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ERRATUM TO :

CARLEMAN ESTIMATES
FOR THE LAPLACE-BELTRAMI EQUATION
ON COMPLEX MANIFOLDS

by ALDO ANDREOTTI and EDOARDO VESENTINI

(Publications Mathématiques de l'Institut des Hautes Études Scientifiques, n° 25)

1. We draw the attention to a mistake which appears in the above quoted paper due to an error in computation. This has to be corrected as indicated below; the rest of the paper remains unchanged.

In the formula (29) p. 102 the curvature tensor $s_{\bar{\beta}_i \beta}^\alpha$ of the metric on the bundle E should be replaced by the expression of the "covariant curvature tensor"

$$\tilde{s}_{\bar{\beta}_i \beta}^\alpha = \nabla_{\bar{\beta}_i} J_{\beta}^\alpha,$$

and this is because we use the Riemannian connection on the bundle $\bar{\Theta}^*$.

Accordingly in formula (31) p. 102 the curvature forms s and L should respectively be substituted by the "covariant" curvature forms \tilde{s} and \tilde{L} .

2. In the following § 5 the use of covariant curvature would however be cumbersome and it is therefore better to abandon the Riemannian connection and make use exclusively of the ∂ -connections and $\bar{\partial}$ -connections (see 2 b) and 3 a)) to establish the inequality we need following the same lines. The argument we shall presently give has to be added as section 15 c) of the paper.

First we note that, in terms of covariant derivatives with respect to the considered connection, the operator $\bar{\partial}$ has the expression

$$\bar{\partial} = \hat{\partial} + S,$$

where for $\varphi \in C^{p,q}(X, E)$:

$$\begin{aligned} (\hat{\partial}\varphi)_{\bar{\alpha}\bar{\beta}_1 \dots \bar{\beta}_{q+1}}^\alpha &= (-1)^p \sum_{r=1}^{q+1} (-1)^{r-1} \nabla_{\bar{\beta}_r} \varphi_{\bar{\alpha}\bar{\beta}_1 \dots \hat{\bar{\beta}}_r \dots \bar{\beta}_{q+1}}^\alpha, \\ (S\varphi)_{\bar{\alpha}\bar{\beta}_1 \dots \bar{\beta}_{q+1}}^\alpha &= (-1)^p \sum_{i,r=1}^{q+1} (-1)^{r-1} \overline{S_{\bar{\beta}_i \beta_r}^\alpha} \varphi_{\bar{\alpha}\bar{\beta}_1 \dots (\bar{\alpha})_i \dots \hat{\bar{\beta}}_r \dots \bar{\beta}_{q+1}}^\alpha, \end{aligned}$$

$S_{\bar{\beta}_i \beta_r}^\alpha$ being the torsion tensor as defined in n° 3 a).

Analogously we see that

$$\theta = \hat{\theta} + T$$

where $T = - * \#^{-1} S \# *$, and for $\varphi \in C^{pq}(X, E)$

$$(\hat{\theta}\varphi)_{A\bar{B}_1 \dots \bar{B}_{q-1}}^a = (-1)^{p-1} \Sigma \nabla_\alpha \varphi_{A\bar{B}_1 \dots \bar{B}_{q-1}}^{a\alpha}$$

We now consider the vectors ξ and η introduced in 14 b) (the covariant derivatives being intended to be with respect to the ∂ - and $\bar{\partial}$ - connections). We have

$$\operatorname{div} \xi - \operatorname{div} \eta = \Sigma \nabla_\beta \xi^\beta - \Sigma \nabla_{\bar{\gamma}} \eta^{\bar{\gamma}} - 2 \Sigma (S_\tau \xi^\tau - \bar{S}_\tau \eta^{\bar{\tau}}),$$

where $S_\tau = \Sigma S_{\tau\beta}^\beta$. A calculation of the same type as the one given in 14 b) gives:

$$(33') \quad \Sigma \nabla_\beta \xi^\beta - \Sigma \nabla_{\bar{\gamma}} \eta^{\bar{\gamma}} = (q-1)! \{A(\bar{\nabla}\varphi, \bar{\nabla}\varphi) + A(\hat{\mathcal{H}}\varphi, \varphi) - A(\hat{\partial}\varphi, \hat{\partial}\varphi) - A(\hat{\theta}\varphi, \hat{\theta}\varphi)\},$$

where now

$$(\hat{\mathcal{H}}\varphi)_{A\bar{B}}^a = \sum_{i=1}^q (-1)^i \{ \Sigma s_{\bar{b}_i \beta}^a \varphi_{A\bar{B}_i}^{b\beta} + \Sigma L_{\beta\bar{b}_i \sigma}^\sigma \varphi_{A\bar{B}_i}^{a\beta} + \Sigma (-1)^k \Sigma L_{\alpha_k \bar{b}_i \beta}^\alpha \varphi_{\alpha A_k \bar{B}_i}^{a\beta} \}.$$

