	6	0.187	518	2	2

In order to model preferences with respect to particular criteria, we introduce indifference and preference thresholds on $c_1, c_2, ..., c_7$. Such models of preferences are called pseudo-criteria. A global model of preferences is a fuzzy outranking relation obtained using ELECTRE III which involves, moreover, veto thresholds and importance indices of criteria (*see* Roy, 1978 and Roy *et al.*, 1986). All these additional data are listed in Table IV.

TABLE IV
Thresholds on criteria and their relative importance.

Criterion	Indifference threshold	Preference threshold	Veto threshold	Importance index
c_1	2	3.5	7	8
c_2	$0.05\ c_2$	$0.1\ c_2$	$0.5\ c_2$	5
c_3	0	4	8	6
c_4	$0.07 + 0.1\ c_4$	$0.1 + 0.2\ c_4$	$0.4 + 0.25\ c_4$	6
c_5	$20 + 0.05\ c_5$	$50 + 0.1\ c_5$	$100 + 0.2\ c_5$	6
c_6	2	4	10	2
c_7	0	2.5	10	1

An exploration of the fuzzy outranking relation using a distillation procedure of ELECTRE III leads to a partial preorder of users shown in figure 5. It is, of course, a final solution of the first problem, *i. e.* the priority of users $\acute{O}_p$. The boxes represent equivalence classes and arrows represent preferences. The water users belonging to two boxes are incomparable if there is no path between them.

The second problem of the programming task concerns the selection of a variant of technical realization of the regional WSS evaluated from technical and economic view-points. In order to create a set of variants, a special generator is used which takes into account water demands, maximum capacities of possible system components and a potential distribution network. This network is composed of all possible main pipeline connections between sources and users. Each generated variant determines a selection of water sources and a subnetwork of pipeline connections which satisfy water demands. The variants are characterized by investment and operating costs depending on the type of water intakes, pumps, water treatment facilities, the way of storing water and the routing of pipeline connections.

Coming back to our example, the potential distribution network and possible location of water sources and reservoirs are shown in figure 6.

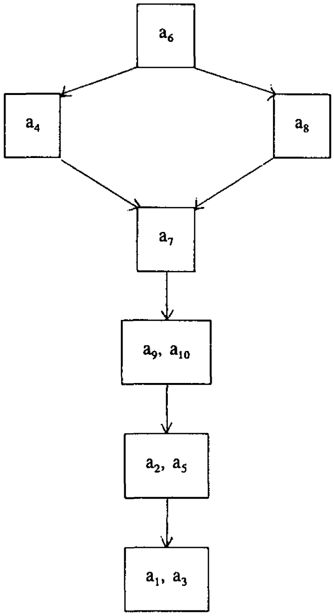


Figure 5. – Priority order of users $\acute{O}_p$ resulting from ELECTRE III.

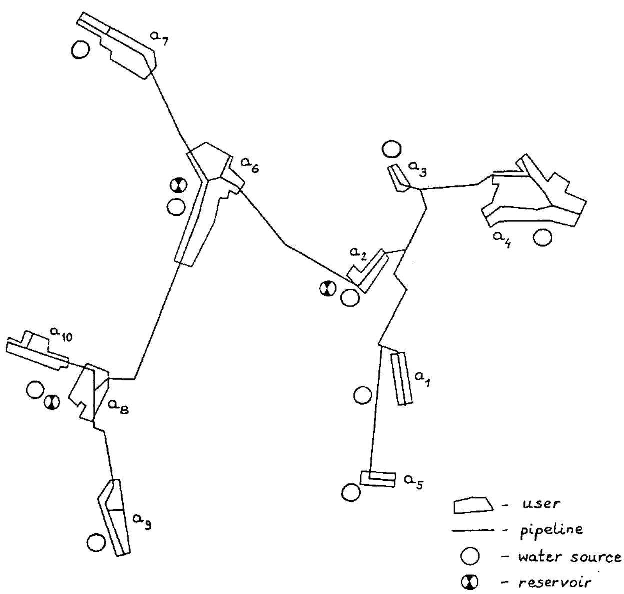


Figure 6. – Potential distribution network.

Nine technical variants which satisfy water demands have been generated from the full network. For each variant, it is possible to determine an order of connections of users to the WSS. A user's rank in the order depends on its location in relation to water sources and others users, as well as on the schedule of technical construction of the WSS according to a given variant. The orders corresponding to 9 variants are partial preorders $\acute{O}_{v1}$, $\acute{O}_{v2}$, ..., $\acute{O}_{v9}$, shown in figure 7.

