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NOTES & DÉBATS

A note on Pérez de Moya's newly discovered Principios de Geometria (1584)

M. Céu Silva & Antoni Malet

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NOTES & DÉBATS

A NOTE ON PÉREZ DE MOYA'S *PRINCIPIOS DE GEOMETRIA* (1584)

M. Céu Silva & Antoni Malet

Abstract. — In 1584, the Spanish mathematician Juan Pérez de Moya published a geometric work strongly practical in character titled *Principios de geometria, de que se pod[r]an aprovechar los estudiosos de artes liberales, y todo hombre que su officio le necessitare a tomar la regla y co[m]pas en la mano. Con el medir, y dividir tierras.* While a copy is now kept in the National Library of Lisbon, as far as we know no Spanish public library holds a copy of it, nor is there any reference to it in the secondary bibliography. In this article we briefly describe the *Principios de geometria*, pointing to the main differences with Moya's other geometric texts.

Résumé (Note sur les *Principios de geometria* (1584) de Pérez de Moya)

En 1584, le mathématicien espagnol Juan Pérez de Moya a publié une géométrie pratique intitulée *Principios de geometria, de que se pod[r]an aprovechar los estudiosos de artes liberales, y todo hombre que su officio le necessitare a tomar la regla y co[m]pas en la mano. Con el medir, y dividir tierras.* Un exemplaire en est conservé à

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M. Céu Silva, Centro de Matemática da Universidade do Porto (CMUP), R. Campo Alegre, 687. 4169-007 Porto, Portugal.

Courrier électronique : mcsilva@fc.up.pt

A. Malet, Universitat Pompeu Fabra, c. Ramon Trias Fargas, 25. 08005 Barcelona, Spain.

Courrier électronique : antoni.malet@upf.edu

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la Bibliothèque nationale de Lisbonne. Pour autant que nous sachions, aucune bibliothèque publique espagnole n'en possède un exemplaire et le texte n'est jamais cité dans la bibliographie secondaire. Dans cet article nous présentons une brève description des *Principios de geometria*, en nous arrêtant tout particulièrement sur les différences avec d'autres travaux géométriques de l'auteur.

1. INTRODUCTION

As is well known, Juan Pérez de Moya (Santisteban del Puerto, 1513 – Granada, 1596) was one of the most popular sixteenth-century mathematical authors in the Spanish Monarchy, then a vast and loose aggregate of European principalities and kingdoms and of newly conquered territories in America. A Roman Catholic priest and a competent mathematics teacher, Pérez de Moya addressed his many writings mostly to a public of mathematical practitioners.[1] Besides his mathematical writings, he also published in 1585 an interesting *Philosophia secreta: donde debaxo de historias fabulosas, se contiene mucha doctrina, provechosa a todos los estudios: con el origen de los idolos o dioses de la gentilidad*, a work that draws inspiring moral lessons from classical sources and mythology. His best known and most influential work doubtless was his *Arithmetica practica y especulativa*, first published in 1562 and countless times reprinted through the mid eighteenth century. The National Library of Lisbon keeps a work by Pérez de Moya titled *Principios de geometria, de que se pod[r]an aprovechar los estudiosos de artes liberales, y todo hombre que su officio le necessitare a tomar la regla y co[m]pas en la mano. Con el medir, y dividir tierras.*[2] Printed in Madrid in 1584 by Francisco Sánchez, it features Moya's authorship in the title page and is dedicated to Juan Baptista Gentil, a patron of Pérez de Moya who supported the publication costs of Moya's *Philosophia secreta.* The *Principios de geometria* is a little octavo volume, bound in parchment, with 127 numbered sheets of which the first is missing. It ends with a non-numbered sheet reading: "Printed in Madrid, in the house of Francisco Sanchez. Year one thousand

1 On Pérez de Moya, see [Leal y Leal 1972], [Clavería 1995] and [Valladares Reguero 1997].

2 The book belongs to the Biblioteca Nacional de Lisboa and is catalogued as Res. 6553P. It is referenced in the catalogue *O Livro Científico dos Séculos XV e XVI, Ciências Físico-Matemáticas na Biblioteca Nacional* [Leitão & Martins 2004]. This catalogue was printed following the exhibition *Conta, Peso e Medida: A Ordem Matemática e a Descrição Física do Mundo*, opened to the public in the National Library of Lisbon from 2 December 2004 through 5 March 2005.

and five hundred eighty four". [3] What makes this little book interesting is the fact that apparently no Spanish public library holds a copy of it, nor does it appear in the *Catálogo Colectivo del Patrimonio Bibliográfico Español* (CCPBE) [4], nor is it mentioned in any bio-bibliography of Pérez de Moya examined by us or in general bibliographies—excepted the very recent one edited by V. Navarro et al. [5] Classical sources like [Fernández de Navarrete 1851], [Picatoste Rodriguez 1999], [Domínguez Berrueta 1899], and López Piñero's *Diccionario histórico de la ciencia moderna en España*, as well as the recent and careful articles by [Leal y Leal 1972], [Clavería 1995], and [Valladares Reguero 1997], which have so much contributed to clarify Pérez de Moya's life and social context, contain no reference to Moya's *Principios de geometria*. Not surprisingly, therefore, there is no secondary literature on it. More generally, it is also true that Moya's geometry has scarcely received any attention so far. In what follows, we present a brief description of the *Principios de geometria* introduced by a few considerations about Pérez de Moya's approach to geometry in general.