For $\varphi \in \mathcal{D}^{pq}(X, E)$, by Stokes formula we get

$$(*) \quad \|\bar{\nabla}\varphi\|^2 + (\hat{\mathcal{H}}\varphi, \varphi) = \|\hat{\partial}\varphi\|^2 + \|\hat{\theta}\varphi\|^2 + 2 \int_X \Sigma (S_\tau \xi^\tau - \bar{S}_\tau \eta^{\bar{\tau}}) dX.$$

We have the following estimates

$$\|\hat{\partial}\varphi\|^2 \leq 2(\|\bar{\partial}\varphi\|^2 + \|S\varphi\|^2) \leq 2\|\bar{\partial}\varphi\|^2 + c \int_X |S|^2 A(\varphi, \varphi) dX,$$

$$\|\hat{\theta}\varphi\|^2 \leq 2(\|\theta\varphi\|^2 + \|T\varphi\|^2) \leq 2\|\theta\varphi\|^2 + c \int_X |S|^2 A(\varphi, \varphi) dX,$$

$$\left| \int_X (\Sigma S_\tau \xi^\tau) dX \right| \leq c \{ \varepsilon \int_X |S|^2 A(\varphi, \varphi) dX + \frac{1}{\varepsilon} \int_X A(\bar{\nabla}\varphi, \bar{\nabla}\varphi) dX \},$$

$$\left| \int_X (\Sigma \bar{S}_\tau \eta^{\bar{\tau}}) dX \right| \leq c \{ \varepsilon \int_X |S|^2 A(\varphi, \varphi) dX + \frac{1}{\varepsilon} \int_X A(\hat{\theta}\varphi, \hat{\theta}\varphi) dX \},$$

where c is a universal positive constant, ε any positive number and $|S|^2$ is the length of the torsion tensor.

We substitute these four estimates, with $\varepsilon = 4c$, in (*). If we set

$$\mathcal{H} = \hat{\mathcal{H}} - (16c^2 + \frac{5}{2}c) |S|^2$$

we obtain an inequality of the type

$$\|\bar{\nabla}\varphi\|^2 + c_1 (\mathcal{H}\varphi, \varphi) \leq c_2 \{ \|\bar{\partial}\varphi\|^2 + \|\theta\varphi\|^2 \}$$

where c_1, c_2 are positive universal constants. Proposition 15 and its corollaries can thus be stated using this expression of \mathcal{H} .

3. The use of identity (33') instead of identity (33) simplifies the argument of section 17 d) in that that no covariant Levi form has to be introduced. What follows

replaces thus 17 d) from line 7 from below of p. 112 onwards. From (33') we get (with the notations of 17 d))

$$\frac{1}{(s-1)!} \int_{\partial Y} (\xi - \eta)_n dS = \|\bar{\nabla} \varphi\|_Y^2 + (\mathcal{K} \varphi, \varphi)_Y - \|\hat{\partial} \varphi\|_Y^2 - \|\hat{\theta} \varphi\|_Y^2 - 2 \int_Y \Sigma (S_\tau \xi^\tau - \bar{S}_\tau \eta^{\bar{\tau}}) dY;$$

hence

$$\|\bar{\nabla} \varphi\|_Y^2 + (\mathcal{K} \varphi, \varphi)_Y + \frac{1}{(s-1)!} \int_{\partial Y} \frac{1}{|\partial f|} \mathcal{L}(f) \{\varphi, \varphi\} dS = \|\hat{\partial} \varphi\|_Y^2 + \|\hat{\theta} \varphi\|_Y^2,$$

and thus as in section 15 c) we derive the inequality

$$\|\bar{\nabla} \varphi\|_Y^2 + c_1 (\mathcal{K} \varphi, \varphi)_Y + \frac{1}{(s-1)!} \int_{\partial Y} \frac{1}{|\partial f|} \mathcal{L}(f) \{\varphi, \varphi\} dS \leq c_2 \{ \|\bar{\partial} \varphi\|_Y^2 + \|\theta \varphi\|_Y^2 \}.$$

If $\mathcal{L}(f)$ has $n - q + 1$ positive eigenvalues at each point of ∂Y we can choose a hermitian metric so that $l_{\mathcal{L}(f)} \geq c_0$ at each point of ∂Y with $c_0 > 0$. Also we can replace the metric h on the fibers of E by $e^{\tau|f|} h$. We will have

$$\frac{1}{(s-1)!} \int_{\partial Y} \frac{1}{|\partial f|} \mathcal{L}(f) \{\varphi, \varphi\} dS \geq c_3 \int_{\partial Y} A(\varphi, \varphi) dS,$$

$$A_\tau(\mathcal{K} \varphi, \varphi) \geq c_4 \tau A_\tau(\varphi, \varphi),$$

with positive constants, c_3 and c_4 , for any $\tau \geq \tau_0$ with a convenient $\tau_0 \geq 0$, and for any $\varphi \in B^{r,s}(Y, E)$ with $\text{supp}(\varphi) \subset U$, provided $s \geq q$. From this we obtain by the same argument as the one given there (p. 113 from line 11 from below onwards) the last inequality of 17 d) (1).

(1) In the expression of $\|\varphi\|_{\frac{n}{s}, \nu}^{(n)}$ there is a misprint: $e^{\tau|f|}$ should replace $e^{\tau f}$.