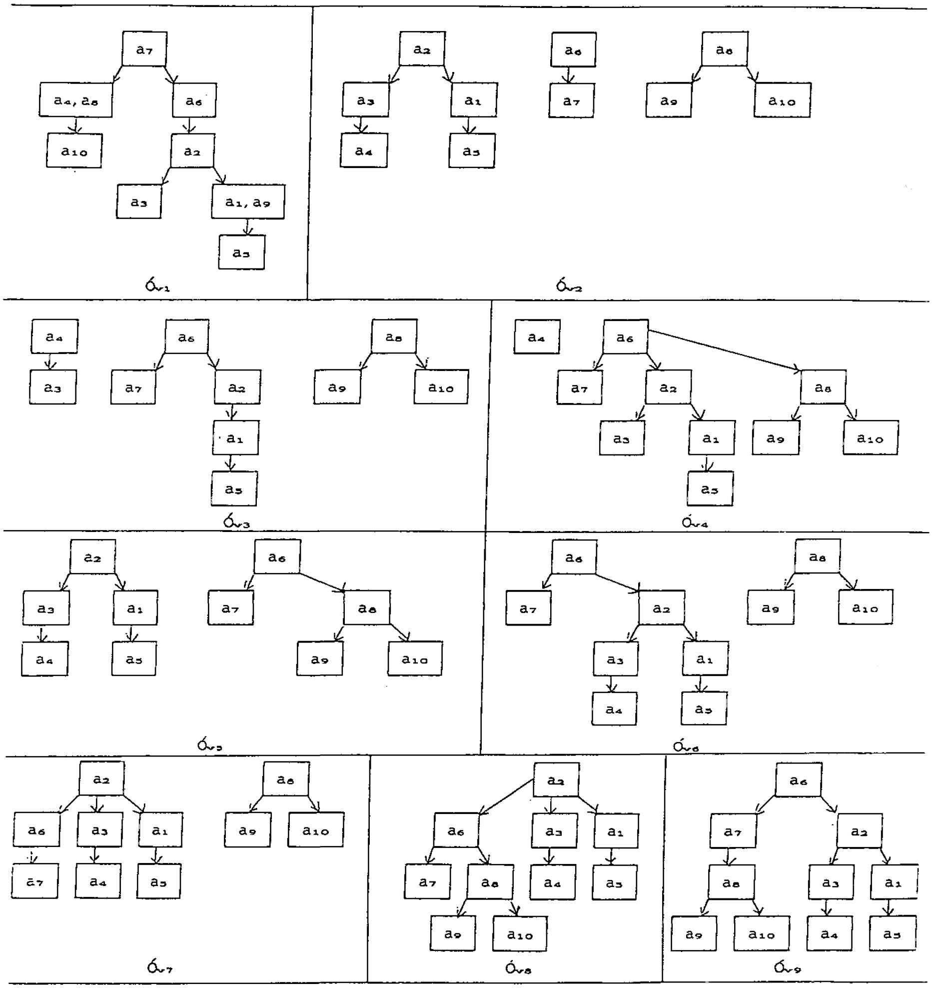


Figure 7. – Technical programming orders $\acute{O}_{v1}$, $\acute{O}_{v2}$, ..., $\acute{O}_{v9}$.

Using the criterion of distance proposed in this paper (with $x=2, y=3$), we can compare technical programming orders $\acute{O}_{v1}, \acute{O}_{v2}, \ldots, \acute{O}_{v9}$ with priority order of users $\acute{O}_p$.

Evaluation of 9 technical variants using investment and operating costs criteria and the distance criterion is shown in Table V.

TABLE V
Evaluation of technical variants.

Criterion / Variant	Investment cost	Operating cost	Distance criterion $g(\acute{O}_p, \acute{O}_v)$
v_1	191	32	87
v_2	88	19	117
v_3	93	19	106
v_4	83	15	94
v_5	94	15	108
v_6	79	16	102
v_7	78	16	119
v_8	94	15	113
v_9	97	13	88

As can be seen from Table V, a final selection of the best variant is not trivial since the cheapest technical variant is not the closest to the priority order of users $\acute{O}_p$ (*see* variant v_7). The selection can be performed using one of available methods, like ELECTRE I (*see* Roy, 1985, and Goicoechea *et al.*, 1982) or PREFCALC (*see* Jacquet-Lagrèze, 1990). This question is, however, beyond the scope of the present paper.

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