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GENERALIZED PICARD LATTICES ARISING FROM HALF-INTEGRAL CONDITIONS

by G. D. MOSTOW (*)

1. Introduction

Set

$$F_{gh}(x_2, \ldots, x_{d+1}) = \int_g^h u^{-\mu_0} (u - 1)^{-\mu_1} \prod_2^{d+1} (u - x_i)^{-\mu_i} \, du$$

where $g, h \in \{\infty, 0, 1, x_2, \ldots, x_{d+1}\}$. Then for fixed $\mu_0, \ldots, \mu_{d+1}$, F_{gh} is a multivalued function on the subset M of $(\mathbf{P}^1)^{d+3}$ defined as

$$M = \{ (x_i) \mid x_i \neq 0, 1, \infty \text{ and } x_i \neq x_j \text{ for } i \neq j \}.$$

For topological reasons, the **C**-linear span of these functions form a $d + 1$ dimensional vector space that is invariant under monodromy. Taking $d + 1$ such functions as the homogeneous coordinates in projective d-space $\mathbf{P}^d$, we get a map

$$\hat{w} : \hat{M} \to \mathbf{P}^d$$

where $\hat{M}$ is the universal covering of the space M. Set

$$\mu_\infty = 2 - (\mu_0 + \mu_1 + \ldots + \mu_{d+1}).$$

Assume hereafter that μ_i is real and strictly positive for all i $(0 \leq i \leq d + 1$ or $i = \infty)$. Let Γ denote the image of $\pi_1(M)$ in $PGL(d + 1, \mathbf{C})$ under the monodromy action. In the preceding paper, the following sufficiency condition was proved:

If for all i, j in $\{\infty, 0, 1, \ldots, d + 1\}$
(INT): $(1 - \mu_i - \mu_j)^{-1}$ *is an integer for all* $i \neq j$ *such that* $\mu_i + \mu_j < 1$, *then* Γ *is a lattice in the projective unitary group* $PU(d, 1)$.

In the case $d = 2$, this condition is essentially equivalent to Picard's, and under condition (INT), I call Γ a Picard lattice.

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The purpose of this paper is to relax condition (INT) in case some of the μ_i's are equal. The main result, proved in § 3, states:

Let $S_1 \subset S = \{\infty, 0, 1, 2, \ldots, d+1\}$ *and assume that* $\mu_s = \mu_t$ *for all* $s, t \in S_1$. *If* $\mu_s > 0$ *for all* $s \in S$ *and* (μ_s) *satisfies the condition*
(Σ INT): *For all* $s \neq t$ *such that* $\mu_s + \mu_t < 1$

$$(1 - \mu_s - \mu_t)^{-1} \text{ is } \begin{cases} \text{an integer if } s \text{ or } t \text{ is not in } S_1, \\ \text{a half-integer if } s, t \in S_1; \end{cases}$$

then Γ *is a lattice in* $PU(d, 1)$.

When condition (Σ INT) is satisfied, we define in § 2 a finite extension Γ_Σ of Γ. The lattice Γ_Σ arises from an extension of order $n!$ of the fundamental group $\pi_1(M)$ where $n = \text{card } S_1$. If (μ_s) satisfies condition (Σ INT) but not (INT), then $\Gamma_\Sigma = \Gamma$; if (μ_s) satisfies (INT) too, then Γ_Σ/Γ is the symmetric group on n letters (cf. (3.11)).

In § 4, it is shown that each lattice $\Gamma(p, t)$ of $PU(2, 1)$ constructed in my paper [2] via three **C**-reflections is contained in the lattice Γ_Σ arising from monodromy of a hypergeometric function satisfying condition (Σ INT) for a three element subset S_1. Conversely, each such lattice Γ_Σ lies in an extension (of order at most 3) of a lattice $\Gamma(p, t)$ for suitable p and t; in § 6 (p, t) is expressed in terms of $(\mu_s)_{s \in S}$. This $\Gamma(p, t)$ description of Γ applies to most of the 27 Picard lattices, since for 22 of them, at least three of the $(\mu_s)_{s \in S}$ are equal.

In § 5 there is a list of all sequences $(\mu_1, \ldots, \mu_N)$ satisfying condition (Σ INT) but not (INT) for $N > 4$. It is seen that $N \leq 12$; that is, one gets lattices Γ in $PU(d, 1)$ satisfying condition (Σ INT) for $d \leq 9$ but not for $d > 9$.

The description of Γ_Σ in terms of $\Gamma(p, t)$ makes it possible to give an explicit, fundamental domain for Γ_Σ (cf. [3]) and a two generator presentation for Γ_Σ in case $d = 2$; this fundamental domain is the one described in [2] for $p \leq 5$.

None of the groups $\Gamma(p, t)$ in [2] coincide with a Picard lattice Γ; the lattice $\Gamma(p, t)$ of [2] is commensurable with a Picard lattice only if p is even (i.e. $p = 4$), in which case $\Gamma/\Gamma \cap \Gamma(p, t)$ has order 1 or 3 and $\Gamma(p, t)/\Gamma \cap \Gamma(p, t)$ has order 6.

2. The Main Theorem

We continue the notation of the preceding paper, referred to hereafter as DM, except that we write $PU(d, 1)$ for $PU(1, d)$.

Let $S = S_1 \cup S_2$ be a decomposition of the set S into disjoint subsets and assume that $\mu_s = \mu_t$ for all $s, t \in S_1$. Let Σ denote the permutation group of S_1. Then Σ operates on P^S by permutation of factors and hence on the set M of injective maps of S into P. It stabilizes the local system L on the family of punctured projective lines over M. The action of Σ on M and $B(\alpha)_M$ descend to an action on Q, Q_{st}, Q_{sst}, and on the bundle $B(\alpha)_Q$. Consequently, the bundle map $B(\alpha)_Q \to Q$ descends to a bundle map $B(\alpha)_{Q/\Sigma} \to Q/\Sigma$. The section w_μ of the bundle $B(\alpha)_Q$ is preserved by Σ; hence it descends to a section, also denoted w_μ, of the bundle $B(\alpha)_{Q/\Sigma}$.

Let Q' denote the subset of Q on which Σ operates freely; Q' is an open dense submanifold of Q. From the flatness of the bundle $B(\alpha)_Q$ over Q we infer the flatness of $B(\alpha)_{Q/\Sigma}$ restricted to Q'/Σ; this latter bundle is denoted by $B(\alpha)_{Q'/\Sigma}$.

