

RECHERCHE COOPÉRATIVE SUR PROGRAMME N° 25

DAVID RUELLE

Addendum to " An Extension of the Theory of Fredholm Determinants "

Les rencontres physiciens-mathématiciens de Strasbourg - RCP25, 1989, tome 40
« Conférences de B. Helffer, J. Sjöstrand, D. Ruelle et J. Fritz », , exp. n° 4, p. 54-57

http://www.numdam.org/item?id=RCP25_1989__40__54_0

© Université Louis Pasteur (Strasbourg), 1989, tous droits réservés.

L'accès aux archives de la série « Recherche Coopérative sur Programme n° 25 » implique l'accord avec les conditions générales d'utilisation (<http://www.numdam.org/conditions>). Toute utilisation commerciale ou impression systématique est constitutive d'une infraction pénale. Toute copie ou impression de ce fichier doit contenir la présente mention de copyright.

NUMDAM

Article numérisé dans le cadre du programme
Numérisation de documents anciens mathématiques
<http://www.numdam.org/>

Addendum to
 "An extension of the theory of Fredholm determinants"
 by D. Ruelle

In deriving (2.12) we have used the inequality

$$\sum_{\vec{b} \in J_k^{(\ell)}} \|\mathfrak{M}_k^{(\ell)} \chi_{\vec{b}}^{\dagger}\| \leq \text{const } (e^{P+\varepsilon})^{\ell} \quad (2.13)$$

which we shall now prove. Given $\beta > 0$, we let $\varphi_{\omega\beta} = |\varphi_{\omega}| + \beta \|\varphi_{\omega}\|$, and define $\mathfrak{M}_{k\beta}^{(\ell)}$ with φ_{ω} replaced by $\varphi_{\omega\beta}$. We first check that

$$\|\mathfrak{M}_k^{(\ell)} \chi_{\vec{b}}^{\dagger}\| \leq C(\beta) \|\mathfrak{M}_{k\beta}^{(\ell)} \chi_{\vec{b}}^{\dagger}\|_0 \quad (2.14)$$

where $C(\beta)$ does not depend on ℓ . We may indeed write

$$(\mathfrak{M}_k^{(\ell)} \chi_{\vec{b}}^{\dagger})_j(x) = \int \mu(d\omega_1) \dots \mu(d\omega_{\ell}) \varepsilon \varphi_{\omega_{\ell}}(x) \dots \varphi_{\omega_1}(\psi_{\omega_2} \dots \psi_{\omega_{\ell}} x)$$

where the integral is restricted to those $(\omega_1, \dots, \omega_{\ell})$ for which there is a permutation π such that

$$\pi(v(j_0, \bar{\omega}), \dots, v(j_k, \bar{\omega})) = \vec{b},$$

and $\varepsilon = \text{sign } \pi$. Therefore

$$\|\mathfrak{M}_k^{(\ell)} \chi_{\vec{b}}^{\dagger}\|_0 \leq \|\mathfrak{M}_{k\beta}^{(\ell)} \chi_{\vec{b}}^{\dagger}\|_0.$$

If $x, y \in X_{j_0} \cap \dots \cap X_{j_k}$, we have also

$$\begin{aligned}
& |(\mathfrak{M}_k^{(\ell)} \chi_b^{\rightarrow})_j^{\rightarrow}(x) - (\mathfrak{M}_k^{(\ell)} \chi_b^{\rightarrow})_j^{\rightarrow}(y)| \leq \int \mu(d\omega_1) \dots \mu(d\omega_\ell) \\
& |\varphi_{\omega_\ell}(x) \dots \varphi_{\omega_1}(\psi_{\omega_2} \dots \psi_{\omega_\ell} x) - \varphi_{\omega_\ell}(y) \dots \varphi_{\omega_1}(\psi_{\omega_2} \dots \psi_{\omega_\ell} y)| \\
& \leq \sum_{i=1}^{\ell} \int \mu(d\omega_1) \dots \mu(d\omega_k) \varphi_{\omega_\ell \beta}(x) \dots \varphi_{\omega_{i+1} \beta}(\psi_{\omega_{i+2}} \dots \psi_{\omega_\ell} x) \\
& \quad |\varphi_{\omega_i}(\psi_{\omega_{i+1}} \dots \psi_{\omega_\ell} x) - \varphi_{\omega_i}(\psi_{\omega_{i+1}} \dots \psi_{\omega_\ell} y)| \\
& \quad \varphi_{\omega_{i-1} \beta}(\psi_{\omega_i} \dots \psi_{\omega_\ell} y) \dots \varphi_{\omega_1 \beta}(\psi_{\omega_2} \dots \psi_{\omega_\ell} y) .
\end{aligned}$$

