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COMBINED REPLACEMENT MODELS (*)

by T. NAKAGAWA (1)

Abstract. — *This paper summarizes ten replacement models which combine three basic replacements: (i) age replacement; (ii) block replacement; (iii) periodic replacement with minimal repair at failure. The expected cost rates of each model are derived, using the usual calculus method of probability. As an example, we give an optimum policy to minimize the expected cost rate of one model.*

Keywords: Replacement; Three policies; Expected cost; Optimization.

Résumé. — *Cet article résume dix modèles de renouvellement combinant : (i) âge de renouvellement; (ii) renouvellement par bloc; (iii) renouvellement périodique avec réparation minimale en cas de panne. Les taux moyens de coût de chaque modèle sont calculés en utilisant les méthodes probabilistes classiques. Comme exemple, nous donnons la politique optimale pour minimiser le taux moyen d'un des modèles.*

1. INTRODUCTION

Failure of a unit during actual operation is sometimes costly or dangerous. It is important to replace an operating unit before its failure. Three replacement policies were defined and studied for an infinite time horizon by Barlow and Proschan [2]. Under these policies, we assume that replacement and repair times are negligible.

(i) Age replacement

A unit is replaced at scheduled time T after its installation or at failure, whichever occurs first. The expected cost rate is:

$$C_1(T)=[c_1+c_2 F(T)] \Big/ \int_0^T \bar{F}(t)\,dt, \qquad (1)$$

where $F(t)$ = distribution of the failure time of a unit, where $\bar{F} \equiv 1-F$, c_1 = cost of replacement, c_2 = additional cost of replacement for a failed unit.

(ii) Block replacement

A unit is replaced at scheduled times kT $(k=1, 2, \ldots)$ and at failure.

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The expected cost rate is:

$$C_2(T)=[c_1+c_3 M(T)]/T, \tag{2}$$

where $M(t)$ = expected number of failures during $(0, t]$, c_3 = cost of replacement for each failed unit.

(iii) Periodic replacement

A unit is replaced at scheduled times $kT (k=1, 2, \ldots)$. Minimal repair is made at failures between successive replacements, so that the failure rate of a unit remains undisturbed by any repair of failures. The expected cost rate is:

$$C_3(T)=[c_1+c_4 R(T)]/T, \tag{3}$$

where $R(t)$ = cumulative failure rate of the failure time distribution $F(t)$, i. e., $R(t) \equiv \int_0^t r(u)\,du$ where $r(t)$ $(\equiv f(t)/\bar{F}(t))$ is a failure rate and f is a density of F, c_4 = cost of minimal repair for each failed unit.

Many authors studied modified or extended models of the above policies and discussed optimum policies; e. g., [6, 8, 17, 20] for age replacement, [4, 9, 12, 22] for block replacement, and [3, 10, 14, 24] for periodic replacement. Similarly, it is of great interest to consider combined models. For example, a unit is replaced at time T or at N-th failure, whichever occurs first, where $(N-1)$-th previous failures are corrected with minimal repair [21]. The model corresponds to age replacement when $N=1$ and to periodic replacement when $N=\infty$. If there exists an N^* $(1<N^*<\infty)$ which minimizes the expected cost rate, this has a lower cost rate than two basic models.

This paper considers ten replacement models which combine age, block and periodic replacements, and obtains the expected cost rates of each model. It is difficult to discuss optimum policies for such models. As an example, we pick up only model 4 and derive optimum replacement times which minimize the expected cost rate.

2. COMBINED POLICIES

(1) The unit is replaced at scheduled time T or at N-th failure, whichever occurs first, where $(N-1)$-th failed units are replaced by a new one. Then, the

expected cost rate is easily given by:

$$C_{12}(T, N)=\frac{c_1+c_2 F^{(N)}(T)+c_3 \sum_{j=1}^{N-1} F^{(j)}(T)}{\int_0^T [1-F^{(N)}(t)]\,dt}, \tag{4}$$

where $F^{(j)}(t)=j$-fold convolution of $F(t)$ with itself ($j=1, 2, \ldots$), and:

$$F^{(0)}(t)=1 \quad \text{for} \quad t \geqq 0, \qquad 0 \quad \text{for} \quad t<0.$$

This corresponds to age replacement when $N=1$ and to block replacement when $N=\infty$.