Let o be a base point in Q', let $\bar{o}$ denote the orbit Σo, and let

$$\theta_\Sigma : \pi_1(Q'/\Sigma, \bar{o}) \to \operatorname{Aut} B(\alpha)_{\bar{o}}$$

denote the monodromy homomorphism. Then

$$B(\alpha)_{Q'/\Sigma} = \widehat{Q'/\Sigma} \underset{\pi_1(Q'/\Sigma,\bar{o})}{\times} B(\alpha)_{\bar{o}} = \widetilde{Q'/\Sigma} \times_{\Gamma_\Sigma} B(\alpha)_{\bar{o}}$$

where $\widehat{Q'/\Sigma}$ denotes the simply connected covering space of Q'/Σ, $\Gamma_\Sigma = \pi_1(Q'/\Sigma, \bar{o})/\operatorname{Ker}\theta_\Sigma$, and

$$(2.1) \qquad \widetilde{Q'/\Sigma} = (\widehat{Q'/\Sigma})/\operatorname{Ker}\theta_\Sigma.$$

Theorem. — Assume that $(\mu_s)_{s\in S}$ *satisfies the condition*

(2.2) (Σ INT) *For all* $s \neq t$ *in* S *such that* $\mu_s + \mu_t < 1$, $(1 - \mu_s - \mu_t)^{-1}$ *is*

$$\begin{cases} \text{an integer, if } s \text{ or } t \text{ is not in } S_1, \\ \text{a half-integer, if } s, t \in S_1. \end{cases}$$

Then $\operatorname{Im}\theta_\Sigma$ *is a lattice in* $PU(\operatorname{card} S - 3, 1)$.

3. Proof of the theorem

(3.1) The basic idea of the proof is to show that under hypothesis (Σ INT) Q'/Σ plays the same role that Q plays in DM under hypothesis (INT). We begin with some remarks about morphisms of completions of spreads.

(3.2) Let Y_i be a locally connected Hausdorff space ($i = 1, 2$) and Y'_i an open dense connected subset in Y_i. Assume that each point $y \in Y_i$ has a base of open neighborhoods $\mathscr{V}_y$ satisfying

(3.2.1) for V in $\mathscr{V}_y$, $V \cap Y'_i$ is connected,

(3.2.2) for $V' \subset V''$ in $\mathscr{V}_y$, $\pi_1(V' \cap Y'_i) \twoheadrightarrow \pi_1(V'' \cap Y'_i)$.

Let $\rho'_i : X'_i \to Y'_i$ denote a covering map. Considered as a map of X'_i to Y_i, ρ'_i is a spread. Let $\rho_i : X_i \to Y_i$ denote the completion of ρ'_i ($i = 1, 2$) (cf. DM 8.1). Then X_i and Y_i are locally connected and ρ_i is a complete spread.

Assume in addition that there are maps $\sigma' : X'_1 \to X'_2$ and $\tau : Y_1 \to Y_2$ such that $\rho_2 \sigma' = \tau\rho_1$. Then by (8.1.1) of DM there is a map $\sigma : X_1 \to X_2$ such that the diagram below is commutative

$$\begin{tikzcd} X_1' \arrow[r, "\sigma'"] \arrow[d, hook] & X_2' \arrow[d, hook] \\ X_1 \arrow[r, "\sigma"] \arrow[d, "\rho_1"'] & X_2 \arrow[d, "\rho_2"] \\ Y_1 \arrow[r, "\tau"] & Y_2 \end{tikzcd}$$

Lemma **(3.3).** — *Assume in addition that*

(3.3.1) *σ' is a surjective covering map,*

(3.3.2) *τ is an open map,*

(3.3.3) *for any $y \in Y_1$ and $V \in \mathscr{V}_y$ (cf. (3.2)), V is connected component of $\tau^{-1}\tau(V)$.*

Then the map σ is open and surjective.

Proof. — Let V be an open connected set in Y_1 small enough so that V is a connected component of $\tau^{-1}\tau(V)$ (cf. (3.3.3)). In order to prove that σ is open, it suffices, by definition of a spread, to prove that for any connected component $\rho_1^{-1}(V)^c$ of $\rho_1^{-1}(V)$, $\sigma(\rho_1^{-1}(V)^c)$ coincides with a connected component of $\rho_2^{-1}\tau(V)$.

Commutativity of the diagram and surjectivity of σ' yields

$$\rho_2^{-1}\tau(V) \cap X_2' = \sigma'(\rho_1^{-1}\tau^{-1}\tau(V) \cap X_1').$$

Set $C_1' = (\rho_1^{-1}\tau^{-1}\tau(V) \cap X_1')^c$, the connected component of $\rho_1^{-1}\tau^{-1}\tau(V) \cap X_1'$ contained in $[\rho_1^{-1}\tau^{-1}\tau(V)]^c$, the connected component of $\rho_1^{-1}\tau^{-1}\tau(V)$ which contains $\rho_1^{-1}(V)^c$. We have $\rho_2\sigma'(C_1') = \tau(V) \cap \rho_2(X_i)$. Inasmuch as σ', ρ_1' and ρ_2' are covering maps, $\sigma'(C_1')$ coincides with a connected component $C_2' = [\rho_2^{-1}\tau(V) \cap X_2']^c$ of $\rho_2^{-1}\tau(V) \cap X_2'$, because one sees easily that $\sigma'(C_1')$ is both open and closed in C_2'. By definition of the completion of a spread, one deduces at once that

$$\sigma([\rho_1^{-1}\tau^{-1}\tau(V)]^c) = [\rho_2^{-1}\tau(V)]^c,$$

the latter denoting the connected component of $\rho_2^{-1}\tau(V)$ containing $\sigma(\rho_1^{-1}(V)^c)$. But

$$[\rho_1^{-1}\tau^{-1}\tau(V)]^c \subset \rho_1^{-1}[\tau^{-1}\tau(V)]^c = \rho_1^{-1}(V)^c,$$

the last equality by (3.3.3). Consequently $\sigma(\rho_1^{-1}(V)^c) = [\rho_2^{-1}\tau(V)]^c$. Hence σ is open. Verification that σ is surjective is direct. This completes the proof.

Remark **(3.4).** — By taking $X_2' = Y_1'$, $X_2 = Y_1$, $\sigma = \rho_1$, $\rho_2 =$ identity, (3.3) implies that the map ρ_1 is open and surjective if Y_1' is connected and X_1' is not empty.

Lemma **(3.5).** — *Let $\sigma' : X_1' \to X_2'$ and $\sigma : X_1 \to X_2$ be as in* (3.3), *and let $\varphi_1 : X_1 \to B$ be a continuous map. Then the commutative diagram of solid arrows*

$$
\begin{tikzcd}
 & X_1' \arrow[dl, hook'] \arrow[dr, "\varphi_1'"] \arrow[dd, "\sigma'"'] & \\
X_1 \arrow[rr, "\varphi_1"'] \arrow[dd, "\sigma"'] & & {}'B \arrow[dd, equal, "\text{identity}"] \\
 & X_2' \arrow[dl, hook'] \arrow[dr, "\varphi_2'"] & \\
X_2 \arrow[rr, dashed, "\varphi_2"'] & & B
\end{tikzcd}
$$

can be completed as shown.