2. PÉREZ DE MOYA'S GEOMETRY

Pérez de Moya first dealt with geometrical topics in the *Libro quarto* of his *Arithmetica practica y speculatiua* [Pérez de Moya 1562]. In keeping with the general orientation of this work, geometry was presented in three short chapters of a utilitarian character under the title "On the rules of practical geometry useful for measuring estates". [6] Essentially, they contain some results related to land measuring. With little modifications, these topics reappear in the many successive editions of Pérez de Moya's influential *Arithmetica practica y speculatiua*. [7]

The brief extension devoted to geometry in the *Arithmetica practica y speculatiua* should not mislead us as to the general interest that Pérez de

3 *Impresso en Madrid en casa de Francisco Sanchez. Ano de mil y quinientos y ocheta y quatro.*

4 The on-line Catálogo Colectivo del Patrimonio Bibliográfico Español is found at `http://www.mcu.es/ccpb/index.html`. Consulted on 31 July 2008.

5 [Navarro Brotons et al. 1999]. The book was already referenced in PORBASE, the union catalogue of Portuguese libraries, which can be consulted on-line at `http://opac.porbase.org`. Consulted on 31 July 2008.

6 *Trata algunas reglas de Geometria pratica necessarias para el medir de las heredades.*

7 Keeping account only of editions whose year of printing is known, there are four editions in the 16th century, eleven editions in the 17th, and ten in the 18th (plus the modern, critical 20th-century edition).

Moya had for the subject. [8] In fact, the quantity and variety of information given in these few pages suggests that he had already in mind the composition of the more comprehensive account of geometrical results that he penned down a few years later. In 1568 he published *Obra intitulada Fragmentos Mathematicos*, a two-volume work entirely devoted to geometrical and astronomical topics. [9] The first volume, dated 1568, is devoted to practical geometry. The second, printed in 1567, deals with astronomy, geography, natural philosophy, navigation, and gnomonics. Pérez de Moya used both volumes as authoritative sources of reference for his other works, including later editions of his popular *Arithmetica practica*. [10]

Pérez de Moya's ambitious goal in writing the *Fragmentos Mathematicos* was to provide a comprehensive encyclopedic treatment of all the mathematics needed in the liberal arts. Yet, according to Moya's preface addressed to the reader, difficulties related to the cost of printing had not allowed him to complete his plan:

> "It happened to me, in this work (benevolent and prudent reader) the same that happened to a man who undressed his petticoat to be able to jump further away, but did jump less. I am saying this because when I determined to begin this work, I was encouraged to include in it all the subjects concerning the liberal arts. But when I saw the cost, and the several casting types required, I decided to shorten it, and I composed a little book entitled Fragmentos Mathematicos" [...]. [11]

This "little" book (Pérez de Moya calls it *obrezilla*, "tiny work") has about 380 sheets, including a 47-sheet index. [12] For our purposes, the most important chapters are those dealing with geometry and planimetry,

8 Notice that in the 1998 edition of the *Aritmética prática y speculativa* [Baranda 1998] this chapter occupies only 12 of the 618 pages of the treatise.

9 *Obra intitulada Fragmentos Mathematicos, en que se tratan cosas de Geometria y Astronomia, y Geographia, y Philosophia natural, y Sphera, y Astrolabio, y Nauegacion, y Reloxes* (Salamanca, 1567–1568). Its two volumes bear the titles, *Libro primeiro — Geometría Práctica* (1568) and *Libro segundo — trata de cosas de astronomia y geographia y philosophia natural y sphera y astrolabio y navegación y reloges* (1567).

10 See, for example [Pérez de Moya 1569] where Moya says: "*Si dixere bolas en lugar de quadrad, lee cubicar. Lee sobre esta Geometria el tractado que intitulamos Fragmentos mathematicos*". ("If you said balls instead of squares, you should read [the computation of] cubatures. About this Geometry, see our treatise titled *Fragmentos mathematicos.*")

11 "*Acontescido me há en esta obra (benévolo y prudente Lector) lo q̃ dizen acontescio al que se quito el sayo para poder saltar mas, y salto menos. Digo esto, porq̃ quando determine començar la, tenia animo de no dexar materia tocãte a las artes liberales de q̃ cũplidamẽte no tratasse. Mas quando cõsidere la costa, y vários moldes q̃ eran menester, acorte el camiño, y cõpuse una obrezilla intitulada Fragmentos Mathematicos* [...]" in "*El Bachiller Ivan Perez de Moya a los lectores*", letter written in 1567, December [Pérez de Moya 1568].

12 There are about 760 pages: 271 in the first book and 487 in the second.

included in the first volume. We briefly resume here their main contents. Pérez de Moya dedicates 40 chapters to geometry proper. He studies there the triangle (definition and construction of triangles) and the circle (how to construct the diameter and the centre of a circle); the determination of the circle circumscribed to a polygon (triangle, square and pentagon); the division of segments in equal parts; the geometric construction of the square root. Moya also deals with elementary problems of quadrature; he shows how to draw a circle whose area equals that of a square whose side is given; he teaches how to divide a circle in parts and conversely how to find a circle that is $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, etc. of a given circle; he also makes reference to the problem of the duplication of the cube and the determination of other multiples of it.

Only one chapter is devoted to planimetry.[13] In it, Pérez de Moya teaches the determination of areas of polygons (from the triangle to the polygon of 15 sides), of circular and oval figures, and applies this knowledge to the solution of such practical problems as the measuring of fields and farms, of cloth tissue, of the material needed to pave a floor or to construct a roof, and so on.