(2) The unit is replaced at each failure during $(0, T_0]$ and at scheduled time T for $T \geqq T_0$. If the unit fails during (T_0, T), it is replaced by a new unit before time T. Then, the probability that the unit fails in an interval (T_0, T) is, from Ross ([19], p. 45),

$$\Pr\{\gamma(T_0) \leqq T-T_0\}=F(T)-\int_0^{T_0} \bar{F}(T-t)\,dM(t),$$

where $\gamma(t)$ = remaining life of the unit at time t in a renewal process. The mean time to replacement after time T_0 is:

$$\int_{T_0}^{T} t d_t \Pr\{\gamma(T_0) \leqq t-T_0\}+T \Pr\{\gamma(T_0)>T-T_0\}$$

$$=T_0+\int_{T_0}^{T}\left[\bar{F}(t)+\int_0^{T_0} \bar{F}(t-u)\,dM(u)\right] dt.$$

Thus, the expected cost rate is:

$$C_{12}(T, T_0)=\frac{c_1+c_2\left[F(T)-\int_0^{T_0} \bar{F}(T-t)\,dM(t)\right]+c_3 M(T_0)}{T_0+\int_{T_0}^{T}\left[\bar{F}(t)+\int_0^{T_0} \bar{F}(t-u)\,dM(u)\right] dt}. \tag{5}$$

This corresponds to age replacement when $T_0=0$ and to block replacement when $T=T_0$.

(3) The unit is replaced at time T or at N-th failure, whichever occurs first, after its installation. The unit undergoes only minimal repair at failures between replacements. Then, the expected number of failures before replacement is, from Morimura [10],

$$\sum_{j=0}^{N-1} jp_j(T)+(N-1)\int_0^T p_{N-1}(t)r(t)\,dt = N-1-\sum_{j=0}^{N-1}(N-1-j)p_j(T),$$

where $p_j(t)=\{[R(t)]^j/j!\}\,e^{-R(t)}$, which represents the probability that j failures occur in $(0, t]$. The mean time to replacement is:

$$\int_0^T tp_{N-1}(t)r(t)\,dt+T\sum_{j=0}^{N-1}p_j(T)=\sum_{j=0}^{N-1}\int_0^T p_j(t)\,dt.$$

Then, the expected cost rate is:

$$C_{13}(T, N)=\frac{c_1+c_2\sum_{j=N}^{\infty}p_j(T)+c_4\left[N-1-\sum_{j=0}^{N-1}(N-1-j)p_j(T)\right]}{\sum_{j=0}^{N-1}\int_0^T p_j(t)\,dt}. \tag{6}$$

This corresponds to age replacement when $N=1$ and to periodic replacement when $N=\infty$. In particular, when $T=\infty$, i.e., the unit is replaced only at N-th failure, the optimum policy was derived by Morimura [10] and Nakagawa [16], and Park [18] in Weibull case.

(4) The unit undergoes only minimal repair at failures during $(0, T_0]$. If the unit fails in (T_0, T), it is replaced by a new unit, while if the unit does not fail in (T_0, T), it is replaced at scheduled time T. The mean time to replacement is:

$$\frac{1}{\bar{F}(T_0)}\int_{T_0}^T t\,dF(t)+T\frac{\bar{F}(T)}{\bar{F}(T_0)}=T_0+\frac{1}{\bar{F}(T_0)}\int_{T_0}^T \bar{F}(t)\,dt.$$

Thus, the expected cost rate is:

$$C_{13}(T, T_0)=\frac{c_1+c_2\{[F(T)-F(T_0)]/\bar{F}(T_0)\}+c_4R(T_0)}{T_0+\int_{T_0}^T \bar{F}(t)\,dt/\bar{F}(T_0)}. \tag{7}$$

This corresponds to age replacement when $T_0=0$ and to periodic replacement when $T=T_0$. The optimum policy was discussed by Tahara and Nishida [21], and Nakagawa [14] when T is constant.

