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N. W. Mc Lachlan
Pierre Humbert

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DIRECTEUR :

Henri VILLAT

Membre de l'Institut,
Professeur à la Sorbonne,
Directeur du « Journal de Mathématiques pures et appliquées ».

FASCICULE C

Formulaire pour le Calcul symbolique

Par MM. N. W. Mc LACHLAN et Pierre HUMBERT

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FORMULAIRE

POUR

LE CALCUL SYMBOLIQUE

Par MM. N. W. Mc LACHLAN
et Pierre HUMBERT.

NOTE LIMINAIRE.

Le calcul symbolique d'Heaviside, souvent appelé calcul opératoire, *operational calculus*, est un instrument de plus en plus utilisé par les physiciens et même par les mathématiciens, qui au début semblaient s'en méfier quelque peu. Mais c'est un outil qu'il faut savoir manier : si les règles opératoires sont simples et faciles à appliquer, il n'en est pas de même pour les correspondances entre originaux et images, et le chercheur, que ses calculs ont conduit à une forme opératoire compliquée, est trop souvent arrêté, faute de savoir à quel original correspond cette image inconnue, de même qu'il lui est parfois très malaisé d'écrire, *a priori*, l'image de telle fonction rencontrée. Les correspondances extrêmement nombreuses, établies par les divers auteurs, sont, en effet, presque toujours disséminées dans des mémoires dont la multiplicité rend toute recherche fort pénible. Le besoin semble urgent de présenter aux amateurs de ce calcul un formulaire contenant le plus grand nombre possible de correspondances, et constituant ainsi un instrument de travail infiniment précieux. Précisément, un ingénieur et mathématicien britannique, M. N. W. McLachlan, a recueilli une collection extraordinairement riche de telles correspondances, liste qu'il avait l'intention de publier en Angleterre, mais pour laquelle il n'a point trouvé d'éditeur. J'ai pensé que ce travail considérable ne devait pas rester ignoré, et grâce à l'accueil que M. Henri Villat a bien voulu lui faire, nous donnons aujourd'hui ce formulaire, que j'ai adapté à l'écriture mathématique usitée en France, et que nous avons, M. McLachlan et moi-même, enrichi d'un certain nombre de résultats inédits. On trouvera donc ici, pour la première fois réunies, près de sept cents formules de calcul symbolique, soit règles opératoires ou correspondances, ces dernières classées d'après la nature de la fonction originale : on pourra ainsi se servir de ce fascicule comme d'un véritable dictionnaire. La tâche ingrate de la vérification de toutes ces formules a été assumée par M. W. T. Howell, que nous sommes heureux de remercier de son aide.

P. H.

ABRÉVIATIONS ET NOTATIONS.

Nous désignerons par t la variable indépendante (réelle) et par p la variable paramétrique. Les lettres m et n représenteront des nombres entiers, les lettres μ et ν des nombres quelconques.

Afin de simplifier l'écriture, dans l'expression des images des fonctions de Bessel et analogues, nous utiliserons les symboles abréviatifs suivants :

$$
\begin{array}{ll}
\mathrm{P} = p + \sqrt{p^2 + a^2}, & q = \sqrt{p^2 + 1}; \\
\mathrm{Q} = p + \sqrt{p^2 + 1}, & r = \sqrt{p^2 + a^2}; \\
\mathrm{R} = p + \sqrt{p^2 - a^2}, & s = \sqrt{p^2 - a^2}; \\
\mathrm{S} = p + \sqrt{p^2 - 1}, & u = \sqrt{p^2 - 1}; \\
\mathrm{T} = p + \sqrt{p^2 - i}, & v = \sqrt{p^2 - \imath a^2}; \\
\mathrm{U} = \dfrac{1 + \sqrt{p^2 + 1}}{p}, & w = \sqrt{p^2 - i}; \\
\end{array}
$$

$$
y = \sqrt{t^2 - b^2}.
$$

Suivant l'usage, nous désignerons par $\mathrm{R}(\nu)$ la partie réelle de ν.

DÉFINITIONS ET NOTATIONS UTILISÉES POUR LES FONCTIONS.

B_n = nombre de Bernoulli d'ordre n

$$B(\mu, \nu) = \int_0^1 x^{\mu-1}(1-x)^{\nu-1}\,dx = \frac{\Gamma(\mu)\Gamma(\nu)}{\Gamma(\mu+\nu)}$$ fonction Bêta eulérienne

$$C(t) = \int_0^t \cos\frac{\pi x^2}{2}\,dx = \frac{1}{\sqrt{2\pi}}\int_0^t \frac{\cos x}{\sqrt{x}}\,dx$$ intégrale de Fresnel

$$C_{m-n}^{n+\frac{1}{2}} = \frac{1}{(2n-1)(2n-3)\dots 3.1}\,\frac{d^n}{dt^n}\,P_m(t)$$ polynome de Gegenbauer

$$ci(t) = -\int_t^\infty \frac{\cos x}{x}\,dx$$ cosinus intégral

$$D_\nu(t) = 2^{\frac{\nu}{2}+\frac{1}{4}}\,t^{-\frac{1}{2}}\,W_{\frac{\nu}{2}+\frac{1}{4},\,\pm\frac{1}{4}}\left(\frac{t^2}{4}\right)$$ fonction de Weber (ou du cylindre parabolique)

$$D_n(t) = e^{-\frac{t^2}{4}}\,He_n(t)$$

$$D_{-1}(t) = \sqrt{\frac{\pi}{2}}\,e^{\frac{t^2}{4}}\,erfc\left(\frac{t}{\sqrt{2}}\right)$$

$$D_{-\frac{1}{2}}(t) = \sqrt{\frac{t}{2\pi}}\,K_{\frac{1}{4}}\left(\frac{t^2}{4}\right)$$

$$E\left(k, \frac{\pi}{2}\right) = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-k^2\sin^2\theta}} \qquad (k^2 < 1)$$ intégrale elliptique complète

$$Ei(t) = \int_{-\infty}^t \frac{e^x}{x}\,dx$$ logarithme intégral

$$Ei(it) = ci(t) + i\,si(t)$$

$$erf(t) = \frac{2}{\sqrt{\pi}}\int_0^t e^{-x^2}\,dx$$ fonction d'erreur intégrale

$$erfc(t) = 1 - erf(t) = \frac{2}{\sqrt{\pi}}\int_t^\infty e^{-x^2}\,dx = \frac{e^{-\frac{t^2}{2}}}{\sqrt{\pi t}}\,W_{-\frac{1}{4},\,\frac{1}{4}}(t^2)$$ fonction complémentaire

$$ {}_r\mathrm{F}_s(\alpha_1, \ldots, \alpha_r; \gamma_1, \ldots, \gamma_s; t) = \sum_{m=0}^{\infty} \frac{(\alpha_1, m) \ldots (\alpha_r, m)}{(\gamma_1, m) \ldots (\gamma_s, m)} \frac{t^m}{m!} $$

fonction hypergéométrique générale

$$ {}_2\mathrm{F}_1(\alpha, \beta; \gamma; t) = 1 + \frac{\alpha\beta t}{1!\,\gamma} + \frac{\alpha(\alpha+1)\,\beta(\beta+1)\,t^2}{2!\,\gamma(\gamma+1)} + \ldots $$

fonction de Gauss

$$ {}_1\mathrm{F}_1(\alpha; \gamma; t) = 1 + \frac{\alpha t}{1!\,\gamma} + \frac{\alpha(\alpha+1)\,t^2}{2!\,\gamma(\gamma+1)} + \ldots $$

fonction de Kummer

$$ \Gamma(t) = \int_0^{\infty} e^{-x} x^{t-1}\, dx \qquad [\mathrm{R}(t) > 0] $$

fonction Gamma eulérienne

$$ \mathrm{H}e_n(t) = (-1)^n e^{\frac{t^2}{2}} \frac{d^n}{dt^n} e^{-\frac{t^2}{2}} = t^n - \frac{n(n-1)}{2} t^{n-2} + \frac{n(n-1)(n-2)(n-3)}{2.4} t^{n-4} - \ldots $$

polynome d'Hermite

$$ \left.\begin{aligned} \mathrm{H}_\nu^{(1)}(t) &= \mathrm{J}_\nu(t) + i\,\mathrm{Y}_\nu(t) = \frac{2}{\pi} i^{-\nu-1} \mathrm{K}_\nu(-it) = -i^{-2\nu} \mathrm{H}_\nu^{(2)}(-t) \\ \mathrm{H}_\nu^{(2)}(t) &= \mathrm{J}_\nu(t) - i\,\mathrm{Y}_\nu(t) = \frac{2}{\pi} i^{\nu+1} \mathrm{K}_\nu(\ it) = -i^{2\nu} \mathrm{H}_\nu^{(1)}(-t) \end{aligned}\right\} $$