In 1573, Pérez de Moya published in Alcalá de Henares the *Tratado de Mathematicas: en que se contienen cosas de Arithmetica, Geometria, Cosmographia y Philisophia* [sic] *natural*, dedicated to the king of the Spanish Monarchy, Philip II [Pérez de Moya 1573]. It is a magisterial treatise that includes all the mathematics the author had previously published in Castilian, but also the main parts of his *Silva*, a work he published in Latin in 1557.[14] In it Pérez de Moya corrected some errors contained in previous works and offered additional information on some subjects [Pérez de Moya 1573, El Bachiller Ivan Perez de Moya, al letor]. The first of its three parts is dedicated to arithmetic and algebra. The second part is a treatise of geometry, *Tratado de Geometria practica y speculativa*. The last part, titled *Tratado de cosas de Astronomia, y Cosmographia, y Philosofia natural*, includes different topics of astronomy, cosmography, and natural philosophy. Because of the variety and quantity of the information offered in the treatise, it appears that the author had eventually managed to find the resources necessary to publish the encyclopedic work he could not publish years before.

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13 This chapter is divided into "*artículos*".

14 In Moya's words: "*Y asi va agora este libro como una summa de lo que se há hecho en lengua vulgar, y lo mejor y mas importante de las cosas naturales que pusimos en un librillo de Latin intitulado* Sylva" [Pérez de Moya 1573, El Bachiller Ivan Perez de Moya, al letor]. The Latin work Moya mentions here bears the title *Silva, eutrapelias id est Comitatis et urbanitatis ex variis probatae fidei Authoribus & vitae experimentis* (Valladolid 1557).

In his prefatory letter to the reader, Pérez de Moya stresses the importance of geometry in "all kinds of letters" ("*todo genero de letras*") and in the "mechanical arts", adding that it is necessary for almost any job. He thinks that geometry is important to make the "Logician" understand some propositions studied at schools; it is useful to the "Legist" because it teaches how to divide fields and farms; it is necessary for the "Soldier" and the "Astrologer" to calculate distances and measure altitudes and depths; finally to the "Architects" and all the other "jobs" (*officios*), which will find many important topics in it.[15]

The second part, *Tratado de Geometria practica, y speculativa*, has some 250-odd sheets divided in four books. The first book concerns elementary geometry; the second deals with altimetry, called "the first kind of measure"; the third deals with planimetry (the second kind of measure); and the fourth and last with stereometry.

The geometrical chapters within the 1568 *Fragmentos Mathematicos* are not easily compared with the 1573 *Tratado de Geometria practica, y speculativa*. From one work to the other the author has rearranged the chapters and, sometimes, reorganized the contents of corresponding chapters. It is nonetheless true that the 1573 *Tratado de Geometria* includes practically everything that appears in the 1568 *Fragmentos*, while some topics studied in the *Tratado de Geometria* are missing from the *Fragmentos*. This is the case with some questions dealing with the construction of polygons; with the determination of the difference in area between the polygons with the same number of sides that are circumscribed and inscribed to a circle; with the "proportion" between the circle inscribed into a geometrical "figure with many sides" and the corresponding circumscribed circle. In planimetry, the 1573 *Tratado* includes considerations about the areas of geometric solids not found in the 1568 *Fragmentos*.

Pérez de Moya's main concern is always practical and didactical, which leads him to put convenience and expediency over everything else. He warns the reader that sometimes his language is not above criticism as concerns strict mathematical truth, and yet he is deliberately using it because it seems the most suitable one for the tyro. This is in particular the case when he deals with the side and the diagonal of the square, or when he explains how to calculate the area of the circle. He is aware that some people might criticize him for "lack of rigor", as he acknowledges his text to contain some "improprieties":

15 Moya states several propositions as, for example: "Every triangle has three angles the sum of which is equal to two right angles" ("*Omnis triangulus habet três ângulos equales duobis rectis*") [Pérez de Moya 1584, pp. 44–45].

> "[...] Please we warned that you may notice some improprieties existing here, as when in chapter nineteenth of the first b[ook] of Geometry it is said that I show how to get to know the diameter of a square by means of its side; in which case as one of them is incommensurable with the other, it is improper to use this word "to know". And further on I say that I show the quadrature of the circle: while it is impossible to know it precisely. But I used these words to make the beginners know what it is possible to do with human ability". [16]

3. THE *PRINCIPIOS DE GEOMETRIA* (1584)

We can now turn to the small geometrical volume now kept in Lisbon. The *Principios de geometria* is divided in two books, the first of which occupies the first 72 sheets and the second the remaining 55. We do not know the title of the first book, as its first sheet is missing. The second book "[d]eals with things concerning that kind of measure called Planimetry, which concerns the measuring and dividing of lands". [17] Both books are organized in chapters (*Capítulos*), and a few chapters are still divided in articles (*Artículos*). [18]

As Moya acknowledges, his *Principios* of 1584 contains a simplified version of the second book of his 1573 *Tratado de Geometria*. The *Principios* of 1584 is a very practical work aiming at an audience of surveyors, whom the author explicitly mentions as the ones most to profit from its second book [Pérez de Moya 1584, Segundo Libro, Cap. XIII]. The small size of the volume, the reduced number of diagrams it contains, the poor quality of the printing generally (particularly as compared with the 1573 *Tratado*), the format and style of its presentation (see below), everything leads us to suggest that the work must have been a cheap one, and one easily packed and carried around by its users. The price is missing in the place left to

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16 (...) *auisoles que podran luego examinar algunas impropriedades que aquí hallaran, assi como decir en el capitulo diez y nueve del primero li[bro]. d[e] Geometria, que muestro saber el diametro de un cuadrado por la costa* [*i.e.* the side], *en lo qual como sea lo uno incommensurable, con lo otro, es impropriedad usar destre nombre saber. Y mas adelãte digo, q̃ muestro la quadratura del circulo: como no sea posible saber-se precissamente. Mas use destes terminos por decir dello a los principiãtes, lo que humanamente se puede hazer.* [Pérez de Moya 1573, El Bachiller Ivan Perez de Moya, al letor].