Proof. — By (3.3), the map σ is a surjective open map. Given $q \in X_2$, it suffices to prove that $\varphi_1(\sigma^{-1} q)$ is a single point, i.e. the map φ_1 descends to a continuous map φ_2 of X_2.

Let $p \in \sigma^{-1} q$, let U be a connected neighborhood of q in X_2 and let $\sigma^{-1}(U)^c$ denote the connected component of p in $\sigma^{-1}(U)$. Then

$$\varphi_1(p) = \lim_{\substack{x \to p \\ x \in \sigma^{-1}(U)^c \cap X_1'}} \varphi_1'(x) = \lim_{\substack{x \to p \\ x \in \sigma^{-1}(U)^c \cap X_1'}} \varphi_2'(\sigma(x)) = \lim_{\substack{y \to q \\ y \in U \cap X_2'}} \varphi_2'(y)$$

since $\sigma(\sigma^{-1}(U)^c \cap X_1') = U \cap X_2'$ because σ' is a surjective covering map. It follows at once that $\varphi_1(p)$ is independent of the choice of p in $\sigma^{-1} q$.

(3.6) We shall apply (3.2) with $X_1' = \tilde{Q}' = \hat{Q}'/\operatorname{Ker}\theta$, the smallest covering space of Q' on which the monodromy acts trivially, $Y_1 = Q_{sst}$ or Q_{st} and $X_1 = \tilde{Q}_{sst}$ or $\tilde{Q}_{st}$, the completion of X_1' over Y_1, $X_2' = \widetilde{Q'/\Sigma}$, the space defined in (2.1), $Y_2 = Q_{sst}/\Sigma$ or Q_{st}/Σ, and X_2 the completion of X_2' over Y_2. We write $\widetilde{Q_{sst}/\Sigma}$ (resp. $\widetilde{Q_{st}/\Sigma}$) for X_2. In both cases the map τ is the orbit map $x \mapsto \Sigma x$, and $\sigma' : \tilde{Q}' \to \widetilde{Q'/\Sigma}$ is the lift of τ given by the map $\hat{Q}'/\operatorname{Ker}\theta \to \hat{Q}'/\operatorname{Ker}\theta_\Sigma$.

Remark. — $Q - Q'$ is a finite union of subvarieties some of which may be of **C**-codimension 1 in Q. Although $\pi_1(Q', o) \to \pi_1(Q, o)$ and $\hat{Q}' \to \hat{Q}$ may fail to be injective, $\tilde{Q}' \to \tilde{Q}$ is injective, because $\operatorname{Ker}\pi_1(Q', o) \to \pi_1(Q, o)$ lies in $\operatorname{Ker}\theta$; this last assertion follows immediately from the fact that the map $\omega_\mu : \tilde{Q} \to B^+(\alpha)_o$ is etale (DM Proposition (3.9)). In particular, $\tilde{Q}$ is the completion of $\tilde{Q}'$ over Q. Here the simply connected $\hat{Q}'$ is identified with $\widehat{Q'/\Sigma}$ via $\hat{\sigma}'$, the lift of σ':

(3.6.1)
$$
\begin{array}{ccc}
\hat{Q}' & \xrightarrow[=]{\hat{\sigma}'} & \widehat{Q'/\Sigma} \\
\downarrow & & \downarrow \\
\hat{Q}'/\operatorname{Ker}\theta = \tilde{Q}' & \xrightarrow{\sigma'} & \widetilde{Q'/\Sigma} = \hat{Q}'/\operatorname{Ker}\theta_\Sigma \\
\downarrow & & \downarrow \\
Q' & \xrightarrow{\tau} & Q'/\Sigma
\end{array}
$$

θ is the monodromy homomorphism of $\pi_1(Q', o)$ to $\operatorname{Aut} B(\alpha)_o$, $\pi_1(Q', o)$ is identified with a subgroup of $\pi_1(Q'/\Sigma, \bar{o})$; $\pi_1(Q'/\Sigma, \bar{o})$ thereby acts on the space $\widetilde{Q}'$ and thus $\operatorname{Ker}\theta_\Sigma \cap \pi_1(Q', o) = \operatorname{Ker}\theta$. It is perfectly clear that the hypotheses of (3.2), (3.3) and (3.5) are satisfied, and that (3.2) and (3.5) are applicable.

(3.7) Let $\mathscr{T}_1$ denote the set of all stable partitions T of S such that

$$\operatorname{card} T = \operatorname{card} S - 1.$$

By definition each $T \in \mathscr{T}_1$ has only one element in each coset except for a single coset with two elements $\{s, t\}$ satisfying $\mu_s + \mu_t < 1$. As in DM, Q_T denotes the subset of all $y \in P^S$ such that for any $s_1, s_2 \in S$, $y(s_1) = y(s_2)$ if and only if s_1, s_2 are in the same coset. For each $T \in \mathscr{T}_1$ let Q'_T denote the subset of elements in Q_T which are fixed by no elements of Σ other than the permutation of the two elements occuring in the same coset of T in case they are both in S_1. Set

$$Q'_1 = Q' \cup \coprod_{T \in \mathscr{T}_1} Q'_T.$$

The degree of the orbit map $Q' \to Q'/\Sigma$ is card Σ, but locally in Q'_1 around a point of Q'_T, the degree of orbit map is 2. Clearly $Q_{sst} - Q'_1$ is a subvariety, $Q'_1 - Q'$ is a smooth divisor in Q'_1, and the same is true for their images in Q_{sst}/Σ, even though Q_{sst}/Σ may have singularities. In fact, Q'_1/Σ is an open non-singular subvariety of the variety Q_{sst}/Σ.

Let $\widetilde{Q}'_1$ denote the completion of Q' over Q'_1 and let $(\widetilde{Q'/\Sigma})_1$ denote the completion of $\widetilde{Q'/\Sigma}$ over Q'_1/Σ. Then $\widetilde{Q}'_1$ is a branched cover with branch locus along the disjoint union of **C**-codimension 1 submanifolds $\coprod_{T \in \mathscr{T}_1} Q'_T$ and ramification along Q'_T given by the order in **R**/**Z** of $1 - \mu_s - \mu_t$ where $\{s, t\}$ is the two-element coset of T.

(3.8) Let $\rho : \widetilde{Q}'_1 \to Q'_1$ (resp. $\rho_\Sigma : (\widetilde{Q'/\Sigma})_1 \to Q'_1/\Sigma)$ denote the completion of the covering map $\rho' : \widetilde{Q}' \to Q'$ over Q'_1, (resp. $\rho'_\Sigma : \widetilde{Q'/\Sigma} \to Q'/\Sigma$ over Q'_1/Σ).