fonctions de Hankel

$$ \mathbf{H}_\nu(t) = \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{t}{2}\right)^{\nu+2r+1}}{\Gamma\left(r + \frac{3}{2}\right) \Gamma\left(\nu + r + \frac{3}{2}\right)} $$

fonction de Struve

$$ \mathbf{H}_\nu(ti\sqrt{i}) = ster_\nu t + i\, stei_\nu t $$

$$ \mathrm{I}_\nu(t) = \sum_{r=0}^{\infty} \frac{\left(\frac{t}{2}\right)^{\nu+2r}}{r!\,\Gamma(\nu + r + 1)} = i^{-\nu} \mathrm{J}_\nu(it) = i^{\nu} \mathrm{J}_\nu(-it) $$

fonction de Bessel d'argument imaginaire

$$ \mathrm{I}_0 (t\sqrt{i})^t = ber\,t + i\,bei\,t $$

fonctions de Kelvin

$$ \mathrm{J}_\nu(t) = \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{t}{2}\right)^{\nu+2r}}{r!\,\Gamma(\nu + r + 1)} = \frac{1}{2}\left[\mathrm{H}_\nu^{(1)}(t) + \mathrm{H}_\nu^{(2)}(t)\right] $$

fonction de Bessel

$$ \mathrm{J}i_\nu(t) = \int_t^{\infty} \frac{\mathrm{J}_\nu(t)}{t}\, dt $$

fonction de Bessel intégrale

$$ \mathrm{J}_\nu(ti\sqrt{i}) = ber_\nu t + i\, bei_\nu t $$

fonctions de Kelvin

$$ \mathrm{J}_n^k(t) = \frac{1}{\pi} \int_0^{\pi} (2\cos\theta)^k \cos(n\theta - t\sin\theta)\, d\theta $$

fonction de Bourget

$$J_{\mu,\nu}(t) = \frac{t^{\mu+\nu}}{3^{\mu+\nu}\Gamma(\mu+1)\Gamma(\nu+1)}\, {}_0F_2\left(\mu+1, \nu+1; -\frac{t^3}{27}\right)$$ fonction de Bessel du troisième ordre

$$K_\nu(t) = \frac{\pi}{2\sin\nu\pi}\left[I_{-\nu}(t) - I_\nu(t)\right]$$

$$= \frac{\pi}{2} i^{\nu+1} H_\nu^{(1)}(it) = \frac{\pi}{2} i^{-\nu-1} H_\nu^{(2)}(-it)$$ fonction K de Bessel

$$i^{-\nu} K_\nu(t\sqrt{i}) = ker_\nu t + i\, kei_\nu t$$

$$Ki_\nu(t) = \int_t^\infty \frac{K_\nu(t)}{t}\, dt$$ fonction K intégrale

$$L_\nu(t) = i^{-\nu-1} H_\nu(it) = \sum_{r=0}^{\infty} \frac{\left(\frac{t}{2}\right)^{\nu+2r+1}}{\Gamma\left(r+\frac{3}{2}\right)\Gamma\left(\nu+r+\frac{3}{2}\right)}$$ fonction de Struve d'argument imaginaire

$$L_n(t) = \frac{e^t}{n!}\frac{d^n}{dt^n}(t^n e^{-t}) = 1 - nt + \frac{n(n-1)t^2}{2!\,2!} + \dots$$ polynome de Laguerre

$$L_n^\alpha(t) = t^\alpha \frac{e^t}{n!}\frac{d^n}{dt^n}(t^{n+\alpha}e^{-t}) = \sum_{r=0}^{\infty} \frac{\Gamma(\alpha-\beta+r)}{r!\,\Gamma(\alpha-\beta)} L_{n-r}^\beta(t)$$ polynome de Laguerre étendu

$$M_{\mu,\nu}(t) = t^{\nu+\frac{1}{2}} e^{-\frac{t}{2}}\, {}_1F_1\left(\frac{1}{2}+\nu-\mu; 2\nu+1; t\right)$$ fonction hypergéométrique confluente

$$M_{0,\nu}(t) = 2^{2\nu}\Gamma(\nu+1)\sqrt{t}\, I_\nu\left(\frac{t}{2}\right)$$

$$O_n(t) = \frac{1}{2}\int_0^\infty e^{-xt}\left[(x+\sqrt{x^2+1})^n + (x-\sqrt{x^2+1})^n\right] dx \qquad [R(t) > 0]$$

$$= \frac{2^{n-1} n!}{t^{n+1}}\left[1 + \frac{t^2}{2(2n-2)} + \frac{t^4}{2.4.(2n-2)(2n-4)} + \dots\right]$$ polynome de Neumann

$$P(t, \nu) = \int_0^t e^{-x} x^{\nu-1}\, dx$$ fonction P de Prym

$$P_n(t) = {}_2F_1\left(-n, n+1; 1; \frac{1-t}{2}\right) \qquad (-1 \leqq t \leqq 1)$$ polynome de Legendre

$$P_\nu(t) = {}_2F_1\left(-\nu, \nu+1; 1; \frac{1-t}{2}\right) \qquad (|1-t| < 2)$$ fonction de Legendre

$$P_\nu^m(t) = (t^2-1)^{\frac{m}{2}} \frac{d^m}{dt_m} P_\nu(t)$$ fonction de Legendre associée

$$Q(t, \nu) = \int_t^\infty e^{-x} x^{\nu-1}\, dx$$ fonction Q de Prym

$$Q_\nu(t) = \frac{\sqrt{\pi}\,\Gamma(\nu+1)}{\Gamma\left(\nu+\frac{3}{2}\right)}(2t)^{-\nu-1}\,{}_2F_1\left(\frac{\nu}{2}+1, \frac{\nu+1}{2}; \nu+\frac{3}{2}; \frac{1}{t^2}\right) \quad |t|>1$$

fonction de Legendre de seconde espèce

$$Q_\nu^m(t) = (t^2-1)^{\frac{m}{2}} \frac{d^m}{dt^m} Q_\nu(t)$$

fonction de Legendre associée de seconde espèce

$$S(t) = \int_0^t \sin\frac{\pi x^2}{2}\,dx = \frac{1}{\sqrt{2\pi}}\int_0^t \frac{\sin x}{\sqrt{x}}\,dx$$

intégrale de Fresnel

$$\left.\begin{aligned} s_1(t) &= \frac{1}{3}(e^{-t} + e^{-jt} + e^{-j^2t}) \\ s_2(t) &= \frac{1}{3}(e^{-t} + j\,e^{-jt} + j^2\,e^{-j^2t}) \qquad (j^3=1) \\ s_3(t) &= \frac{1}{3}(e^{-t} + j^2\,e^{-jt} + j\,e^{-j^2t}) \end{aligned}\right\}$$

sinus du troisième ordre

$$(s_1^3 + s_2^3 + s_3^3 - 3s_1s_2s_3 = 1;\ s''' + s = 0)$$

$$S_n(t) = \int_0^\infty e^{-xt}\left[(x+\sqrt{x^2+1})^n - (x-\sqrt{x^2+1})^n\right]\frac{dx}{\sqrt{x^2+1}}$$

polynome de Schläfli

$$S(\nu;\,t) = \int_0^\infty e^{-xt}\frac{dx}{(1+x)^\nu}$$

$$= t^{\nu-1}e^t\int_t^\infty e^{-x}x^{-\nu}\,dx = t^{\frac{\nu}{2}-1}e^{\frac{t}{2}}\,W_{-\frac{\nu}{2},\frac{1-\nu}{2}}(t)$$

fonction de Schlömilch

$$S(1,-t) = -e^{-t}\,Ei(t)$$

$$si(t) = -\int_t^\infty \frac{\sin x}{x}\,dx$$

sinus intégral

$$T_n(t) = \frac{1}{2}\left[(t+i\sqrt{1-t^2})^n + (t-i\sqrt{1-t^2})^n\right] = \cos(n \arccos t)$$