17 "*Trata cosas pertenecientes al genero de medida q̃ dizen Planimetria, q̃ pertenece al medir, y dividir tierras*" [Pérez de Moya 1584, Segundo Libro, p. 72v].

18 The first *Libro* has 28 chapters, of which only *Capitulo* XXVII is further subdived in 10 articles. The *Libro segundo* has 23 *Capítulos*, and only the XVII is divided in 6 *Artículos*.

announce it.[19] In any case, in different places we find (and we do not find it in Moya's other geometrical works) an explicit preoccupation with providing detailed, specific, practical, and non-equivocal rules to solve specific mensuration problems. Here Moya also explicitly dismisses problems with a mere theoretical interest. For instance, in dealing with the determination of the area of a triangle, he gives but one single method, because, he says, he wants the user not to be confused about which method to use (in other works Moya provided different methods):

> "There are so many procedures for, and particularities in measuring triangles, that it could be confusing to the understanding of the measurers to refer to them here; as I put them in another book, now I only include a general rule to measure a triangle, whatever it is, if the measure of its sides is known".[20]

In another place, Pérez de Moya mentions the calculation of the areas of equal parts of a circle, which calculation he has provided in another work. Here, he does not want to deal with the subject since it is not of interest to surveyors: "[this calculation] is here not included, since it is of no use for surveyors, at whom I mostly thought when I wrote this second book".[21] As said above, the *Principios de geometria* summarizes the contents of the second book of the 1573 *Tratado de Geometria.* Yet, it is not the case that Moya just took some chapters from his great treatise and put them together to produce his new book. In fact, although it is true that the contents of *Principios de geometria* are found in the *Tratado de Geometría*, there are enough differences between them to make the *Principios* a new or different work.

We notice among the differences the more detailed explanation of some subjects in the *Principios de geometria.* For instance, it devotes three chapters (IX, XII and XIII) to carefully develop the properties of the triangles, while the *Tratado de Geometria* quickly summarizes them in only one chapter (XVI). We notice also Moya's concern with the applications of geometry to practical matters. Thus he teaches how to make an *esquadra*, a material instrument based on the concept of perpendicularity [Pérez de Moya 1584, pp. 46–47]. He takes care that no unknown notion becomes

19 In the front page we can see "*Tassado a ... el pliego*", although the price is not mentioned.

20 "*Para medir triangulos, ay tantos modos, y primores que quererlos referir aqui seria confundir los entendimientos de algunos medidores, con los muchos preceptos, los quales por averlos puesto en outro volumen, solo pondre una regla general para medir cualquier triangulo de qualquiera suerte y genero que sea, con solo la noticia de sus lados*" [Pérez de Moya 1584, Segundo Libro, p. 85].

21 "*[Este cálculo] no pongo aqui, porque no haze al propósito al medidor de tierras, para quien mi intento principal fue escrevir este segundo libro*" [Pérez de Moya 1584, Segundo Libro, Capítulo XIII, p. 89v]).

an obstacle for anyone lacking the basics. He thinks, for instance, that in the *Principios de geometria* he must introduce angles before dealing with the division of a segment in two or more parts—while in the *Tratado de Geometria* he introduced them afterwards. Furthermore, the *Tratado de Geometria* as well as the *Fragmentos Mathematicos* include a concluding section of solved problems (31 in the first and 28 in the second), where geometric concepts are applied to altimetry, planimetry and stereometry. However, in the *Principios de geometria* the examples follow immediately the study of each corresponding subject.

4. SOURCES

Pérez de Moya quotes a rather large number of sources along the text, sometimes by mentioning only the author's name, without reference to his work. With difference, Euclid is the author most often quoted (sometimes, the *Elements* is not explicitly mentioned but only the number of book and proposition). The first Book of the *Elements* is the one most cited in the *Principios de geometria*, but there are also references to Books III, IV and VI (in the first book of *Principios*) and to Books II and V (in the second book). Pérez de Moya does not specify which edition of the *Elements* he uses, but both Campanus's and Zamberti's are good candidates. In the *Fragmentos Mathematicos*, Campanus's is explicitly mentioned in one comment [Pérez de Moya 1568, p. 61], while Zamberti's is mentioned in the *Arithmetica practica y speculatiua* [Baranda 1998, p. 83].

Besides Euclid, Pérez de Moya mentions other authors. Aristotle's *Physics*, Book II, is mentioned in relation to the sum of the angles of a triangle. Archimedes appears in connection with the relation between the area and the circumference of a circle (see below). Ptolemy's *Almagest* is mentioned in a geometrical demonstration. Sebastiano Serlio and one Pinola [Giacomo Barozzi da Vignola ?] appear in references to architecture.[22] Vitruvius (book 3, cap. 1), Columella (book 5, cap.1) and Pliny (book II, cap. 23, doubtless of the *Natural History*) are mentioned in

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22 Sebastiano Serlio (1475-1554) was a Bolognese architect and painter. Pérez de Moya does not mention the title of Serlio's work, but the third and fourth books of Serlio's architecture were published in Castilian in 1552 as *Tercero y cuarto libro de architectura de Sebastiano Serlio. En los quales se trata de las maneras de como se pude adornar los hedificios: con los exemplos de las antiguedades. Agora nuevamente traducido de Toscano en Romance Castellano por Francisco de Villalpando* (Toledo, en casa de Iuan de Ayala, 1552); further editions in Toledo, 1565 and 1573.

relation to units of measure. [23] We know that Pérez de Moya knew many more sources, for in the geometrical part of the *Fragmentos Mathematicos* of 1568 he quoted Peletier, Tartaglia, Cardano, Dürer, and Sacrobosco.