Consider the commutative diagram

$$\begin{array}{ccc} \widetilde{Q}'_1 & \xrightarrow{\sigma} & (\widetilde{Q'/\Sigma})_1 \\ \downarrow{\scriptstyle\rho} & & \downarrow{\scriptstyle\rho_\Sigma} \\ Q'_1 & \xrightarrow{\tau} & Q'_1/\Sigma. \end{array}$$

The action of $\dfrac{\pi_1(Q'/\Sigma, \bar{o})}{\operatorname{Ker}\theta} := \Gamma'_\Sigma$ on $\widetilde{Q}'$ extends to $\widetilde{Q}'_1$ by the universal property of completions (cf. DM (8.1.1)) and σ may be regarded as a morphism of Γ'_Σ spaces.

Let $y \in Q'_1 - Q'$ and let V be a neighborhood of y in Q'_1 small enough so that the image of $\pi_1(V \cap Q')$ in $\pi_1(Q', o)$ is the decomposition group D_y of y and the image of $\pi_1(\tau(V \cap Q'))$ in $\pi_1(Q'/\Sigma, \bar{o})$ is the decomposition group $D_{\tau(y)}$ of $\tau(y)$. We have $y \in Q'_T$ where $T \in \mathscr{E}_1$. As V one can take the product of a disc in Q_r with a disc transversal to Q_r and stable under the permutation of the two-element coset of T. Clearly $\mathbf{Z} \cong D_y \hookrightarrow D_{\tau(y)} \cong \mathbf{Z}$, the injection being $z \mapsto 2z$. We recall (cf. DM (8.2)) that $\rho^{-1}(y) = \operatorname{Ker} \theta \backslash \pi_1(Q', o)/D_y$, and thus the stabilizer in $\pi_1(Q', o)$ of a point in $\rho^{-1}(y)$ is a conjugate of $D_y \operatorname{Ker} \theta$, and it equals $D_y \operatorname{Ker} \theta$ for a suitable choice base of point o.

Lemma. — Suppose

(3.8.1) $$D_y \operatorname{Ker} \theta_\Sigma \supset D_{\tau(y)}$$

Then any element of $\operatorname{Ker} \theta_\Sigma$ *which fixes the point* $y \in Q'_T$ *fixes each point of* $\rho^{-1}(y)$.

Proof. — Let $\tilde{y} \in \rho^{-1}(y)$ and let $\tilde{V}$ denote the connected component of $\tilde{y}$ in $\rho^{-1}(V)$. Since $\sigma' : Q' \to Q'/\Sigma$ is a covering map, $\sigma(\tilde{V})$ is the connected component of $\sigma(\tilde{y})$ in $\rho_\Sigma^{-1} \tau(V)$. By hypothesis (3.8.1), we can assume that the stabilizer of $\tilde{y}$ in $\pi_1(Q', o)$ contains the stabilizer of $\sigma(\tilde{y})$ in $\pi_1(Q'/\Sigma, \bar{o})$ modulo $\operatorname{Ker} \theta_\Sigma$.

Let h be an element of $\operatorname{Ker} \theta_\Sigma$ with $hy = y$. Then $h\tilde{y} = g\tilde{y}$ with $g \in \pi_1(Q', o)$. Hence $g\sigma(\tilde{y}) = \sigma(h\tilde{y}) = \sigma(\tilde{y})$. Consequently g is in the stabilizer of $\tilde{y}$ in $\pi_1(Q', o) \bmod \theta_\Sigma$. Since $g \in \pi_1(Q', o)$, we get $g = g_1 h_1$ with $h_1 \in \pi_1(Q', o) \cap \operatorname{Ker} \theta_\Sigma = \operatorname{Ker} \theta$ and $g\tilde{y} = g_1 h_1 \tilde{y} = g_1 \tilde{y} = \tilde{y}$. Therefore $h\tilde{y} = g\tilde{y} = \tilde{y}$.

Remark. — From (3.11.1), one can see that (3.8.1) holds if μ satisfies (Σ INT) but not (INT).

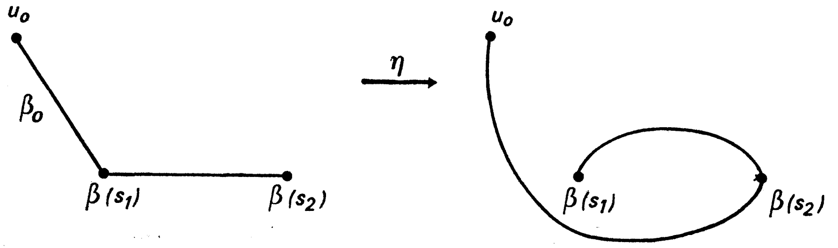
Lemma **(3.9)**. — *Let* $S_1 \subset S$, *let* Σ *denote the permutation group of* S_1, *and assume that* $\mu_s = \mu_t$ *for all* $s, t \in S_1$. *Let* s_1, s_2 *be distinct elements of* S_1, *and let* $[s_1, s_2]$ *denote the element of* $\pi_1(Q'/\Sigma, \bar{o})$ *coming from a positive loop in* Q'/Σ *around the* $\mathbf{C}$*-codimension* 1 *submanifold of* Q'_1/Σ *lying below the submanifold of* Q'_1 *on which the* s_1 *and* s_2 *coordinates coincide. Suppose that*

(3.9.1) $$1 - 2\mu_s = \frac{2}{k}, \quad k \text{ integer, all } s \in S_1.$$

Then

$$\operatorname{order} \theta_\Sigma([s_1, s_2]) = k.$$

Proof. — The proof is very much like the proof of Proposition (9.1.1) in DM. Let T_1 be the tree with vertices $\{s_1, s_2\}$ and let T_2 be a tree with vertices in $S - \{s_1, s_2\}$. Let $\beta : T_1 \amalg T_2 \to P$ be an embedding with $\beta \mid S = o$, the base point of Q'. Without loss of generality we may assume that $\beta(T_i) \subset D_i$ $(i = 1, 2)$ where D_1 and D_2 are discs having disjoint closures. Choose a base $\{\ell(a) . \beta \mid a;\ a \text{ an oriented edge of } T_1 \amalg T_2\}$ of $H_1^{lf}(P - o(S), \check{L})$ as in (2.5) of DM. The monodromy, being the result of horizontal transport, is effected by an isotopy η of P_o which is the identity map on $P - D_1$ and turns $(o(s_2), o(s_1))$ into $(o(s_1), o(s_2))$ by one positive half-turn. This isotopy has no effect on $\ell(a)\ \beta \mid a$ for an oriented edge $a \subset T_2$. To keep track of the change in the sections of the local system along varying arcs, fix a point $u_0 \in P_0 - D_1$, let β_0 denote the singular chain given by an arc from u_0 to the point $\beta(s_1)$, let a denote the oriented edge from s_1 to s_2, and let $\ell(\beta_0)$ be an extension of the section $\ell(a)$.