polynome de Tchebychev de première espèce

$$T_m^n(t) = (-1)^n\frac{L_n^m(t)}{\Gamma(m+n+1)}$$

$$= \frac{t^n}{n!\,(m+n)!} - \frac{t^{n-1}}{(n-1)!\,(m+n-1)!\,1!} + \ldots$$

polynome de Sonine

$$U_n(t) = \frac{1}{2i}\left[(t+i\sqrt{1-t^2})^n - (t-i\sqrt{1-t^2})^n\right] = \sin(n \arccos t)$$

polynome de Tchebychev de seconde espèce

$$W_{\mu,\nu}(t) = \frac{\Gamma(-2\nu)}{\Gamma\left(\frac{1}{2}-\mu-\nu\right)}M_{\mu,\nu}(t) + \frac{\Gamma(2\nu)}{\Gamma\left(\frac{1}{2}-\mu+\nu\right)}M_{\mu,-\nu}(t)$$

fonction hypergéométrique confluente de Whittaker

$$W_{0,\nu}(t)=\sqrt{\frac{t}{\pi}}\,K_\nu\left(\frac{t}{2}\right)$$

$$W_{\mu,\pm\frac{1}{4}}(t)=2^{\frac{1}{4}-\mu}\,t^{\frac{1}{4}}\,D_{2\mu-\frac{1}{2}}(\sqrt{2t})$$

$$W_{\nu+\frac{1}{2},\,\nu}(t)=t^{\nu+\frac{1}{2}}\,e^{-\frac{t}{2}}$$

$$Y_\nu(t)=\frac{\cos\nu\pi\,J_\nu(t)-J_{-\nu}(t)}{\sin\nu\pi}=\frac{i}{2}\left[H_\nu^{(2)}(t)-H_\nu^{(1)}(t)\right]$$ fonction de Bessel de seconde espèce

$$Yi_\nu(t)=\int_t^\infty\frac{Y_\nu(t)}{t}\,dt$$ fonction Y intégrale

$$\zeta(\nu,t)=\sum_{r=0}^{\infty}\frac{1}{(t+r)^\nu}\qquad[R(\nu)>1]$$ fonction Zêta de Riemann généralisée

$$\varpi(t)=\log\Gamma(t)-\left(t-\frac{1}{2}\right)\log t+t-\log\sqrt{2\pi}$$

$$=\int_0^\infty\frac{e^{-xt}}{x}\left[\frac{1}{2}-\frac{1}{x}+\frac{1}{e^x-1}\right]dx$$ fonction de Binet

$$\Phi_m(t)={}_1F_1(-m;\,1;\,t)$$ polynome d'Abel

$$\Psi(t)=\frac{d}{dt}\log\Gamma(t)=\frac{\Gamma'(t)}{\Gamma(t)}$$ dérivée logarithmique de la fonction Gamma

$$\gamma=-\Psi(1)=\lim_{n\to\infty}\left[1+\frac{1}{2}+\ldots+\frac{1}{n}-\log n\right]=0{,}5772156649\ldots$$ constante d'Euler

A. — DÉFINITION ET THÉORÈMES GÉNÉRAUX.

Le calcul symbolique substitue à une fonction $f(t)$ de la variable réelle t une fonction $\varphi(p)$ de la variable paramétrique p définie par l'*intégrale de Laplace*

$$(1)\quad (a)\ \varphi(p)=p\int_0^\infty e^{-pt}f(t)\,dt;\qquad (b)\ \varphi_b(p)=p\int_b^\infty e^{-pt}f\left(a\sqrt{t^2-b^2}\right)dt;$$

$\varphi(p)$ est dite *image* de $f(t)$; $f(t)$ est l'*original* de $\varphi(p)$.

Nous écrirons, symboliquement ([1]),

$$f(t)\supset\varphi(p),$$
$$\varphi(p)\subset f(t).$$

Théorème d'inversion (Mellin). — *On peut obtenir $f(t)$ à partir de son image par l'intégrale*

$$(2)\quad (a)\ f(t)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{e^{zt}\varphi(z)}{z}dz;\qquad (b)\ f\left(a\sqrt{t^2-b^2}\right)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{e^{zt}\varphi_b(z)}{z}dz,$$

si les conditions suivantes sont vérifiées (*voir* référence **27**).

a. Toutes les singularités de $\varphi(z)$ sont à la gauche de $c\pm i\infty$, $c>0$.

b. L'intégrale

$$(a)\ \int_{c-i\infty}^{c+i\infty}\left|\frac{\varphi(z)}{z}\right|dz=\int_{-\infty}^{+\infty}\left|\frac{\varphi(c+iy)}{c+iy}\right|dy;\qquad (b)\ \int_{c-i\infty}^{c+i\infty}\left|\frac{\varphi_b(z)}{z}\right|dz$$

est convergente.

c. $R(p)>0$; (a) t réel et >0; (b) t réel et $>b$.

d. L'intégrale (1) *est absolument convergente.*

e. (a) $\left|\frac{\varphi(p)}{p}\right|$; (b) $\left|\frac{e^{bp}\varphi_b(p)}{p}\right|$ *tende vers zéro uniformément*

---

[1] On emploie très souvent la notation $f(t)\doteqdot\varphi(p)$. Un grand nombre de raisons (*voir* référence 24 à la bibliographie) rendent préférable celle que nous adoptons ici, et qui commence à se répandre dans les publications britanniques.

quand $|p|$ *tend vers l'infini, c'est-à-dire que l'intégrale*

$$(a)\quad \int \frac{\varphi(z)\,dz}{z(z-p)}; \qquad (b)\quad \int e^{(z-p)b}\frac{\varphi_h(z)\,dz}{z(z-p)}$$

doit être nulle le long d'un demi-cercle de rayon infini, à droite de $c \pm i\infty$. *L'intégrale* (2) *est alors nulle pour* (*a*) $t < 0$, (*b*) $t < b$.

f. L'intégrale (2) *représente une fonction continue de* t, *pour* $t > 0$. *La condition* (*b*) *est inutile lorsque la plus grande puissance de* z *dans le développement de* $\varphi(z)$ *est* z^ν, *avec* $0 < \mathrm{R}(\nu) < 1$.

Théorème d'impulsion. — *Soit* $y = f(t)$ *une impulsion de courte durée,* $0 < t < h$; *supposons la fonction* $f(t)$ *uniforme, finie en ses discontinuités, lesquelles sont en nombre limité; supposons, en outre, que l'intégrale* $\int_0^h f(t)\,dt$ *soit égale à une constante* A. *Alors, si* h *tend vers zéro, on aura* (voir références 26, 27)

$$f(t) \supset \mathrm{A}p.$$

Ainsi l'on a, pour $0 < t < \frac{\pi}{2\omega}$,

$$\omega \cos \omega t \supset \frac{\omega p}{p^2+\omega^2}\left[p + \omega e^{-\frac{\pi p}{2\omega}}\right]$$

et, pour ω *tendant vers l'infini, le second membre tend vers* p.

Théorème du produit. — *Si* $f_1(t) \supset \varphi_1(p)$ *et* $f_2(t) \supset \varphi_2(p)$, *on a*

$$\int_0^t f_1(\lambda) f_2(t-\lambda)\,d\lambda = \int_0^t f_2(\lambda) f_1(t-\lambda)\,d\lambda \supset \frac{1}{p}\varphi_1(p)\,\varphi_2(p)$$

ou encore

$$\begin{aligned} &f_1(t) f_2(0) + \int_0^t f_1(\lambda) f_2'(t-\lambda)\,d\lambda \\ &= f_2(t) f_1(0) + \int_0^t f_2(\lambda) f_1'(t-\lambda)\,d\lambda \supset \varphi_1(p)\,\varphi_2(p) \end{aligned}$$

et, s'il s'agit de n *fonctions, avec* $f_i(t) \supset \varphi_i(t)$,

$$\begin{aligned} &\int_0^t \cdots \int_0^t f_1(\lambda_1) f_2(\lambda_2) \ldots f_{n-1}(\lambda_{n-1}) f_n(t - \lambda_1 - \ldots - \lambda_{n-1})\,d\lambda_1 \ldots d\lambda_{n-1} \\ &\supset \frac{1}{p^{n-1}}\varphi_1(p) \ldots \varphi_n(p) \qquad (\lambda_1 \geqq 0, \ldots, \lambda_{n-1} \geqq 0;\ \lambda_1 + \ldots + \lambda_{n-1} \leqq t). \end{aligned}$$