(5) Consider the unit with two types of failures [3]. When the unit fails, type 1 failure occurs with probability $\alpha(0 \leqq \alpha \leqq 1)$ and is removed by minimal repair, and type 2 failure occurs with probability $\beta(\equiv 1-\alpha)$ and is removed by replacement. The unit is replaced at time T or at time of type 2 failure, whichever occurs first. The expected number of type 1 failures before replacement is:

$$\sum_{j=0}^{\infty} j\alpha^j p_j(T)+\sum_{j=0}^{\infty} j\alpha^j \beta \int_0^T p_j(t)\, r(t)\, dt=(\alpha/\beta)\, F_\beta(T),$$

where $F_\beta(t) \equiv 1-e^{-\beta R(t)}$. The mean time to replacement is:

$$T\sum_{j=0}^{\infty} \alpha^j p_j(T)+\sum_{j=0}^{\infty} \alpha^j \beta \int_0^T t p_j(t)\, r(t)\, dt=\int_0^T \bar{F}_\beta(t)\, dt.$$

Thus, the expected cost rate is:

$$C_{13}(T;\ \alpha)=\frac{c_1+c_2 F_\beta(T)+c_4(\alpha/\beta)\, F_\beta(T)}{\int_0^T \bar{F}_\beta(t)\, dt}. \tag{8}$$

This becomes the same age replacement model by replacing $F_\beta(t)$ and $c_2+c_4(\alpha/\beta)$ into $F(t)$ and c_2, respectively. Further, this corresponds to age replacement when $\alpha=0$ and to periodic replacement when $\alpha=1$.

(6) Consider a system with two types of units which operate statistical independently. When unit 1 fails, it undergoes minimal repair and begins to operate again. When unit 2 fails, the system is replaced. Unit 1 has a failure time distribution $G(t)$ with failure rate $h(t)$, i. e., $h(t) \equiv g(t)/\bar{G}(t)$, where g is a density of G, and unit 2 has $F(t)$. Then, the expected cost rate is easily given by:

$$C_{13}(T;\ G)=\frac{c_1+c_2 F(T)+c_4 \int_0^T \bar{F}(t)\, h(t)\, dt}{\int_0^T \bar{F}(t)\, dt}. \tag{9}$$

This corresponds to age replacement when $G(t) \equiv 0$ and to periodic replacement when $F(t) \equiv 0$.

(7) The unit is replaced at failures during $(0, T_0]$ and at scheduled time T for $T \geqq T_0$. If the unit fails during (T_0, T), it undergoes minimal repair. The expected number of failures during (T_0, T) is:

$$\int_0^{T_0} \left[\int_{T_0}^{T} r(t-u)\, dt \right] d_u \Pr\{\delta(T_0) \leqq T_0 - u\}$$

$$= \bar{F}(T_0)[R(T) - R(T_0)] + \int_0^{T_0} [R(T-u) - R(T_0 - u)]\, \bar{F}(T_0 - u)\, dM(u),$$

where $\delta(t)$ = age of the unit at time t in a renewal process. Thus, the expected cost rate is, from [12],

$$C_{23}(T, T_0) = \frac{\left\{ \begin{array}{l} c_1 + c_3 M(T_0) + c_4 \left\{ \bar{F}(T_0)[R(T) - R(T_0)] \right. \\ \left. \quad + \int_0^{T_0} [R(T-u) - R(T_0 - u)]\, \bar{F}(T_0 - u)\, dM(u) \right\} \end{array} \right\}}{T} \tag{10}$$

This corresponds to periodic replacement when $T_0 = 0$ and to block replacement when $T = T_0$.

(8) In periodic replacement, we have assumed that the failure rate of an operating unit remains undisturbed by any repair of failures. Suppose that the age of the unit after minimal repair becomes at $(a \geqq 0)$ when it was t before failure. The expected number of failures during $(0, T]$ is, from [15],

$$R(T;\, a) = \sum_{j=1}^{\infty} \int_{t_1 < t_2 < \ldots < t_j \leqq T} \int \frac{f(at_1 + t_2 - t_1)}{\bar{F}(at_1)} \frac{f[a^2 t_1 + a(t_2 - t_1) + t_3 - t_2]}{\bar{F}[a^2 t_1 + a(t_2 - t_1)]}$$

$$\ldots \frac{f[a^{j-1} t_1 + a^{j-2}(t_2 - t_1) + \ldots + a(t_{j-1} - t_{j-2}) + t_j - t_{j-1}]}{\bar{F}[a^{j-1} t_1 + a^{j-2}(t_2 - t_1) + \ldots + a(t_{j-1} - t_{j-2})]}\, dt_1\, dt_2 \ldots dt_j$$

The expected cost rate is:

$$C_{23}(T;\, a) = [c_1 + c_4 R(T;\, a)]/T. \tag{11}$$

Noting that $R(T;\ 0)=M(T)$ and $R(T;\ 1)=R\ (T)$, we easily see that this corresponds to block replacement when $a=0$ and to periodic replacement when $a=1$. If $a<1$ then the unit is younger at each minimal repair and if $a>1$ then it is worce than before failure.

