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Addendum

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R.A.I.R.O.
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ADDENDUM

Consider the following stationary renewal process. There are $k + 1$ channels, each occupied by an individual whose survival function is $s(t)$. Each time an occupant dies he is replaced immediately by a newborn baby (with survival function $s(t)$).

Under stationary conditions the channels must be independent of each other. i.e. the joint age distribution must be a product of the individual distributions. Therefore, when one occupant dies (and is replaced by a newborn) all we know about the k survivors is that they are a randomly distributed sample, with respect to age, from a stationary population with the survivability function $s(t)$. If we wait till the next death we still know nothing more precise about the age of the survivors than their being a randomly distributed sample. We can imagine the $k + 1$ individuals within a black box out of which the dead ones exit and into which the replacements enter. And this is the scenario, repeated at each instant of a death, of our Renewal Theorem!

The independence of the channels implies that the age density at the next observed death (as described in the Theorem) is that of the stationary population, characterized by $s(t)$, which supplies the newborn babies, and that the expected time between successive deaths must obviously equal the average lifespan divided by $k + 1$. *If our Renewal Theorem were not valid then the age distribution of the survivors and the total lifespan distribution could be influenced by a passive observer watching the system from death to death.*

However, the Theorem is not intuitively obvious to most people. The reason is, no doubt, that the $k + 1$ individuals are not an independent sample, after a *specified* time lapse following the last death (and replacement by a newborn), *conditional* upon there being no death till then. Somehow, this dependency is felt to carry over to the survivors at the instant of death. (Of course, the $k + 1$ individuals have randomly distributed ages following a specified time interval after a death if no condition is imposed which excludes additional deaths before the lapse of this interval.)

The Renewal Theorem is therefore of didactic and methodological interest by showing that the scenario is a segment of a stationary renewal process with independent channels. This imbedding into a continuous process allows to dispense with the detailed derivation and to use instead the general reasoning of this Addendum. Admittedly, some people have found this general reasoning unconvincing and elusive, and insisted on the detailed analytical proof.