B. — FORMULES OPÉRATOIRES.

$$f(t) \supset \varphi(p)$$

$$f(at) \supset \varphi\left(\frac{p}{a}\right) \qquad (a \text{ réel, } > 0\text{; dans certains cas, } a \text{ complexe})$$

$$f(t-h) \supset e^{-hp}\varphi(p) \qquad (t > h) \qquad (h \text{ réel} > 0)$$

$$e^{at}f(bt) \supset \frac{p}{p-a}\varphi\left(\frac{p-a}{b}\right)$$

$$a^{t}f(bt) \supset \frac{p}{p-\log a}\varphi\left(\frac{p-\log a}{b}\right)$$

$$f(t-\log a) \supset a^{-p}\varphi(p) \qquad (a > 1)$$

$$f^{(n)}(t) \supset p^{n}\varphi(p) - \sum_{s=0}^{n-1} p^{n-s}f^{(s)}(0)$$

$$\supset p^{n}\varphi(p), \qquad \text{si } f(0) = f'(0) = \ldots = f^{(n-1)}(0) = 0$$

$$f'(t) \supset p\varphi(p), \qquad \text{si } f(0) = 0$$

$$t^{n}f(t) \supset (-1)^{n}p\frac{d^{n}}{dp^{n}}\left[\frac{\varphi(p)}{p}\right]$$

$$\left(t\frac{d}{dt}\right)^{n}f(t) \supset (-1)^{n}\left(p\frac{d}{dp}\right)^{n}\varphi(p)$$

$$\int_0^t\int_0^t \ldots \int_0^t f(t)\,(dt)^{n} \supset \frac{\varphi(p)}{p^{n}}$$

$$\int_0^t t\int_0^t \ldots t\int_0^t t\,f(t)\,(dt)^{n} \supset (-1)^{n}p\left(\frac{1}{p}\frac{d}{dp}\right)^{n}\left[\frac{\varphi(p)}{p}\right]$$

$$\left(\frac{1}{t}\frac{d}{dt}\right)^{n}f(t) \supset p\int_p^\infty p\int_p^\infty \ldots p\int_p^\infty \varphi(p)\,(dp)^{n}, \qquad \text{si } \left[\left(\frac{1}{t}\frac{d}{dt}\right)^{s}f(t)\right]_{t=0} = 0$$

$$[s = 0, 1, \ldots, (n-1)]$$

$$\frac{f(t)}{t^{n}} \supset p\int_p^\infty \ldots \int_p^\infty \frac{\varphi(p)}{p}\,(dp)^{n}$$

$$\int_0^t \frac{f(t)}{t}\,dt \supset \int_p^\infty \frac{\varphi(p)}{p}\,dp$$

$$\int_t^\infty \frac{f(t)}{t}\,dt \supset \int_0^p \frac{\varphi(p)}{p}\,dp$$

$$\int_0^\infty \frac{f(t)}{t}\,dt = \int_0^\infty \frac{\varphi(p)}{p}\,dp$$

$$\int_0^t f(\lambda)f(t-\lambda)\,d\lambda \supset \frac{\varphi^2(p)}{p}$$

$$f(t^2) \supset \frac{p}{\sqrt{\pi}}\int_0^\infty e^{-\frac{p^2x^2}{4}}\varphi\left(\frac{1}{x^2}\right)dx$$

$$\frac{1}{\sqrt{\pi t}}\int_0^\infty e^{-\frac{x^2}{4t}}f(x)\,dx \supset \varphi(\sqrt{p})$$

$$t^{\frac{\nu}{2}}\int_0^\infty x^{-\frac{\nu}{2}}\,\mathrm{J}_\nu(2\sqrt{xt})f(x)\,dx \supset p^{1-\nu}\varphi\left(\frac{1}{p}\right) \qquad \mathrm{R}(\nu) > -1.$$

$$\frac{t}{2}\sqrt{\pi}\int_0^\infty \mathrm{J}_{1,\frac{1}{2}}\left[3\sqrt[3]{\frac{t^2x}{4}}\right]f(x)\frac{dx}{\sqrt{x}} \supset \varphi\left(\frac{1}{p^2}\right)$$

$$\int_0^\infty \frac{t^x\,f(x)\,dx}{\Gamma(x+1)} \supset \frac{\varphi(\log p)}{\log p}$$

$$\int_0^\infty \frac{t^x f'(x)\,dx}{\Gamma(x+1)} \supset -\varphi'(\log p)$$

$$\int_0^t \mathrm{J}_0[2\sqrt{x(t-x)}]f(x)\,dx \supset \frac{\varphi\left(p+\frac{1}{p}\right)}{p+\frac{1}{p}}$$

$$\frac{1}{2^n t^{\frac{n+1}{2}}\sqrt{\pi}}\int_0^\infty e^{-\frac{x^2}{4t}}\mathrm{H}e_n\left(\frac{x}{2\sqrt{t}}\right)f(x)\,dx \supset p^{\frac{n}{2}}\varphi(\sqrt{p})$$

$$\frac{1}{\sqrt{\pi t}}\int_0^\infty e^{-\frac{x^2}{4t}}f^{(2n+1)}(x)\,dx + \frac{1}{4\sqrt{\pi t}}\sum_{s=0}^{n}(-1)^{n-s}\frac{(2n-2s)!\,f^{(2s)}(0)}{(n-s)!\,t^{n-s}} \supset p^{n+\frac{1}{2}}\varphi(\sqrt{p})$$

$$\frac{1}{\sqrt{\pi t}}\int_0^\infty e^{-\frac{x^2}{4t}}\;f^{(2n)}(x)\,dx + \frac{1}{4\sqrt{\pi t}}\sum_{s=0}^{n}(-1)^{n-s}\frac{(2n-2s)!\,f^{(2s-1)}(0)}{(n-s)!\,t^{n-s}} \supset p^n\varphi(\sqrt{p})$$

$$f(t) - \int_0^t f(\sqrt{t^2-x^2})\,\mathrm{J}_1(x)\,dx \supset \frac{p}{\sqrt{p^2+1}}\varphi(\sqrt{p^2+1})$$

$$\frac{1}{\sqrt{\pi t}}\int_0^\infty e^{-\frac{x^2}{4t}}\left[f(x) - \int_0^x y f(y)\frac{\mathrm{J}_1(\sqrt{x^2-y^2})}{\sqrt{x^2-y^2}}dy\right]dx \supset \sqrt{\frac{p}{p+1}}\varphi(\sqrt{p+1})$$

$$\frac{1}{\sqrt{\pi t}}\int_0^\infty e^{-\frac{x^2}{4t}}x^{\frac{\nu}{2}}\,dx\int_0^\infty \mathrm{J}_\nu(2\sqrt{xy})f(y)\,y^{-\frac{\nu}{2}}\,dy \supset p^{\frac{1-\nu}{2}}\varphi\left(\frac{1}{\sqrt{p}}\right)$$

$$\frac{1}{2^n\sqrt{\pi}\,t^{\frac{n}{2}+1}}\int_0^\infty e^{-\frac{x^2}{4t}}\,\mathrm{H}e_n\left(\frac{x}{2\sqrt{t}}\right)dx\int_0^\infty f(y)\,\mathrm{J}_\nu(2\sqrt{xy})\left(\frac{x}{y}\right)^{\frac{\nu}{2}}dy \supset p^{\frac{n-\nu-1}{2}}\varphi\left(\frac{1}{\sqrt{p}}\right)$$

$$t^n f(t^2) \supset \frac{\nu p}{2^{n+1}\sqrt{\pi}}\int_0^\infty e^{-\frac{p^2x^\nu}{4}}\mathrm{H}e_n\left(\frac{px^{\frac{\nu}{2}}}{2}\right)\varphi(x^{-\nu})\,x^{\frac{n\nu+\nu}{2}-1}\,dx \qquad \text{quel que soit } \nu \quad (\neq 0).$$

Si $f(t) \supset \varphi(p)$ et $g(t) \supset f(p)$, on a

$$p\int_0^\infty \frac{g(t)\,dt}{(p+t)^2} = \varphi(p).$$

Si $f(t) \supset \varphi(p)$ et $g(t) \supset \sqrt{p}\,f\left(\frac{1}{p}\right)$, on a

$$\frac{\sqrt{\pi}}{2}\,t\,g\left(\frac{t^2}{4}\right) \supset \varphi(p^2).$$

Si $f(t) \supset \varphi(p)$ et $g(x, t) \supset p^\nu e^{-xp^\mu}$, on a

$$\int_0^\infty g(x, t) f(x)\,dx \supset p^{\nu-\mu}\,\varphi(p^\mu).$$

Si $f_1(t) \supset \varphi_1(p)$ et $f_2(t) \supset \varphi_2(p)$, on a

$$f_1(t) f_2(t) \supset p\,f_1\left(-\frac{d}{dp}\right)\left[\frac{\varphi_2(p)}{p}\right] = p\,f_2\left(-\frac{d}{dp}\right)\left[\frac{\varphi_1(p)}{p}\right],$$

si f_1 ou f_2 sont développables en série ordonnée suivant les puissances positives de t.