(9) Consider the replacement model of a cumulative damage model [11, 23]: Assume that random variables $X_j\,(j=1, 2, \ldots)$ are associated with a sequence of inter-arrival times between successive shocks, and random variables $W_j\,(j=1, 2, \ldots)$ denote the amount of damage produced by the j-th shock. It is assumed that $\{W_j\}$ are non-negative, independent and identically distributed, and W_j is independent of $X_i\,(i\neq j)$. The unit fails only when the total amount of damage exceeds a failure level K.

Suppose that $\Pr\{X_j\leqq t\}\equiv F(t)$ and $\Pr\{W_j\leqq x\}\equiv G(x)$ $(j=1, 2, \ldots)$. The unit is replaced at scheduled time T or at failure, whichever occurs first. Then, the probability that the unit is replaced at failure is:

$$\sum_{j=1}^{\infty} F^{(j)}(T)\,[G^{(j-1)}(K)-G^{(j)}(K)],$$

and the expected number of shocks before replacement is:

$$\sum_{j=1}^{\infty} (j-1)\,F^{(j)}(T)\,[G^{(j-1)}(K)-G^{(j)}(K)] + \sum_{j=1}^{\infty} j\,G^{(j)}(K)\,[F^{(j)}(T)-F^{(j+1)}(T)] = \sum_{j=1}^{\infty} F^{(j)}(T)\,G^{(j)}(K).$$

The mean time to replacement is:

$$\sum_{j=1}^{\infty} [G^{(j-1)}(K)-G^{(j)}(K)]\int_0^T t\,dF^{(j)}(t) + T\sum_{j=0}^{\infty} G^{(j)}(K)\,[F^{(j)}(T)-F^{(j+1)}(T)] = \sum_{j=1}^{\infty} [G^{(j-1)}(K)-G^{(j)}(K)]\int_0^T [1-F^{(j)}(t)]\,dt.$$

Thus, the expected cost rate is:

$$C_{12}(T;\ K)=\frac{c_1+c_2\sum_{j=1}^{\infty}F^{(j)}(T)[G^{(j-1)}(K)-G^{(j)}(K)]+c_3\sum_{j=1}^{\infty}F^{(j)}(T)\,G^{(j)}(K)}{\sum_{j=1}^{\infty}[G^{(j-1)}(K)-G^{(j)}(K)]\int_0^T[1-F^{(j)}(t)]\,dt},\qquad (12)$$

where c_3 = cost suffered for one schock. This corresponds to age replacement when $K=0$ and to block replacement when $K=\infty$.

(10) In model 9, we assume that each shock occurs in a non-homogeneous Poisson process (e. g., *see* Çinlar [5], p. 97), i. e.,

$$\Pr\{X_1+X_2+\ldots+X_j\leqq t\}=\sum_{n=j}^{\infty}p_n(t).$$

In a similar way of obtaining (12), the expected cost rate is:

$$C_{13}(T;\ K)=\frac{c_1+c_2\sum_{j=1}^{\infty}p_j(T)[1-G^{(j)}(K)]+c_3\sum_{j=1}^{\infty}p_j(T)\sum_{n=1}^{j}G^{(n)}(K)}{\sum_{j=1}^{\infty}[G^{(j-1)}(K)-G^{(j)}(K)]\sum_{n=0}^{j-1}\int_0^T p_n(t)\,dt}.\qquad (13)$$

This corresponds to age replacement when $K=0$ and to periodic replacement when $K=\infty$. Further, note that models 9 and 10 become models 1 and 3, respectively, in particular cases of $G^{(j)}(K)=1$ for $j\leqq K$, 0 for $j>K$ and $K=N-1$.