We can assume that the value of $\ell(u_0)$ remains unchanged during the isotopy. We have

$$\begin{aligned}\eta_*(\ell(a).\beta \mid a) &= \eta_*(\ell(\beta_0)\,\beta_0 + \ell(a).\beta \mid a) - \eta_*(\ell(\beta_0).\beta_0)\\ &= -\,\alpha_{s_2}^{-1}\ell(a).\beta \mid a.\end{aligned}$$

Inasmuch as the local system $\check{L}$ is stable under Σ, the monodromy $[s_1, s_2]$ effects on $H_1^{lf}(P - S, \check{L})$ a linear transformation with matrix relative to the base $\{\ell(a).\beta \mid a;\ a$ an oriented edge of T_1 or $T_2\}$

$$\operatorname{diag}(-\,\alpha_{s_2}^{-1}\,1, 1, \ldots, 1).$$

By hypothesis, $1 - 2\mu_{s_2} = \frac{2}{k}$ with k an integer. Hence

$$\alpha_{s_2} = \exp 2\pi i \mu_{s_2} = \exp 2\pi i \left(\frac{1}{2} - \frac{1}{k}\right)$$

and $-\,\alpha_{s_2}^{-1} = \exp \dfrac{2\pi i}{k}$. From this result follows.

Corollary **(3.10)**. — *Let* $\mathscr{E}_{1,1}$ *denote the set of partitions in* $\mathscr{E}_1$ *whose two-element coset lies in* S_1. *Assume* (3.9.1). *Let* $\sigma : \tilde{Q}'_1 \to \widetilde{Q'_1/\Sigma}$ *be defined as in* (3.6) *and* (3.8). *Then*

(1) *if k is even, σ is a covering map;*

(2) *if k is odd σ has local degree 2 at each point of* $\rho^{-1}(Q'_T)$ *for all* $T \in \mathscr{E}_{1,1}$.

Proof. — The map σ is open and surjective by (3.3). Consider $\rho : \tilde{Q}'_1 \to Q'_1$ at a point $\tilde{y}$ of $\rho^{-1}(y)$ with $y \in Q'_T$ where $T \in \mathscr{E}_{1,1}$. Then

$$\begin{aligned}\text{local degree of } \rho &= \text{order}\,\frac{D_y}{D_y \cap \operatorname{Ker}\theta} = \text{order}\,\frac{D_y \operatorname{Ker}\theta_\Sigma}{\operatorname{Ker}\theta_\Sigma}\\ &= \text{order}\,\theta_\Sigma([s_1, s_2]^2)\end{aligned}$$

where $\{s_1, s_2\}$ determines T. Hence

$$\text{local degree of } \rho = \begin{cases} k/2 & \text{if } k \text{ is even,}\\ k & \text{if } k \text{ is odd.}\end{cases}$$

Similarly, the local degree of $\rho_\Sigma : (\widetilde{Q'/\Sigma})_1 \to Q'_1/\Sigma$ is the order of $\theta_\Sigma([s_1, s_2])$ above any point of $\tau(Q'_T)$, where $\tau : Q'_1 \to Q'_1/\Sigma$ is the orbit map. Since the local degree of τ at y is 2, one can verify from the commutative diagram of (3.8) the asserted local degree of σ at points of $\rho^{-1}(Q'_T)$ for all $T \in \mathscr{E}_{1,1}$. Since σ is a covering map on $\widetilde{Q}'$, the result follows.

(3.11) The exact homotopy sequence of the fibration of Q' by Σ orbits gives the exact sequence

$$1 \to \pi_1(Q', o) \to \pi_1(Q'/\Sigma, o) \to \Sigma \to 1.$$

Assume (3.9.1) with *k odd*. Then, by Lemma (3.9), $\theta_\Sigma([s_1, s_2])$ lies in the group generated by $\theta_\Sigma([s_1, s_2]^2)$ for any 2-element coset $\{s_1, s_2\}$ of a partition in $\mathscr{E}_{1,1}$. It follows at once that

$$\textbf{(3.11.1)} \qquad \theta_\Sigma(\pi_1(Q', o)) = \theta_\Sigma(\pi_1(Q'/\Sigma, \bar{o}), \text{ or equivalently}$$

$$\pi_1(Q', o) \operatorname{Ker} \theta_\Sigma = \pi_1(Q'/\Sigma, \bar{o}), \text{ or equivalently,}$$

$$\frac{\operatorname{Ker} \theta_\Sigma}{\operatorname{Ker} \theta} \cong \Sigma.$$

Hence the action of Σ on Q'_1 has a faithful lift to the action of $\dfrac{\operatorname{Ker} \theta_\Sigma}{\operatorname{Ker} \theta}$ on $\widetilde{Q}'_1$ and to $\widetilde{Q}_{sst}$ as well. Thus if k is odd, we may write, by abuse of notation

$$\textbf{(3.11.1)}' \qquad \widetilde{Q}_{sst}/\Sigma = \widetilde{Q_{sst}/\Sigma}.$$

The action of the transposition of two elements of S on $\widetilde{Q}_{sst}$ is clear from (3.8).

If on the other hand (3.9.1) holds with *k even*, then for all $T \in \mathscr{E}_{1,1}$ and $y \in Q'_T$ (under the identification of $\pi_1(Q', o)$ with a subgroup of $\pi_1(Q'/\Sigma, \bar{o})$) $D_{\tau(y)}/D_y = \theta_\Sigma(D_{\tau(y)})/\theta(D_y)$, since each side is isomorphic to $\mathbf{Z}/2\mathbf{Z}$, by Lemma (3.9) for the right side and by the local degree of τ being 2. It follows that $D_{\tau(y)} \cap \operatorname{Ker} \theta_\Sigma \subset D_y$. Hence

$$D_y \cap \operatorname{Ker} \theta = D_{\tau(y)} \cap \operatorname{Ker} \theta_\Sigma.$$

Since these subgroups together with $\{D_y;\, y \in Q - Q'\}$ generate $\operatorname{Ker} \theta$ and $\operatorname{Ker} \theta_\Sigma$ (because Q_{st} and $\widetilde{Q}_{st}$ are simply connected), we get $\operatorname{Ker} \theta = \operatorname{Ker} \theta_\Sigma$. Consequently

$$\textbf{(3.11.2)} \qquad \frac{\text{Image } \theta_\Sigma(\pi_1(Q'/\Sigma, \bar{o}))}{\text{Image } \theta(\pi_1(Q', o))} = \Gamma_\Sigma/\Gamma = \Sigma$$

and

$$\textbf{(3.11.2)}' \qquad \widetilde{Q}_{st} = \widetilde{Q_{st}/\Sigma}.$$

Theorem **(3.12)**. — *Let* S_1 *be a subset of* S *and let* Σ *denote the permutation group of* S_1. *Assume that* $(\mu_s)_{s \in S}$ *satisfies condition* (Σ INT) (*cf.* (2.21)). *Then* $\operatorname{Im} \theta_\Sigma$ *is a lattice in* PU(card S $-$ 3, 1).