Si $f_m(t) \supset \varphi_m(p)$, on a

$$\sum_{m=1}^{n} f_m(t) \supset \sum_{m=1}^{n} \varphi_m(t).$$

Si $n = \infty$, la formule subsiste pourvu que les deux membres soient convergents.

$$\int_t^\infty f(t)\,dt \supset \left[\frac{\varphi(p)}{p}\right]_{p=0} - \frac{\varphi(p)}{p},$$

si l'intégrale est convergente, $t \geqq 0$.

$$\int_t^\infty f(a\sqrt{t^2 - b^2})\,dt \supset e^{-bp}\left[\frac{\varphi(p)}{p}\right]_{p=0} - \frac{\varphi(p)}{p},$$

si l'intégrale est convergente, $t \geqq b$.

Si $f(a\sqrt{t^2 - b^2}) + c \supset \varphi(p)$, on a

$$f(a\sqrt{t^2 - b^2}) \supset \varphi(p) - c\,e^{-bp}, \qquad t > b.$$

C. — LISTE D'IMAGES.

D'une façon générale, le théorème d'inversion de Mellin s'applique à toutes les correspondances réunies ci-après. Nous signalons par un ✱ les correspondances qui ne s'appliquent qu'à la partie réelle de l'intégrale de Mellin.

1. — Fonctions algébriques.

$$t^\nu \supset \frac{\Gamma(\nu+1)}{p^\nu} \qquad R(\nu) > -1$$

$$(t^2-b^2)^\nu \supset \frac{(2b)^{\nu+\frac{1}{2}}\Gamma(\nu+1)\,b^{2\nu+1}}{\sqrt{\pi}\,p^{\nu-\frac{1}{2}}} K_{\nu+\frac{1}{2}}(bp) \qquad (t>b), \quad R(\nu) \geqq -\frac{1}{2}$$

$$(2bt-t^2)^{n-\frac{1}{2}} \supset \sqrt{\pi}\,\Gamma\left(n+\frac{1}{2}\right) 2^n b^n p^{1-n} e^{-pb} I_n(bp) \qquad (0<t<2b;\ n>0)$$

$$(t^2+2bt)^\nu \supset \frac{\Gamma(\nu+1)}{\sqrt{\pi}} 2^{\nu+\frac{1}{2}} b^{\nu+\frac{1}{2}} p^{\frac{1}{2}-\nu} e^{pb} K_{\nu+\frac{1}{2}}(bp) \quad (t>0) \quad R(\nu) \geqq -\frac{1}{2}$$

$$\frac{1}{1+t} \supset -p\,e^p\,\mathrm{E}i(-p)$$

$$\frac{1}{1-t} \supset p\,e^{-p}\,\mathrm{E}i(p)$$

$$\frac{1}{(1+t)^\nu} \supset p\,S(\nu,p) = p^{\frac{\nu}{2}} e^{\frac{p}{2}}\,W_{-\frac{\nu}{2},\frac{1-\nu}{2}}(p)$$

$$\frac{1}{\sqrt{1+t}} \supset \sqrt{\pi p}\,e^p\,\mathrm{erfc}\sqrt{p}$$

$$\frac{t}{(1+t)^\nu} \supset 1-(p+\nu-1)\,S(\nu,p)$$

$$\frac{1}{1+t^2} \supset p[\sin p\,ci(p) - \cos p\,si(p)]$$

$$\frac{t}{1+t^2} \supset -p[\cos p\,ci(p) + \sin p\,si(p)]$$

$$\frac{1}{\sqrt{1+t^2}} \supset \frac{\pi p}{2}[\mathbf{H}_0(p) - Y_0(p)]$$

$$\frac{t}{\sqrt{1+t^2}} \supset \frac{\pi p}{2}[\mathbf{H}_1(p) - Y_1(p)] - 1 \qquad ✱$$

$$(1-t^2)^{\nu-\frac{1}{2}} \supset \frac{\sqrt{\pi}\,\Gamma\left(\nu+\frac{1}{2}\right)}{p^{\nu-1}} 2^{\nu-1}[\mathrm{I}_\nu(p)-\mathbf{L}_\nu(p)] \quad * \; (0<t<1), \quad \mathrm{R}(\nu)>-\frac{1}{2}$$

$$\frac{1}{t} \supset -p\,\mathrm{E}\,i(-p) \qquad (t>1)$$

$$(t+\sqrt{t^2+1})^n+(t-\sqrt{t^2+1})^n \supset 2p\,\mathrm{O}_n(p)$$

$$\frac{(t+\sqrt{t^2+1})^n-(t-\sqrt{t^2+1})^n}{\sqrt{t^2+1}} \supset p\,\mathrm{S}_n(p)$$

$$\sum_{n=1}^{\infty}(-1)^n \frac{\mathrm{B}_n t^{2n}}{2n(2n)!} \supset \varpi'(p)$$

2. — Exponentielles et logarithmes.

$$e^{at} \supset \frac{p}{p-a}$$

$$a^{t} \supset \frac{p}{p-\log a} \qquad (a>1)$$

$$e^{at}-1 \supset \frac{a}{p-a}$$

$$\frac{e^{bt}-e^{at}}{b-a} \supset \frac{p}{(p-a)(p-b)}$$

$$\frac{b\,e^{bt}-a\,e^{at}}{b-a} \supset \frac{p^2}{(p-a)(p-b)}$$

$$\frac{e^{bt}-e^{at}}{t} \supset p\log\frac{p-a}{p-b} \qquad \star$$

$$\frac{1-e^{at}}{t} \supset p\log\frac{p-a}{p} \qquad \star$$

$$e^{at}t^{\nu-1} \supset \Gamma(\nu)\frac{p}{(p-a)^{\nu}} \qquad R(\nu)>0$$

$$\frac{t^{\nu-1}}{\Gamma(\nu)(1-e^{-t})} \supset p\,\zeta(\nu,\,p)$$

$$\frac{e^{t}}{\sqrt{t}} \supset \frac{p\sqrt{\pi}}{\sqrt{p-1}}$$

$$\frac{e^{-t}}{\sqrt{t}} \supset \frac{p\sqrt{\pi}}{\sqrt{p+1}}$$

$$\frac{e^{-\frac{a^2}{4t}}}{\sqrt{t}} \supset \sqrt{\pi p}\,e^{-a\sqrt{p}}$$

$$\frac{a^{2\nu+2}}{(2t)^{\nu+1}}e^{-\frac{a^2}{4t}} \supset a^{\nu+2}p^{\frac{\nu}{2}+1}K_{\nu}(a\sqrt{p})$$

$$\frac{e^{-2\sqrt{t}}}{\sqrt{\pi t}} \supset \sqrt{p}\,e^{\frac{1}{p}}\,erfc\frac{1}{\sqrt{p}}$$

$$\frac{1}{1+e^{-t}} \supset \frac{p}{2}\left[\Psi\left(\frac{p+1}{2}\right)-\Psi\left(\frac{p}{2}\right)\right]$$

$$\frac{1-e^{-at}}{1-e^{-t}} \supset p[\Psi(p+a)-\Psi(p)] \qquad R(p+a)>0,\ R(a)>0$$

$$\frac{(1-e^{-at})(1-e^{-bt})}{t(1-e^{-t})} \supset p\log\frac{\Gamma(p)\,\Gamma(p+a+b)}{\Gamma(p+a)\,\Gamma(p+b)}$$

$$\frac{(1-e^{-at})(1-e^{-bt})}{1-e^{-t}} \supset p[\Psi(p+b)+\Psi(p+a)-\Psi(p+a+b)-\Psi(p)] \qquad \left.\begin{matrix} R(p)>0\\ R(p+a)>0\\ R(p+b)>0\\ R(p+a+b)>0\end{matrix}\right.$$