5. SOME COMMENTS ON THE TEXT

5.1. *Definitions*

The first book of Moya's *Principios de geometria* presents the elements of geometry (point, line, surface, solid, angle, term, figure, circle, diameter, semicircle, portion of circle, sector of circle, area) and its foundations (definitions, petitions and common notions). As other contemporary authors did, Pérez de Moya accompanies the mathematical or abstract definition of a concept with its practical meaning or interpretation. For instance, in his definition of point we read:

> "A point is a thing that has no part, this is to say [i]t is so small a thing that its length, and breadth, and deepness cannot be seen or divided, nor in a half, in the third part, or in any other part however small, because the point is no quantity, but only a single term deliberately imagined to denote the beginning, the middle, or the end of a line. And this is why they say that a point is a thing that does not fill a place nor can it be seen or divided in parts, and this is how the Mathematician understands it. The layman understands the point as a sign done with ink, or other thing, in this way . which, however small, and faint it is made, can be seen and divided". [24]

We find the same pattern in Moya's definition of line:

> "Line, that in Spanish is called a scratch (*raya*) in common speech, is a length without breadth or deepness. The ends or extremities of a line are points. We say that its ends or extremities are two points, to distinguish it from the circular line, which has no extremities. We imagine the origin of the line from the expanse that is deliberately assumed to run from point to point. And the line is a thing so thin regarding its breadth (and since it is an imagined *raya* [it can

23 Pérez de Moya does not mention titles of these authors' works, but in all probability he was making reference to Vitruvius's *De architectura,* Columella's *De re Rustica,* and Pliny's *Naturalis Historia.*

24 "*Punto es una cosa que no tiene parte, quiere dezir. Que es una cosa tan pequeña, q̃ su largura, y anchura, y profundidad, no se puede ver ni diuidir: en mitad, ni tercio, ni en otra ninguna parte por pequeña que sea: porque el pũto no es quantidad, mas un termino simple imaginado intencionalmente: para denotar el principio, medio o fin de alguna linea. Y por esto dizen, que el punto es una cosa que no ocupa lugar ni se puede ver, ni dividir en partes, y desta s[u]erte lo entiende el Mathematico. El natural entiende por punto, una señal hecha con tinta o con otra cosa deste modo . El qual por muy pequeño, y delicado que se haga se puede ver, y dividir*". [Pérez de Moya 1584, p. 2v].

be assumed] as long as we want) that anything, however delicate it be, is wider and deeper. The line, as the point was, can be understood in two ways, one is the Mathematician's, and the other is the layman's. The Mathematician understands by line a thing with length but without breadth or deepness, and in which these two things cannot be seen, because its breadth is only imagined [by] the understanding to indicate with it the beginning, the middle or the end of a surface. The layman understands by line a scratch [*raya*] done with ink, or without, like this ——— Which mark, however delicate and subtle it is made, has breadth that can be seen and divided". [25]

As we have seen, Pérez de Moya's first definitions of point and line follow those of the *Elements*, although we also find a reference to the line as being the "path" of a point in motion. His definition of a straight line makes it the shortest path between two points:

"Straight line, which means unbent [line], is a (sic) shortest expanse from a point to another, whose extremities are those points. In such a way that, if from one point to the other, lines are drawn in the mind or in ink or in other ways, the one that makes the shortest possible path will be called recta or straight line and all the others that do not go by this shortest path, will be called curved, or tortuous, or crooked". [26]

5.2. *Angles and polygons*

The *Principios de geometria* deals with elementary questions about angles, segments of straight line (their division in two or more equal parts, subtraction, etc), and triangles (its construction and classification). For a given triangle, the fundamental relation between its sides is specified (each side is less than the sum of the other two), and between its angles (the sum of

25 "*Linea, que en Español dezimos raya generalemente hablãdo[,] es una largura sin anchura ni profundidad. Los terminos o fines dela qual son dos puntos. Dize que sus fines o estremos son dos puntos: para differencia dela linea circular, que carece de terminos. El origem dela linea se imagina dela estension que intẽncionalmente se finxe correr de un punto a otro. Y es una cosa la linea tan pequeña segũ su anchura (que puesto que es raya imaginada quan larga quisieremos) que no ay cosa por delicada que sea, que no tenga mayor groseza y anchura. Entiendesse la linea como el punto en dos modos uno según el Mathematico, y otro según el natural. El Mathematico entiende por linea una cosa que teniendo largura no tiene anchura ni profundidad, ni en ellas estas dos cosas se pueden ver, porque solo se imagina su anchura intẽncionalmente como el entendim[i]ento para denotar cõ ella el principio, o m[e]di[o], o fin de una superficie. El natural entiende por linea una raya hecha con tinta, o sin ella desta manera. ——— La qual por delicada y sutil que se haga se puede su anchura ver y dividir*". [Pérez de Moya 1584, p. 3v].