3. OPTIMUM POLICY

Almost all models considered here become a problem of minimizing an objective function with two independent variables, which extends three basic replacement problems. It is very difficult to discuss optimum policies for such models. As an example, we take up an optimization problem which minimizes the expected cost rate $C_{13}(T,\ T_0)$ of model 4. Tahara and Nishida [21] and Adachi and Kodama [1] tried to obtain optimum times T_0^* and T^*, however, the proof was partially incomplete and troublesome to understand.

We seek optimum replacement times T_0^* and T^* which minimize $C_{13}(T,\ T_0)$ in (7). Suppose that the failure rate $r(t)$ is strictly increasing to infinity and differentiable. Then, differentiating $C_{13}(T, T_0)$ with T and T_0 and setting them equal to zero, respectively, we have:

$$\left[r(T)\int_{T_0}^{T}\bar{F}(t)\,dt+\bar{F}(T)\right]/\bar{F}(T_0)=c_4/c_2, \tag{14}$$

$$c_2 T_0 r(T)-c_4 R(T_0)=c_1+c_2-c_4. \tag{15}$$

A necessary condition that finite T and T_0 minimize $C_{13}(T, T_0)$ are that they satisfy (14) and (15).

(i) Suppose that $c_2<c_4<c_1+c_2$. Letting:

$$q(T;\ T_0)\equiv\left[r(T)\int_{T_0}^{T}\bar{F}(t)\,dt+\bar{F}(T)\right]/\bar{F}(T_0),$$

we evidently have:

$q(T_0;\ T_0)=1<c_4/c_2$,

$\lim_{T\to\infty} q(T;\ T_0)=\infty$,

$$dq(T;\ T_0)/dT=r'(T)\int_{T_0}^{T}\bar{F}(t)\,dt/\bar{F}(T_0)>0.$$

Thus, there exists a finite and unique T^* $(T_0<T^*<\infty)$ which satisfies (14) for a fixed T_0. Further,

$$\frac{dq(T;\ T_0)}{dT_0}=-r(T)+\frac{r(T_0)}{\bar{F}(T_0)}\left[r(T)\int_{T_0}^{T}\bar{F}(t)\,dt+\bar{F}(T)\right]<0, \tag{16}$$

since $r(T_0)<[F(T)-F(T_0)]/\int_{T_0}^{T}\bar{F}(t)\,dt$ for $T>T_0$ from the assumption that $r(t)$ is strictly increasing. This implies that T^* is an increasing function of T_0.

Next, prove that a solution T_0^* to (15) exists and is unique, when $T^*(T_0)$ is given by a function of T_0 from (14). First, noting that $r(T)/r(T_0)>q(T;\ T_0)$ for $0\leqq T_0<T<\infty$ from (16), we have the inequality;

$$r(T^*(T_0))/r(T_0)>c_4/c_2. \tag{17}$$

Differentiating the left-hand side of (15) with T_0 and recalling that $T^*(T_0)$ is an increasing function of T_0, we have:

$$c_2\, r(T^*(T_0)) - c_4\, r(T_0) + c_2\, T_0\, r'(T^*(T_0))\,[T^*(T_0)]' > 0.$$

Further, for some $\tilde{T}_0 < T_0$,

$$c_2\, T_0\, r(T^*(T_0)) - c_4\, R(T_0) > c_2\, \tilde{T}_0\, r(T^*(T_0)) - c_4\, R(\tilde{T}_0) \to \infty \text{ as } T_0 \to \infty,$$

because, in this case, $T^*(T_0) \to \infty$. Therefore, the left-hand side of (15) is strictly increasing from 0 to infinity, and hence, there exists a finite and unique T_0^* which satisfies (15).

(ii) If $c_4 \geqq c_1 + c_2$ then $T_0^* = 0$ from (15) and the model becomes age replacement.

(iii) If $c_4 \leqq c_2$ then $T^* = T_0$ from (14) and the model becomes periodic replacement.

From the above results, if $c_2 < c_4 < c_1 + c_2$ then model 4 has a lower cost rate than two basic models, and finite optimum times T_0^* and T^* $(0 < T_0^* < T^* < \infty)$ are given by the unique solutions of two equations (14) and (15).

4. CONCLUSIONS

We have considered ten replacement models for an infinite time horizon, which combine three basic replacements. As further problems, these models would offer interesting topics to reliability theoreticians. However, it is very difficult to discuss optimum policies for such models. One method of obtaining the optimum policy for model 4 could be helpful in solving a problem of minimizing an objective function with two variables.

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