Proof. — We can assume that S_1 has more than one element. Set $1 - 2\mu_s = \frac{2}{k}$ for $s \in S_1$. By hypothesis (Σ INT), k is an integer. If k is even, then $\operatorname{Im} \theta_\Sigma$ is a finite extension of $\operatorname{Im} \theta$ by (3.11.2) and moreover condition (INT) of DM is satisfied. Hence $\operatorname{Im} \theta$ is a lattice by the main theorem of DM. Thus $\operatorname{Im} \theta_\Sigma$ is a lattice if k is even.

Assume now that k is odd. Set

$$\begin{array}{lll} U_\Sigma = Q_{st}/\Sigma, & U_{\Sigma,0} = Q'/\Sigma, & U_{\Sigma,1} = Q_1/\Sigma \\ U \ \ = Q_{st}, & U_0 \ \ = Q', & U_1 \ \ = Q_1 \\ \widetilde{U}_\Sigma = \widetilde{Q_{st}/\Sigma}, & \widetilde{U}_{\Sigma,0} = \widetilde{Q'/\Sigma}, & \widetilde{U}_{\Sigma,1} = \widetilde{Q_1/\Sigma} \end{array}$$

where $Q_1 = Q \cup \coprod_{T \in \mathscr{E}} Q_T$, and $\widetilde{Q_1/\Sigma}$ is the completion of Q'/Σ over Q_1/Σ. By (3.5) we have a commutative diagram

$$\text{(3.12.1)} \qquad \begin{tikzcd} \widetilde{Q}_{st} \arrow[r, "w_\mu"] \arrow[d, "\sigma"'] & B^+(\alpha)_o \arrow[d, equal] \\ \widetilde{Q_{st}/\Sigma} \arrow[r, "w_\mu"] & B^+(\alpha)_{\bar{o}} \end{tikzcd}$$

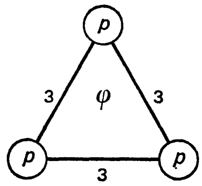
Inasmuch as w_μ is etale on $\widetilde{Q}$ by Proposition (3.9) of DM, it follows at once that w_μ is etale on $\widetilde{Q/\Sigma}$, the completion of $\widetilde{Q'/\Sigma}$ over Q/Σ and that $\widetilde{Q/\Sigma}$ is non-singular even though Q/Σ may have singularities. As in DM, we take a stratification $\mathscr{S}$ of Q_{st} with strata Q_T where T ranges over the stable partitions of S. Let $\mathscr{S}_\Sigma$ denote the image of $\mathscr{S}$ under σ. We wish to apply Proposition (10.16.1) of DM to the diagram

$$\begin{tikzcd} \widetilde{U}_{\Sigma,0} \arrow[r] \arrow[d] & \widetilde{U}_{\Sigma,1} \arrow[r] \arrow[d] & \widetilde{U}_\Sigma \arrow[r, "w_\mu"] \arrow[d, "\rho"] & B^+(\alpha)_{\bar{o}} \\ U_{\Sigma,0} \arrow[r] & U_{\Sigma,1} \arrow[r] & U_\Sigma & \end{tikzcd}$$

All of the hypothesis of Proposition (10.16.1) descend from U_i to $U_{\Sigma,i}$ except possibly the assertion in 1(*e*) : $w_\mu \mid \widetilde{U}_{\Sigma,1}$ is a local homeomorphism. This last condition follows directly at all points except those in $\sigma(Q_T)$ with $T \in \mathscr{E}_{1,1}$. However, at such points we use in diagram (3.12.1) that σ has local degree 2 by Corollary (3.10). Consequently at $\sigma(Q_T)$ with $T \in \mathscr{E}_{1,1}$, the map $w_\mu : \widetilde{Q_{st}/\Sigma} \to B^+(\alpha)_{\bar{o}}$ has local degree $\frac{1}{2}$ (the degree of $w_\mu : \widetilde{Q}_{st} \to B^+(\alpha)_o$ at Q_T). The computation in DM § 9 shows that $w_\mu \mid \widetilde{U}_{\Sigma,1}$ has local degree 1 at points of $\sigma(Q_T)$ for $T \in \mathscr{E}_{1,1}$. By Proposition (10.16.1), $w_\mu : \widetilde{U}_\Sigma \to B^+(\alpha)_{\bar{o}}$ is a local homeomorphism. The proof of Theorem (10.18.2) of DM applies verbatim to yield that $\widetilde{w}_\mu : \widetilde{Q_{sst}/\Sigma} \to \overline{B}^+(\alpha)$ is a homeomorphism onto an open subset of $\overline{\mathrm{B}}^+(\alpha)_{\bar{o}}$ in the DM (5.4) topology and maps $\widetilde{\mathrm{Q}_{st}/\Sigma}$ homeomorphically onto $\mathrm{B}^+(\alpha)_{\bar{o}}$. The image is a lattice, by the same reasoning as in DM. This completes the proof.

4. RCP

In [2], there is a geometric construction of a fundamental domain for groups $\Gamma(\varphi)$ in $\mathrm{PU}(2, 1)$ generated by **C**-reflections on a 3 dimensional complex vector space $\mathrm{V}(\varphi)$ with Coxeter diagram



$(p = 3,4,5)$

and ibid p. 248 there is a list of the groups $\Gamma(\varphi)$ which satisfy the condition (CD2) ensuring discreteness. Let $\mathrm{A}\Gamma(\varphi)$ denote the group obtained by adjoining to $\Gamma(\varphi)$ the group of cyclic permutations of its generators. Then $\mathrm{card}(\mathrm{A}\Gamma(\varphi)/\Gamma(\varphi)) = 1$ or 3.

Theorem. — Let $d = 2$, $\mu_0 = \mu_1 = \mu_2$, *and let* Σ *denote the symmetric group on* $\{0, 1, 2\}$. *Then each of the groups* $\mathrm{A}\Gamma(\varphi)$ *satisfying condition* (CD2) *coincides with the group* Γ_Σ *for suitable* $\{\mu_i \mid i = 0, \ldots, 4\}$ *satisfying condition* (Σ INT).