26 "*Linea recta, q̃ quiere dezir derecha, es una brevissima ẽxtension de un punto a otro, que recibe a los dichos puntos en sus extremos. De suerte, que si de un punto a otro con el entendimiento, o con tinta o con otra cosa se echaren rayas, la que fuere por el mas breve camino que ser puede se dira recta, o derecha, y todas las que no fueren por este mas breve camino se diran curvas, o tuertas, o torcidas*" [Pérez de Moya 1584, p. 3v].

the angles is equal to two right angles). It also deals with the construction of quadrilaterals, the regular pentagon, and other regular polygons with more than five sides (see below).

5.3. *Measuring fields*

As pointed out, Moya pays much attention to planimetry and the determination of areas, the transformation of a triangle in a circle with the same area, the division in equal parts of terrains of various shapes (triangular, quadrangular, hexagonal, circular); and the addition and subtraction of polygonal figures. As is typical in works of this kind, Moya also deals with the computation of distances (between two distant points, between heights, and so on). We also find a rather comprehensive list of units of measure, with their equivalences: "*onça, palmo, dicha, espithema, deunx, pie, paso, passada, pertica, orgya, codo, plethrum, iugero, estádio, diaulos, dolicos, schenus, parafanga, stathmos, milha romana, milla alemã, milla grade, lapis*". See the Appendix for more details. [Pérez de Moya 1584, pp. 73–75].

The applied orientation of the work is most evident when Moya distinguishes between two ways of measuring a sloppy, not horizontal field. Sloping fields are to be measured as if they were horizontal when they are to be used for growing purposes. However, if they are meant for building or housing purposes, then only the "base" of the hilly field is to be measured:

> "Notice that, in hilly fields if the field is to be sowed, it is to be measured as any other flat surface. But if the field is to bear up habitations, [then] it is to be measured according to its base [...]".[27]

In his drawing (see Fig. 1), *ab* is the sloping field seen from the side, and its "base" is *db*.

To measure *db*, Moya suggests to hold out horizontally a stick of known length, *ae*, in the highest point of the field (which he assumes has a regular form). Then, from the end of the stick, *e*, a plumb line must be dropped until it meets the sloping ground, say at *f*. Then we repeat the procedure until we reach the bottom line at *b*. The basis *db* will be equal to the addition of the lengths *ae*, *fg*, *hi*, etc. [Pérez de Moya 1584, pp. 100v – 101v].

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27 *Nota mas, que donde ay monte, si el monte fuere para sembrar, midase como las demas superfícies planas. Y si fuere para dar termino a algun pueblo midase según su basis (si la venta no mandare otra cosa)* [Pérez de Moya 1584, p. 100v].

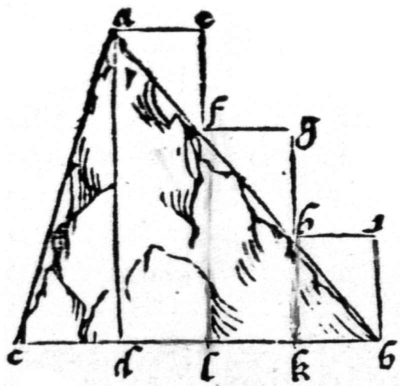

FIGURE 1. We reproduce here the figure Moya used in his *Tratado de Geometria* of 1573 (p. 144), identical to the one he used in the *Principios* of 1584.

5.4. *Square roots and incommensurability*

Moya teaches how to determine square roots geometrically, as the mean proportional between two segments. His examples are $\sqrt{12}$, which he determines as the mean proportional between 4 and 3, and $\sqrt{7}$, mean proportional between 1 and 7. He finds the mean proportional via the semicircle whose diameter is the "addition" of the known proportional terms ($3+4$ and $1+7$ in the foregoing examples). In his *Fragmentos Mathematicos*, Pérez de Moya remarked that while in general it is not possible to find the exact root of a number, roots could be obtained exactly geometrically, if numbers "were reduced to line quantities" (*reduziendo el tal numero a quantidades de línea*) [Pérez de Moya 1568, pp. 63–64]. Moya mentioned here "book I of volume II of Nicolo Tartaglia", in a likely reference to Tartaglia's *General Trattato di numeri, et misure* (1556).

In measuring the diagonal of the square, Moya provides a notion of incommensurability. In fact, he claims that the diagonal of the square can never be measured by the unit that measures its side without leaving a "fraction" (sic). By "fraction" in the 16th century one usually understood "rational fraction", but Moya does not distinguish between rational and irrational remainders:

> "And you will notice that the diagonals of squares will never be commensurable with their sides, that is to say, no diagonal will ever have an [exact number of] units that is equal to that [unit that measures] the side of the square exactly. I do not mean that the diagonal will [not] be equal to the side: nobody doubts

this, because it is obvious that the diagonal is always greater, or longer, than the side; but if the side of the square contains exactly 10 units, or whatever number it be, then I say that the diagonal of the square will not count an exact whole number of those units that the side contains, but that it will always have a fraction with them".[28]

5.5. *Circle and circumference*

Pérez de Moya studies in detail the circle and the circumference, pointing to their useful role in the construction of astrolabe sheets, gnomonics, and many other things:

> "For sundials, and for drawing the winds [i.e. geographical directions], and for making lanterns according to carpenters, and for many other things, it is often necessary to divide the circumference of the circle in equal parts".[29]

Pérez de Moya gives rule and compass constructions for the division the circumference in n equal parts, $n = 2, 3, 4, 5, 6, 7, 8, 10, 12, 16, 24, 32$ and 36, which only in some cases are exact, obviously. These constructions do not appear in the *Fragmentos Mathematicos*, and while they are included in the *Tratado de Geometria practica y speculativa*, some are given differently. An interesting case in point is the division of the circumference in five equal parts, where Moya's approximate construction is easy and simple and yields a good approximation. He does not make any reference to Euclid's or Ptolemy's exact constructions.[30] His procedure to divide the circumference in five equal parts draws two perpendicular diameters (Fig. 2). Next, he divides one of the diameters (say, $[AB]$) in three equal parts, getting the points E and F. Then, from one end of the second diameter, say X, he draws two chords through E and F, which cut the circumference at G and H. Points G and H define an arc that Moya says is the fifth part of the circumference [Pérez de Moya 1584, pp. 62v–68].