$$\frac{t\,e^{-t}}{1-e^{-t}} \supset p\,\zeta(2,\,p+1)$$

$$(1-e^{-t})^{\nu} \supset p\,\mathrm{B}(p,\,\nu+1) \qquad R(\nu)>-1$$

$$\frac{1}{t(e^{t}-1)}-\frac{1}{t^2}+\frac{1}{2t} \supset p\,\varpi(p)$$

$$\frac{1}{e^{t}-1}-\frac{1}{t}+\frac{1}{2} \supset -p\,\varpi'(p)$$

$$\frac{1}{\sqrt{\pi t}}\frac{d^n}{dx^n}e^{-\frac{x^2}{4t}} \supset (-1)^n p^{\frac{n+1}{2}}e^{-x\sqrt{p}}$$

$$e^{-t^2} \supset \frac{\sqrt{\pi}}{2}p\,e^{\frac{p^2}{4}}\,erfc\frac{p}{2} = \frac{\sqrt{\pi}}{2}p\,e^{\frac{p^2}{4}}\left(1-erf\frac{p}{2}\right)$$

$$e^{-t^2}t^{\nu-1} \supset \frac{\Gamma(\nu)}{2^{\frac{\nu}{2}}}p\,e^{\frac{p^2}{8}}\,\mathrm{D}_{-\nu}\left(\frac{p}{\sqrt{2}}\right) \qquad R(\nu)>0$$

$$e^{-e^{t}} \supset \frac{1}{e}-\mathrm{Q}(1,\,1-p)$$

$$e^{-e^{-t}} \supset \frac{1}{e}+\mathrm{P}(1,\,p+1)$$

$$1+2\sum_{n=1}^{\infty}(-1)^n e^{-\frac{n^2\pi^2 t}{a^2}}\cos\left(n\pi\frac{b}{a}\right) \supset a\sqrt{p}\,\frac{\mathrm{ch}\,b\sqrt{p}}{\mathrm{sh}\,a\sqrt{p}} \qquad (0<b<a)$$

$$\frac{b}{a}+2\sum_{n=1}^{\infty}(-1)^n e^{-\frac{n^2\pi^2 t}{a^2}}\frac{\sin\left(n\pi\frac{b}{a}\right)}{n\pi} \supset \frac{\mathrm{sh}\,b\sqrt{p}}{\mathrm{sh}\,a\sqrt{p}} \qquad (0<b<a)$$

$$1-2\sum_{n=1}^{\infty}e^{-\frac{\alpha_n^2 kt}{a^2}}\frac{\mathrm{J}_0\left(\alpha_n\frac{r}{a}\right)}{\alpha_n\,\mathrm{J}_1(\alpha_n)} \supset \frac{\mathrm{J}_0\left[ir\sqrt{\frac{p}{k}}\right]}{\mathrm{J}_0\left[ia\sqrt{\frac{p}{k}}\right]} \qquad [r<a\,;\ \alpha_n = n^{\text{ième}} \text{ racine de } \mathrm{J}_0(y)=0]$$

$$\log t \supset -\log p-\gamma$$

$$t^{\nu}\log t \supset \frac{\Gamma(1+\nu)}{p^{\nu}}[\Psi(\nu+1)-\log p] \qquad R(\nu)>-1$$

$$\log\sqrt{t^2+1} \supset -\cos p\,ci(p)-\sin p\,si(p)$$

$$\frac{\log(1+t^2)}{t} \supset p[ci^2(p)+si^2(p)] = -p\,\mathrm{S}(1,\,ip)\,\mathrm{S}(1,\,-ip)$$

$$\frac{\log(1-t^2)}{t} \supset -p\,\mathrm{S}(1, p)\,\mathrm{S}(1, -p) = p\,\mathrm{E}i(p)\,\mathrm{E}i(-p)$$

$$\log(e^t - 1) \supset -\gamma - \Psi(p)$$

$$\log^2 t \supset (\log p + \gamma)^2 + \frac{\pi^2}{6}$$

$$\mathrm{Q}(t, n) \supset (n-1)!\left[1 - \frac{1}{(p+1)^n}\right]$$

3. — Fonctions circulaires.

$$\sin at \supset \frac{pa}{p^2+a^2}$$

$$\cos at \supset \frac{p^2}{p^2+a^2}$$

$$e^{-at} \sin bt \supset \frac{pb}{(p+a)^2+b^2}$$

$$e^{-at} \cos bt \supset \frac{p(p+a)}{(p+a)^2+b^2}$$

$$\sin(at+\theta) \supset \frac{p}{\sqrt{p^2+a^2}} \sin\left(\theta + \operatorname{arc\,tg}\frac{a}{p}\right)$$

$$\cos(at+\theta) \supset \frac{p}{\sqrt{p^2+a^2}} \cos\left(\theta + \operatorname{arc\,tg}\frac{a}{p}\right)$$

$$e^{-bt} \sin(at \pm \theta) \supset \frac{pa\cos\theta \pm p(p+b)\sin\theta}{(p+b)^2+a^2}$$

$$e^{-bt} \cos(at \pm \theta) \supset \frac{p(p+b)\cos\theta \mp pa\sin\theta}{(p+b)^2+a^2}$$

$$1 - 2\sin at \supset \frac{(p-a)^2}{p^2+a^2}$$

$$1 - \cos at \supset \frac{a^2}{p^2+a^2}$$

$$\sin^2 t \supset \frac{2}{p^2+4}$$

$$\cos^2 t \supset \frac{p^2+2}{p^2+4}$$

$$\sin^{2n} t \supset \frac{(2n)!}{(p^2+2^2)\ldots[p^2+(2n)^2]}$$

$$\sin^{2n+1} t \supset \frac{(2n+1)!\,p}{(p^2+1^2)\ldots[p^2+(2n+1)^2]}$$

$$\frac{\cos\sqrt{t}}{\sqrt{t}} \supset \sqrt{\pi p}\, e^{-\frac{1}{4p}}$$

$$e^{-at}\frac{\cos\sqrt{t}}{\sqrt{t}} \supset \sqrt{\pi}\,\frac{p\, e^{-\frac{1}{4(p+a)}}}{\sqrt{p+a}}$$

$$\frac{\sin at}{t} \supset p \operatorname{arc\,tg}\frac{a}{p}$$

$$t^n \sin at \supset n!\,\frac{p}{(p^2+a^2)^{\frac{n+1}{2}}} \sin\left[(n+1)\operatorname{arc\,tg}\frac{a}{p}\right] \qquad (n>-1)$$

$$t^n \cos at \supset n!\,\frac{p}{(p^2+a^2)^{\frac{n+1}{2}}} \cos\left[(n+1)\operatorname{arc\,tg}\frac{a}{p}\right] \qquad \text{»}$$

$$t^\nu \sin t \supset \frac{\Gamma(\nu+1)p}{2i(p^2+1)^{\nu+1}}\left[(p+i)^{\nu+1}-(p-i)^{\nu+1}\right] \qquad R(\nu)>-1$$

$$t^\nu \cos t \supset \frac{\Gamma(\nu+1)p}{2(p^2+1)^{\nu+1}}\left[(p+i)^{\nu+1}+(p-i)^{\nu+1}\right] \qquad \text{»}$$

$$t \sin^2 \omega t \supset \frac{2\omega^2(3p^2+4\omega^2)}{p(p^2+4\omega^2)^2}$$

$$t^2 \sin \omega t \supset \frac{2\omega p(3p^2-\omega^2)}{(p^2+\omega^2)^3}$$

$$\frac{\sin^2 t}{t} \supset \frac{p}{2}\log\sqrt{1+\frac{4}{p^2}}$$

$$\sin b\sqrt{t} \supset \frac{b}{2}\sqrt{\frac{\pi}{p}}\,e^{-\frac{b^2}{4p}}$$

$$\sqrt{t}\cos b\sqrt{t} \supset \frac{1}{2}\sqrt{\frac{\pi}{p}}\,e^{-\frac{b^2}{4p}}\left(1-\frac{b^2}{2p}\right)$$

$$\frac{\sin\sqrt{t}}{t} \supset \pi p\, erf\,\frac{1}{2\sqrt{p}}$$

$$\frac{e^{at}\sin bt}{t} \supset p\operatorname{arc\,tg}\frac{b}{p-a}$$

$$t^{-\frac{3}{4}}\sin\sqrt{8t} \supset 2^{\frac{1}{4}}\pi\sqrt{p}\,e^{-\frac{1}{p}}\,I_{\frac{1}{4}}\left(\frac{1}{p}\right)$$

$$t^{-\frac{3}{4}}\cos\sqrt{8t} \supset 2^{\frac{1}{4}}\pi\sqrt{p}\,e^{-\frac{1}{p}}\,I_{-\frac{1}{4}}\left(\frac{1}{p}\right)$$