Proof. — Set $\eta = e^{\pi i/p}$, $\rho = \text{order } \bar{\eta} i \varphi^3$, $\sigma = \text{order } \bar{\eta} i \bar{\varphi}^3$, $t = \frac{1}{\pi} \arg \varphi^3$. The list of $\Gamma(\varphi)$ is specified by the values of t, ρ, σ with $0 \leq t < 3\left(\frac{1}{2} - \frac{1}{p}\right)$. We write $k_{ij} = (1 - \mu_i - \mu_j)^{-1}$, $0 \leq i < j \leq 4$ and $\Gamma(p, t) = \Gamma(\varphi)$.

Set

$$\mu_0 = \frac{1}{2} - \frac{1}{p},$$

$$k_{03} = \rho,$$

$$k_{04} = \begin{cases} \sigma \text{ if } 0 \leq t \leq \frac{1}{2} - \frac{1}{p}, \\ -\sigma \text{ if } \frac{1}{2} - \frac{1}{p} < t < 3\left(\frac{1}{2} - \frac{1}{p}\right), \end{cases}$$

$$t = \frac{1}{k_{03}} - \frac{1}{k_{04}}.$$

By a lenghty but straightforward calculation (cf. [3]), the map $\mathrm{R}_1(\varphi) \mapsto \theta_\Sigma([01])$, $\mathrm{R}_2(\varphi) \mapsto \theta_\Sigma([12])$, $\mathrm{R}_3(\varphi) \mapsto \theta_\Sigma([20])$ yields an isomorphism of $\mathrm{A}\Gamma(\varphi)$ onto Γ_Σ induced by an isometry of $\mathrm{V}(\varphi)$ onto $(\mathrm{H}^1(\mathrm{P}_o, \mathrm{L}), \psi)$. (For a geometric proof, cf. [4]).

We list the groups $\Gamma(\varphi)$ and the corresponding (μ_1). From p. 248 of [2] we see where $\mathrm{A}\Gamma(\varphi)/\Gamma(\varphi)$ has order 1 or 3. In the last five cases, $\mathrm{A}\Gamma$ contains a Picard lattice as a subgroup of index 6 by (3.11.2). In the last column, write $\mathrm{A}\Gamma$ if $\Gamma_\Sigma \neq \Gamma(p, t)$.

#	p	k_{03}	k_{04}	t	μ_0	μ_3	μ_4	Arith	$\Gamma_\Sigma = \Gamma$ or $A\Gamma$
1	3	12	12	0	1/6	9/12	9/12		$A\Gamma$
2	3	10	15	1/30	1/6	22/30	23/30	NA	Γ
3	3	9	18	1/18	1/6	13/18	14/18		$A\Gamma$
4	3	8	24	1/12	1/6	17/24	19/24	NA	Γ
5	3	7	42	5/42	1/6	29/42	34/42	NA	Γ
6	3	6	∞	1/6	1/6	4/6	5/6		$A\Gamma$
7	3	5	-30	7/30	1/6	19/30	26/30		Γ
8	3	4	-12	1/3	1/6	7/12	11/12		Γ
9	5	5	10	1/10	3/10	5/10	6/10		Γ
10	5	4	20	1/5	3/10	9/20	13/20	NA	Γ
11	5	3	-30	11/30	3/10	11/30	22/30	NA	$A\Gamma$
12	5	2	-5	7/10	3/10	2/10	9/10		Γ
13	4	8	8	0	1/4	5/8	5/8		Γ
14	4	6	12	1/12	1/4	7/12	8/12	NA	$A\Gamma$
15	4	5	20	3/20	1/4	11/20	14/20	NA	Γ
16	4	4	∞	1/4	1/4	2/4	3/4		Γ
17	4	3	-12	5/12	1/4	5/12	10/12		$A\Gamma$

5. Lattices Γ_Σ in $PU(N-3, 1)$ for $N \geq 5$ satisfying (Σ INT), p odd

(5.1) $N > 5$.

There are groups Γ_Σ satisfying condition (Σ INT) only for $N \leq 12$. We list all cases with $6 \leq N \leq 12$, p odd. All are arithmetic. For $p = 3$, all are centralizers of a subgroup of the first one except for $\left(\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{7}{12}, \frac{7}{12}\right)$.

N	p	μ_0	Multiplicity of μ_0	Remaining μ_i
12	3	$\frac{1}{6}$	12	
11	3	$\frac{1}{6}$	10	$\frac{2}{6}$
	3	$\frac{1}{6}$	9	$\frac{3}{6}$ $\frac{3}{6}$
10	3	$\frac{1}{6}$	8	$\frac{2}{6}$ $\frac{2}{6}$
9	3	$\frac{1}{6}$	8	$\frac{4}{3}$

N	p	μ_0	Multiplicity of μ_0	Remaining μ
	3	$\frac{1}{6}$	7	$\frac{2}{6}$ $\frac{3}{6}$
	3	$\frac{1}{6}$	6	$\frac{2}{6}$ $\frac{2}{6}$ $\frac{2}{6}$
8	3	$\frac{1}{6}$	7	$\frac{5}{6}$
	3	$\frac{1}{6}$	6	$\frac{4}{6}$ $\frac{2}{6}$
	3	$\frac{1}{6}$	6	$\frac{3}{6}$ $\frac{3}{6}$
	3	$\frac{1}{6}$	5	$\frac{2}{6}$ $\frac{2}{6}$ $\frac{3}{6}$
7	3	$\frac{1}{6}$	5	$\frac{3}{6}$ $\frac{4}{6}$
	3	$\frac{1}{6}$	5	$\frac{2}{6}$ $\frac{5}{6}$
	3	$\frac{1}{6}$	5	$\frac{7}{12}$ $\frac{7}{12}$
	3	$\frac{1}{6}$	4	$\frac{2}{6}$ $\frac{2}{6}$ $\frac{4}{6}$
	3	$\frac{1}{6}$	4	$\frac{2}{6}$ $\frac{3}{6}$ $\frac{3}{6}$
	5	$\frac{3}{10}$	6	$\frac{2}{10}$
6	3	$\frac{1}{6}$	4	$\frac{4}{6}$ $\frac{4}{6}$
	3	$\frac{1}{6}$	4	$\frac{3}{6}$ $\frac{5}{6}$
	3	$\frac{1}{6}$	3	$\frac{2}{6}$ $\frac{3}{6}$ $\frac{4}{6}$
	3	$\frac{1}{6}$	3	$\frac{3}{6}$ $\frac{3}{6}$ $\frac{3}{6}$
	5	$\frac{3}{10}$	5	$\frac{5}{10}$
	5	$\frac{3}{10}$	4	$\frac{2}{10}$ $\frac{6}{10}$

(5.2) $N = 5$.