We may assume that $\|\varphi_\omega\|$ is bounded uniformly with respect to ω (this can be achieved by a change of the measure μ). We may then write

$$\begin{aligned}
& |\varphi_{\omega_i}(\psi_{\omega_{i+1}} \dots \psi_{\omega_\ell} x) - \varphi_{\omega_i}(\psi_{\omega_{i+1}} \dots \psi_{\omega_\ell} y)| \\
& \leq \|\varphi_{\omega_i}\| (\theta^{\ell-i} d(x,y))^\alpha \leq \text{const } \varphi_{\omega_i \beta}(\psi_{\omega_{i+1}} \dots \psi_{\omega_\ell} x) \theta^{\alpha(\ell-i)} d(x,y)^\alpha
\end{aligned}$$

and similarly

$$\begin{aligned}
& \varphi_{\omega_r \beta}(\psi_{\omega_{r+1}} \dots \psi_{\omega_\ell} y) \\
& \leq \varphi_{\omega_r \beta}(\psi_{\omega_{r+1}} \dots \psi_{\omega_\ell} x) (1 + \text{const } \theta^{\alpha(\ell-r)}) . \quad (2.15)
\end{aligned}$$

Therefore

$$\frac{|(\mathfrak{M}_k^{(\ell)} \chi_b^{\rightarrow})_j^{\rightarrow}(x) - (\mathfrak{M}_k^{(\ell)} \chi_b^{\rightarrow})_j^{\rightarrow}(y)|}{d(x,y)^\alpha} \leq \text{const } \|\mathfrak{M}_k^{(\ell)} \chi_b^{\rightarrow}\|_0$$

and (2.14) follows.

From (2.15) we also obtain

$$(\mathfrak{M}_{k\beta}^{(\ell)} \chi_{\vec{b}}^{\vec{j}})^{\vec{j}}(y) \leq C'(\beta) (\mathfrak{M}_{k\beta}^{(\ell)} \chi_{\vec{b}}^{\vec{j}})^{\vec{j}}(x)$$

where $C'(\beta)$ does not depend on ℓ . Therefore

$$\begin{aligned} \sum_{\vec{b} \in J_k^{(\ell)}} \|\mathfrak{M}_{k\beta}^{(\ell)} \chi_{\vec{b}}^{\vec{j}}\|_0 &\leq C'(\beta) \sum_{\vec{j}} \sup_x |(\mathfrak{M}_{k\beta}^{(\ell)} 1)^{\vec{j}}(x)| \\ &\leq C''(\beta) \|\mathfrak{M}_{k\beta}^{\ell} 1\|_0 \end{aligned}$$

and finally

$$\begin{aligned} \sum_{\vec{b} \in J_k^{(\ell)}} \|\mathfrak{M}_k^{(\ell)} \chi_{\vec{b}}^{\vec{j}}\| &\leq C(\beta) C''(\beta) \|\mathfrak{M}_{k\beta}^{\ell} 1\|_0 \\ &\leq C'''(\beta) (e^{P(\beta)+\varepsilon/2})^{\ell} \end{aligned}$$

where $e^{P(\beta)}$ is the spectral radius of $\mathfrak{M}_{k\beta}$. Note that $\mathfrak{M}_{k\beta}$ is close in norm to $|\mathfrak{M}_k|$ for β small :

$$\|\mathfrak{M}_{k\beta} - |\mathfrak{M}_k|\| \leq \beta \int \mu(d\omega) \|\varphi_{\omega}\|.$$

Using the upper semicontinuity of the spectral radius we may thus choose β such that

$$\sum_{\vec{b} \in J_k^{(\ell)}} \|\mathfrak{M}_k^{(\ell)} \chi_{\vec{b}}^{\vec{j}}\| \leq C'''(\beta) (e^{P+\varepsilon})^{\ell}$$

i.e., (2.13) holds as announced.

A similar argument may be used to obtain the inequality

$$\sum_{b \in J^{(\ell)}} \|\mathfrak{m}^{(\ell)}((\cdot - x(b))^n \chi_b)\| \leq \text{const}(e^{P+\varepsilon})^\ell (\theta^\ell)^{|\ln \ell|}$$

which is needed in the proof of (3.11).