$$\frac{\sin t}{\sqrt{t}} \supset \sqrt{\frac{\pi}{2}}\;\frac{p}{\sqrt{p^2+1}\sqrt{p+\sqrt{p^2+1}}}$$

$$\frac{\cos t}{\sqrt{t}} \supset \sqrt{\frac{\pi}{2}}\;\frac{p}{\sqrt{p^2+1}\sqrt{-p+\sqrt{p^2+1}}}$$

$$\sin at\,\sin bt \supset \frac{2abp^2}{[p^2+(a-b)^2][p^2+(a+b)^2]}$$

$$\cos at \cos bt \supset \frac{p^2(p^2+a^2+b^2)}{[p^2+(a-b)^2][p^2+(a+b)^2]}$$

$$\sin at \cos bt \supset \frac{ap(p^2+a^2-b^2)}{[p^2+(a-b)^2][p^2+(a+b)^2]}$$

$$\cos at \sin bt \supset \frac{bp(p^2-a^2+b^2)}{[p^2+(a-b)^2][p^2+(a+b)^2]}$$

$$\frac{2}{\pi}\,\frac{\sin t^2}{t} \supset p\left[\mathrm{C}\left(\frac{p^2}{4}\right)-\frac{1}{2}\right]^2+p\left[\mathrm{S}\left(\frac{p^2}{4}\right)-\frac{1}{2}\right]^2$$

$$\operatorname{arc\,tg} t \supset \sin p\; ci(p)-\cos p\; si(p) \qquad \bigstar$$

$$\arcsin t \supset \frac{\pi}{2}[\,\mathrm{I}_0(p)-\mathbf{L}_0(p)] \qquad \bigstar$$

$$t \arcsin t \supset \frac{\pi}{2}\left[\frac{\mathbf{L}_0(p)-\mathrm{I}_0(p)}{p}+\mathbf{L}_{-1}(p)-\mathrm{I}_1(p)\right]+1 \qquad \bigstar$$

$$s_1(t) \supset \frac{p^3}{p^3+1}$$

$$s_2(t) \supset \frac{p}{p^3+1}$$

$$s_3(t) \supset \frac{-p^2}{p^3+1}$$

$$\arcsin\frac{a}{t} \supset \frac{\pi}{2}\left[e^{-ap}-1+pa\left\{\mathrm{K}_0(pa)\mathbf{L}_1(pa)+\mathbf{L}_0(pa)\mathrm{K}_1(pa)+\frac{2}{\pi}\mathrm{K}_0(pa)\right\}\right]$$

$$=\frac{\pi}{2}e^{-ap}-a\int_p^\infty \mathrm{K}_0(ap)\,dp$$

$$\arcsin\frac{a}{\sqrt{t^2-b^2}} \supset \frac{\pi}{2}\,e^{-p\sqrt{a^2+b^2}}-a\int_p^\infty \operatorname{ch} b(p-x)\,\mathrm{K}_0\left(x\sqrt{a^2+b^2}\right)dx$$

$$\left(t>\sqrt{a^2+b^2}\right)$$

4. — Fonctions hyperboliques.

$$\operatorname{sh} at \supset \frac{pa}{p^2 - a^2}$$

$$\operatorname{ch} at \supset \frac{p^2}{p^2 - a^2}$$

$$\operatorname{sh}(\nu \arg \operatorname{ch} t) \supset \nu \, K_\nu(p) \qquad (t > 1)$$

$$\frac{\operatorname{ch}(\nu \arg \operatorname{ch} t)}{\sqrt{t^2 - 1}} \supset p \, K_\nu(p) \qquad \text{»}$$

$$e^{-bt} \operatorname{sh} at \supset \frac{pa}{(p+b)^2 - a^2}$$

$$e^{-bt} \operatorname{ch} at \supset \frac{p(p+b)}{(p+b)^2 - a^2}$$

$$\operatorname{sh}^2 t \supset \frac{2}{p^2 - 4}$$

$$\operatorname{ch}^2 t \supset \frac{p^2 - 2}{p^2 - 4}$$

$$\operatorname{sh}^{2n} t \supset \frac{(2n)!}{(p^2 - 2^2)(p^2 - 4^2)\dots(p^2 - 4n^2)}$$

$$\operatorname{ch}^{2n} t \supset \frac{(2n+1)!\,p}{(p^2 - 1^2)(p^2 - 3^2)\dots[p^2 - (2n+1)^2]}$$

$$\operatorname{sh}^{2n+1} t \supset \frac{\sqrt{\pi}}{2\sqrt{p}} e^{\frac{1}{4p}}$$

$$\frac{\operatorname{ch} \sqrt{t}}{\sqrt{t}} \supset \sqrt{\pi p} \, e^{\frac{1}{4p}}$$

$$e^{-at} \operatorname{sh} 2b\sqrt{t} \supset \frac{\sqrt{\pi}\, bp \, e^{\frac{b^2}{p+a}}}{(p+a)^{\frac{3}{2}}}$$

$$e^{-at} \frac{\operatorname{ch} 2b\sqrt{t}}{\sqrt{t}} \supset \frac{\sqrt{\pi}\, p \, e^{\frac{b^2}{p+a}}}{\sqrt{p+a}}$$

$$\frac{\operatorname{sh} at}{t} \supset \frac{p}{2} \log \frac{p+a}{p-a} = p \arg \coth \frac{p}{a} \qquad R(p) > R(a)$$

$$\frac{(2t)^{\nu-1}}{\Gamma(\nu) \operatorname{sh} t} \supset p\, \zeta\left(\nu, \frac{p+1}{2}\right) \qquad R(\nu) > 1$$

$$\frac{(2t)^{\nu-1}}{\Gamma(\nu)} \coth t \supset p\left[\zeta\left(\nu, \frac{p}{2}\right) - \frac{2^{\nu-1}}{p^{\nu-1}}\right] \qquad \text{»}$$

$$\frac{\mathrm{sh}^2 t}{t} \supset \frac{p}{2}\log\sqrt{1-\frac{4}{p^2}}$$

$$\frac{\mathrm{sh}\, t}{\sqrt{t}} \supset \sqrt{\frac{\pi}{2}}\, \frac{p\left(\sqrt{p^2-1}+p\right)^{-\frac{1}{2}}}{\sqrt{p^2-1}}$$

$$\frac{\mathrm{ch}\, t}{\sqrt{t}} \supset \sqrt{\frac{\pi}{2}}\, \frac{p\left(p-\sqrt{p^2-1}\right)^{-\frac{1}{2}}}{\sqrt{p^2-1}}$$

$$t^{\nu}\,\mathrm{sh}\, t \supset \frac{\Gamma(\nu+1)p}{2(p^2-1)^{\nu+1}}\left[(p+1)^{\nu+1}-(p-1)^{\nu+1}\right] \qquad R(\nu) > -1$$

$$t^{\nu}\,\mathrm{ch}\, t \supset \frac{\Gamma(\nu+1)p}{2(p^2-1)^{\nu+1}}\left[(p+1)^{\nu+1}+(p-1)^{\nu+1}\right] \qquad »$$

$$\frac{1}{\mathrm{ch}\, t} \supset \frac{1}{2}p\left[\Psi\left(\frac{p+3}{4}\right)-\Psi\left(\frac{p+1}{4}\right)\right]$$

$$\mathrm{th}\, t \supset \frac{1}{2}p\left[\Psi\left(\frac{p+2}{4}\right)-\Psi\left(\frac{p}{4}\right)\right]-1 = p\left[\Psi\left(\frac{p}{2}\right)-\Psi\left(\frac{p}{4}\right)-\log 2\right]-1$$

$$\frac{1}{\mathrm{ch}^2 t} \supset \frac{p^2}{2}\left[\Psi\left(\frac{p+2}{4}\right)-\Psi\left(\frac{p}{4}\right)\right]-p \qquad \star$$

$$\frac{\mathrm{th}\, t}{t} \supset p\log\frac{p}{4}+2p\log\Gamma\left(\frac{p}{4}\right)-2p\log\Gamma\left(\frac{p+2}{4}\right) \qquad \star$$

$$\frac{\mathrm{ch}\, t-1}{t\,\mathrm{ch}\, t} \supset -p\log\frac{p}{4}+2p\log\Gamma\left(\frac{p+3}{4}\right)-2p\log\Gamma\left(\frac{p+1}{4}\right) \qquad \star$$