In addition to lattices listed in § 4 which satisfy condition (Σ INT) but not condition (INT), we have the following.

p	μ_0	Multiplicity	Remaining μ_i	Arith
5	$\frac{3}{10}$	4	$\frac{8}{10}$	
5	$\frac{3}{10}$	2	$\frac{9}{20}, \frac{9}{20}, \frac{1}{2}$	NA
7	$\frac{5}{14}$	4	$\frac{8}{14}$	
9	$\frac{7}{18}$	4	$\frac{8}{18}$	NA
	$\frac{7}{18}$	3	$\frac{5}{18}, \frac{10}{18}$	NA

The lattice corresponding to $\mu = \left(\frac{3}{10}, \frac{3}{10}, \frac{3}{10}, \frac{3}{10}, \frac{8}{10}\right)$ deserves mention.

1. Let M_{st} denote the subset of μ-stable points in $(\mathbf{P}^1)^5$ and let $\pi : M_{st} \to Q_{st}$ denote the map to PGL orbits. The group Σ_4 of permutations on the first four coordinates descends to an action on P_{st}. We have

$$\begin{aligned}(x_1, x_2, 1, 0, \infty) &\equiv (1 - x_1, 1 - x_2, 0, 1, \infty) \bmod \mathrm{PGL}\\ &\equiv \sigma(1 - x_2, 1 - x_1, 1, 0, \infty) \bmod \mathrm{PGL}\end{aligned}$$

where σ denotes the permutation $(1, 2)(3, 4)$. Hence σ fixes each point of the line $L = \{\pi(x, 1 - x, 1, 0, \infty) : x \neq \infty\}$ and this line punctured at $x = 0, \frac{1}{2}, 1$ lies in the set $Q - Q'$ (cf. Remark of (3.6)). In this example, Q_{st} is the projective plane and σ descends to the involution $[x_1, x_2, 1] \to [1 - x_2, 1 - x_1, 1]$ in the line $x_1 + x_2 = 1$.

2. The lattice Γ_μ is the lattice $\Gamma\left(5, \frac{1}{2}\right)$ of [2] by the result in § 4 above. On the other hand, it is proved in [2] that $\Gamma\left(5, \frac{1}{2}\right)$ is isomorphic to $\Gamma\left(5, \frac{7}{10}\right)$. Using the result in § 4, $\Gamma\left(5, \frac{7}{10}\right)$ coincides with the group Γ_ν, $\nu = \left(\frac{3}{10}, \frac{3}{10}, \frac{3}{10}, \frac{2}{10}, \frac{9}{10}\right)$. Consequently, $\Gamma_\mu \cong \Gamma_\nu$. It is clear that Γ_ν contains a complex reflection of order 2, a fact that is not so obvious for Γ_μ. The existence of this reflection in Γ_μ is related to the involution in the line L above.

We take this opportunity to insert 3 errata for the proof that $\Gamma\left(5, \frac{1}{2}\right) \cong \Gamma\left(5, \frac{7}{10}\right)$ in [2]:

Read on page 273, Equation (21.1): $\ldots - \alpha\varphi \dfrac{1 - \eta + 2\eta^{-2}}{1 + \eta + \bar{\eta}}$

line 12: Γ_{12} not F_{12}

line 13: ... subgroup of $\Gamma \cap PU(2)$.

6. $A\Gamma(\varphi)$ as extensions of Picard lattices in $PU(2, 1)$

The 27 Picard lattices are listed in (14.3) of DM. For all except five of these lattices, at least three of the μ's are equal; we relabel these μ_0, μ_1, μ_2. The corresponding extended lattice Γ_Σ with Σ the permutation group on $\{0, 1, 2\}$ coincides with the group $A\Gamma(\varphi)$ by § 4. We list below the p and t-parameters of the corresponding Γ_Σ, labelling each Picard lattice by its position on the list of DM (14.3).

Clearly $p = \left(\frac{1}{2} - \mu_0\right)^{-1}$. By § 4,

$$t = k_{03}^{-1} - k_{04}^{-1} = (1 - \mu_0 - \mu_3) - (1 - \mu_0 - \mu_4) = \mu_4 - \mu_3.$$

We order the indices so that $\mu_3 \leq \mu_4$. As a result $k_{03} > 0$ and $k_{03} < |k_{04}|$.

(Of the five Picard lattices not on the list, two are non-arithmetic.)

DM#	D	$D\mu_0$	$D\mu_3$	$D\mu_4$	p	t	k_{03}	k_{04}	$\Gamma_2 = A\Gamma$ or Γ
1	3	1	1	2	6	$\frac{1}{3}$	3	∞	$A\Gamma$
2	4	2	1	1	∞	0	4	4	Γ
3	4	1	2	3	4	$\frac{1}{4}$	4	∞	Γ
4	5	2	2	2	10	0	5	5	Γ
5	6	2	3	3	6	0	6	6	$A\Gamma$
6	6	3	1	2	∞	$\frac{1}{6}$	3	6	$A\Gamma$
8	6	2	1	5	6	$\frac{2}{3}$	2	-6	Γ
9	8	3	3	4	8	$\frac{1}{8}$	4	8	Γ
10	8	2	5	5	4	0	8	8	Γ
11	8	3	1	6	8	$\frac{5}{8}$	2	-8	Γ
12	9	4	2	4	18	$\frac{4}{18}$	3	9	$A\Gamma$

DM#	D	$D\mu_0$	$D\mu_3$	$D\mu_4$	p	t	k_{03}	k_{04}	$\Gamma_2 = A\Gamma$ or Γ
13	10	4	1	7	10	$\frac{6}{10}$	2	-10	Γ
14	12	5	4	5	12	$\frac{1}{12}$	4	6	Γ
16	12	5	3	6	12	$\frac{3}{12}$	3	12	$A\Gamma$
17	12	4	5	7	6	$\frac{2}{12}$	4	12	Γ
21	12	5	1	8	12	$\frac{7}{12}$	2	-12	Γ
22	12	3	7	8	4	$\frac{1}{12}$	6	12	$A\Gamma$
23	12	3	5	10	4	$\frac{5}{12}$	3	-12	$A\Gamma$
24	15	6	4	8	10	$\frac{10}{30}$	3	15	$A\Gamma$
25	18	8	1	11	18	$\frac{10}{18}$	2	-18	Γ
26	20	5	11	14	4	$\frac{3}{20}$	5	20	Γ
27	24	9	7	14	8	$\frac{7}{24}$	3	24	$A\Gamma$

BIBLIOGRAPHY

[1] Deligne, P., and Mostow, G. D., Monodromy of Hypergeometric Functions and Non-lattice Integral Monodromy Groups, *Publ. Math. I.H.E.S.*, this volume, 5-90.

[2] Mostow, G. D., On a Remarkable Class of Polyhedra in Complex Hyperbolic Space, *Pacific J. of Math.*, **86** (1980), 171-276.

[3] Mostow, G. D., On Polyhedra in Complex Hyperbolic Space Associated to Hypergeometric Functions in *n*-Variables (to appear).

[4] Mostow, G. D., Braids, Hypergeometric Functions and Lattices (to appear).

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