28 "*E notaras que jamas las diagonales de los quadrados vendran a ser comensurables com sus lados, quiero dezir, q̃ la diagonal nunca tendra tamanos semejantes a los que el quadrado tuviere por lado justamente. No quiero dezir, que la diagonal hade ser tanto como el lado, q̃ esto ninguno lo duda, porq cosa clara es ser la diagonal siempre mayor, o mas larga, sino q̃ si por lado tuviere el cuadrado 10 tamanos o lo q̃ fuere justamete, digo q̃ la diagonal del cuadrado, no vedra a tener numero entero justamete de tamanos semejantes a los q̃ tiene por lado, sino q̃ vendra alguna fracion co los tamanos que tuviere.*" [Pérez de Moya 1584, pp. 51–52].

29 "*Para la matéria de relojes, y para asentar en plano los vie[n]tos y hazer lanternas segun dizen los carpinteros, y para otras varias cosas se ofrece muchas vezes necesidad de dividir la circunferencia de un circulo en algunas partes yguales*" [Pérez de Moya 1584, p. 63].

30 Euclid, *Elements* IV, 11; Ptolemy, *Ptolemy's Almagest,* G.J. Toomer, ed. and trans. (London: Duckworth, 1984), I.10, pp. 48-9.

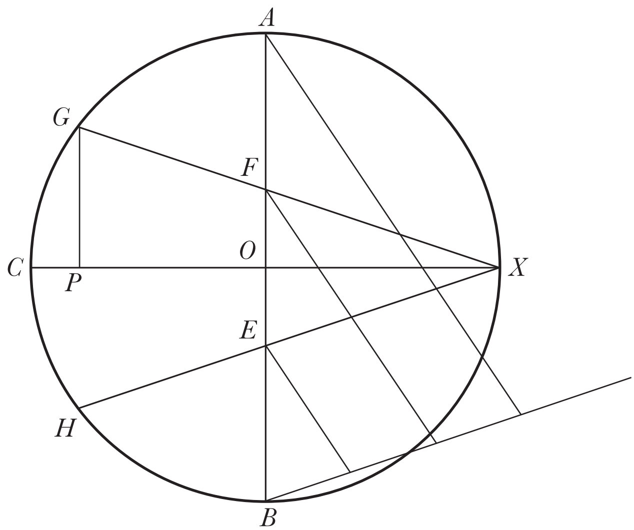

FIGURE 2.

If the construction were correct, the segment GH would be equal to the side, l, of the pentagon inscribed in the circle and X would be a vertex of the pentagon. Let us call d the diagonal, GX, of the pentagon and P the intersection of GH and CX. By construction, $OF = \frac{1}{3}OX$, therefore by the similarity of triangles GPX and FOX, $\frac{GP}{GX} = \frac{\frac{1}{3}OX}{\sqrt{\left(\frac{1}{3}OX\right)^2 + OX^2}}$. Hence, $\frac{\frac{1}{2}\ell}{d} = \frac{\frac{1}{3}}{\frac{\sqrt{10}}{3}}$, and therefore $\frac{d}{l} = \frac{\sqrt{10}}{2}$. However we know that for the side and diagonal of the true regular pentagon, we have $\frac{d}{l} = \frac{1+\sqrt{5}}{2}$.

Pérez de Moya's procedures to divide the circle in three, seven, and eleven parts start by drawing the radius, EH (see Fig. 3). Next, with centre E and radius EH, Moya draws the *arc AHD*. Obviously, arc AED is a third of the circumference. Then, according to Pérez de Moya, half of the *chord* AD provides the side of the regular heptagon, and one third of the cord AD provides the side of the 11-sided regular polygon. Pérez de Moya adds that it is possible to prove with the help of the compass that these constructions are in practice nearly correct. [Pérez de Moya 1584, pp. 62v–68].

To conclude, let us turn to Pérez de Moya's determination of the area of the circle. Pérez de Moya points out that to measure the area of any

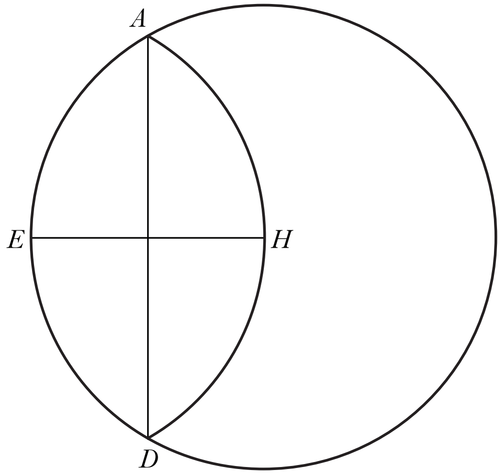

FIGURE 3.

circle is to find out how many "little squares" (*quadradicos*) of side equal to one foot it is possible to mark or make up within the circle. Then he adds that to determine that number, it is enough to know the diameter or the perimeter of the circumference:

> "To measure the area of a circle, we must know either its diameter or its circumference, as it can be measured with whichever of the two". [31]

For instance, Moya calculates the area of a circle whose perimeter is equal to 22 feet by making the area equal to 7/88 times 22 square, which is a standard and quite good approximate value for π. Pérez de Moya grounds this practical rule on Archimedes' authority:

> "The reason of this operation is that Archimedes demonstrates that the whole area of any circle is the seven eighty-eighth parts of the square of its circumference". [32]

Later on, in another example, he mentions Archimedes's theorem (from *On the Measure of the circle*) that every circle is equivalent to a right triangle whose base is equal to the circumference of the circle and whose height is equal to the radius. The rule above can obviously be deduced

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31 "*Para medir la área de un circulo, se há de tener noticia de su diâmetro o de su circunferência porque con qualquiera cosa destas dos se media*" [Pérez de Moya 1584, pp. 62[v]-68].

32 "*La razon desta operación es, que Archimides demuestra que toda la area de un circulo es los siete, ochenta y ocho abos del cuadrado de su circunferencia*" [Pérez de Moya 1584, p. 90].

from it. Moya does not make this point here although it is explicit in Moya's treatises of geometry of 1568 and 1573.

6. CONCLUSION

When compared to other Renaissance books of practical geometry the contents and methods of Moya's *Principios de geometria* (1584) may not be original, but they are in the Spanish context. As far as we know, Juan Ortega's geometrical work is the main one printed before Moya's. First published in 1512 [Ortega 1512], Ortega's treatise was often reissued through the 16th century and beyond. One of its improved versions, the *Tractado subtilissimo d'Arismetica y de Geometria* of 1552 [Ortega 1552] contains a very elementary geometrical text that offers little more than numerical examples in problems of everyday life. By contrast, Moya's geometrical treatises, and his 1584 *Principios de geometria* in particular, include a fair amount of theoretical results along with rule-and-compass constructions organized deductively and grounded on Archimedes as well as on Euclid's *Elements*. To our knowledge, Moya provided the most sophisticated 16th-century Spanish geometrical treatises. His 1584 *Principios*, moreover, suggests the existence of an important demand for cheap books of a highly applied nature to which Moya answered by refashioning his previous, more theoretically oriented treatises.

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APPENDIX A
PÉREZ DE MOYA'S UNITS OF MEASURE

According to Moya, his measures are grounded upon the basic Roman unit, the "finger" (*dedo*):

> "The origin whence all the different measures —those that Geometers and Cosmographers use, and those used by the Romans as well— come from is a

measure called finger (*dedo*); and by finger they understood the space of this line M ——— L".[33]

We notice that the line has an approximate length of 2 cm. In his *Fragmentos Mathematicos*, Moya describes *dedo* as the space occupied by 4 oat seeds: "*espacio que occupan quatro granos de cevada*". He also adds a figure whose length is very similar to the one we find here, accompanied by a similar explanation. The following equivalences appear in [Pérez de Moya 1584, pp. 73v–75].

Onza — 3 *dedos* (in his 1568 book, he has *onça* = 12 oat seeds)
Palmo — 4 *dedos* (Pérez de Moya, following Vitruvius, stresses that his "*palmo*" is not equal to the width of the hand with extended fingers, but just the width of the flat hand surface: not the "*mano estendida*" but just "*la palma de la mano*")
Dicha — 2 *palmos*
Deunx — 10 *dedos* (Moya says this comes from Columella, *De re rustica*, book 5, ch. 1)
Espithema — 3 *palmos*
Pie — 4 *palmos*, or 16 *dedos*
Paso — 2 *pies*
Pasada — there are two kinds of it: the *pasada comun* is the same as the *paso*. The *pasada geométrica* is equal to 2 *pasos*
Pertica — 10 *pies*
Orgia — 6 *pies*
Plethrum — 100 *pies*
Iugero — 100 *pies* ("*Iugero*" and "*Plethrum*" are two different names for the same thing which Moya mentions without further ado; he had mentioned them already in his *Tratado de Geometria* of 1573, p. 97)
Estadio — 125 *pasos geométricos* (Moya says this comes from Pliny)
Diaulos — 2 *estadios*
Dolicos — 12 *estadios*
Parafanga — 30 *estadios*
Schenus — 60 *estadios*
Milla or *Milla Romana* — 8 *estadios* = 1000 *pasos*
Milla Alemana comun (German common mile) — 32 *estadios* = 4000 *pasos*
Milla grande (Long mile) — 5000 *pasos*
Lápis — equal to the *milla romana*

33 "*El origen de donde salen las diversas medidas, de que los Geometras, y Cosmographos se sirven, y de las que usaron los Romanos, es una medida q̃ dizẽ dedo, y por dedo entẽdian espacio desta linea M———L*" [Pérez de Moya 1584, p. 73v].

Estadal — a square whose side equals to of a *vara* (Pérez de Moya does not give here the length of the "*vara*, but in his *Tratado de Geometria, practica, y especulativa* ([1573, p. 97]), 1 *vara* = 3 *pies*)
Ulna comun — 4 *pies*
Ulna agreste — 6 *pies*
Cubito, or *codo pequenõ* — 1 and a half *pie*
Codo comum — 8 *palmos*
Codo grande — 16 *palmos*
Legua — one and a half *milla* = 12 *estadios*
Legua comum — 3 *millas* = 24 *estadios*
Legua Española — 4 *millas* = 32 *estadios* = 4000 *passos*

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