$$\frac{1-\mathrm{ch}\, t}{t} \supset p\log\frac{p}{\sqrt{p^2-1}}$$

$$\frac{1-\mathrm{ch}\, t}{t^2} \supset p\log\frac{\sqrt{p^2-1}}{p}+p\,\arg\coth p \qquad \star$$

$$\sqrt{t}\,\mathrm{ch}\, b\sqrt{t} \supset \frac{\sqrt{\pi}\, e^{\frac{b^2}{4p}}}{2\sqrt{p}}\left(1+\frac{b^2}{2p}\right)$$

$$t^{-\frac{1}{4}}\,\mathrm{sh}\sqrt{8t} \supset 2^{\frac{1}{4}}\pi\sqrt{p}\, e^{\frac{1}{p}}\, I_{\frac{1}{4}}\left(\frac{1}{p}\right)$$

$$t^{-\frac{3}{4}}\,\mathrm{ch}\sqrt{8t} \supset 2^{\frac{1}{4}}\pi\sqrt{p}\, e^{\frac{1}{p}}\, I_{-\frac{1}{4}}\left(\frac{1}{p}\right)$$

$$\mathrm{sh}\, at\,\mathrm{sh}\, bt \supset \frac{2abp^2}{\left[p^2-(a-b)^2\right]\left[p^2-(a+b)^2\right]}$$

$$\mathrm{ch}\, at\,\mathrm{ch}\, bt \supset \frac{p^2(p^2-a^2-b^2)}{\left[p^2-(a-b)^2\right]\left[p^2-(a+b)^2\right]}$$

$$\operatorname{sh} at \operatorname{ch} bt \supset \frac{ap(p^2 - a^2 + b^2)}{[p^2 - (a-b)^2][p^2 - (a+b)^2]}$$

$$\operatorname{ch} at \operatorname{sh} bt \supset \frac{bp(p^2 + a^2 - b^2)}{[p^2 - (a-b)^2][p^2 - (a+b)^2]}$$

$$\cos t \operatorname{ch} t \supset \frac{p^4}{p^4 + 4}$$

$$\operatorname{sh} at - \sin at \supset \frac{2pa^3}{p^4 - a^4}$$

$$\operatorname{sh}\sqrt{t} + \sin\sqrt{t} \supset \sqrt{\frac{\pi}{p}} \operatorname{ch}\frac{1}{4p}$$

$$\operatorname{sh}\sqrt{t} - \sin\sqrt{t} \supset \sqrt{\frac{\pi}{p}} \operatorname{sh}\frac{1}{4p}$$

$$\frac{\operatorname{ch}\sqrt{t} + \cos\sqrt{t}}{\sqrt{t}} \supset 2\sqrt{\pi p} \operatorname{ch}\frac{1}{4p}$$

$$\frac{\operatorname{ch}\sqrt{t} - \cos\sqrt{t}}{\sqrt{t}} \supset 2\sqrt{\pi p} \operatorname{sh}\frac{1}{4p}$$

$$\sqrt{t}(\operatorname{ch}\sqrt{t} + \cos\sqrt{t}) \supset \sqrt{\frac{\pi}{p}}\left(\operatorname{ch}\frac{1}{4p} + \frac{1}{2p}\operatorname{sh}\frac{1}{4p}\right)$$

$$\sqrt{t}(\operatorname{ch}\sqrt{t} - \cos\sqrt{t}) \supset \sqrt{\frac{\pi}{p}}\left(\operatorname{sh}\frac{1}{4p} + \frac{1}{2p}\operatorname{ch}\frac{1}{4p}\right)$$

$$2^{\frac{3}{4}} t^{-\frac{3}{4}}(\operatorname{sh}\sqrt{8t} + \sin\sqrt{8t}) \supset \pi\sqrt{p}\; I_{\frac{1}{4}}\left(\frac{1}{p}\right) \operatorname{ch}\frac{1}{p}$$

$$2^{\frac{3}{4}} t^{-\frac{3}{4}}(\operatorname{sh}\sqrt{8t} - \sin\sqrt{8t}) \supset \pi\sqrt{p}\; I_{\frac{1}{4}}\left(\frac{1}{p}\right) \operatorname{sh}\frac{1}{p}$$

$$2^{\frac{3}{4}} t^{-\frac{3}{4}}(\operatorname{ch}\sqrt{8t} + \cos\sqrt{8t}) \supset \pi\sqrt{p}\, I_{-\frac{1}{4}}\left(\frac{1}{p}\right) \operatorname{ch}\frac{1}{p}$$

$$2^{\frac{3}{4}} t^{-\frac{3}{4}}(\operatorname{ch}\sqrt{8t} - \cos\sqrt{8t}) \supset \pi\sqrt{p}\, I_{-\frac{1}{4}}\left(\frac{1}{p}\right) \operatorname{sh}\frac{1}{p}$$

$$\log \operatorname{sh} t \supset -\Psi\left(\frac{p}{2}\right) - \frac{1}{p} - \gamma - \log 2$$

$$\log \operatorname{ch} t \supset \Psi\left(\frac{p}{2}\right) - \Psi\left(\frac{p}{4}\right) - \frac{1}{p} - \log 2$$

$$\log \operatorname{th} t \supset -2\Psi\left(\frac{p}{2}\right) + \Psi\left(\frac{p}{4}\right) - \gamma$$

$$\log\frac{\operatorname{sh} t}{t} \supset -\varpi'\left(\frac{p}{2}\right)$$

$$\arg\operatorname{ch}\frac{t}{a} \supset K_0(pa) \qquad (t > a)$$

$$e^{-bt}\arg\operatorname{ch}\frac{t}{a} \supset \frac{p}{p+b}K_0[a(p+b)] \qquad (t > a)$$

$$e^{-b(t+a)}\arg\operatorname{ch}\frac{t+a}{a} \supset p\,e^{ap}\,\frac{K_0[a(p+b)]}{p+b} \qquad *$$

$$\arg\operatorname{sh} t \supset \frac{\pi}{2}[\mathbf{H}_0(p) - Y_0(p)]$$

$$t\arg\operatorname{sh} t \supset \frac{\pi}{2}\left[\frac{\mathbf{H}_0(p) - Y_0(p)}{p} + \mathbf{H}_1(p) - Y_1(p)\right] - 1$$

$$\int_1^t \frac{\operatorname{ch}(\nu\arg\operatorname{ch} t)}{\sqrt{t^2-1}}\,dt \supset K_\nu(p) \qquad (t > 1)$$

5. — Logarithme intégral et fonctions analogues.

$$Ei(t) \supset -\log(p-1)$$

$$si(t) \supset -\operatorname{arc\,tg} p$$

$$ci(t) \supset -\log\sqrt{p^2+1}$$

$$2[\sin t - t\,ci(t)] \supset \frac{\log(p^2+1)}{p}$$

$$\sin t\, ci(t) - \cos t\, si(t) \supset -\frac{p}{p^2+1}\left(\log p + \frac{\pi}{2}p\right)$$

$$\cos t\, ci(t) + \sin t\, si(t) \supset -\frac{p}{p^2+1}\left(p\log p - \frac{\pi}{2}\right)$$

$$S(t) \supset \frac{\left(\sqrt{p^2+1}-p\right)^{\frac{1}{2}}}{2\sqrt{p^2+1}}$$

$$C(t) \supset \frac{\left(\sqrt{p^2+1}-p\right)^{-\frac{1}{2}}}{2\sqrt{p^2+1}} = \frac{\left(\sqrt{p^2+1}+p\right)^{\frac{1}{2}}}{2\sqrt{p^2+1}}$$

$$t\,S(t) \supset \frac{\left(\sqrt{p^2+1}-p\right)^{\frac{1}{2}}}{2p\sqrt{p^2+1}}\left(\frac{p}{2\sqrt{p^2+1}} + \frac{p^2}{p^2+1} + 1\right)$$

$$si(t^2) + \frac{\pi}{2} \supset \pi\left[C\left(\frac{p^2}{4}\right) - \frac{1}{2}\right]^2 + \pi\left[S\left(\frac{p^2}{4}\right) - \frac{1}{2}\right]^2$$

6. — Fonctions d'erreur intégrale.

$$erf\, t \supset e^{\frac{p^2}{4}}\, erfc \frac{p}{2}$$

$$erf \frac{a}{2\sqrt{t}} \supset 1 - e^{-a\sqrt{p}}$$

$$erf \sqrt{t} \supset \frac{1}{\sqrt{p+1}}$$

$$e^{t}\, erf \sqrt{t} \supset \frac{\sqrt{p}}{p-1}$$

$$e^{-t}\, erf \sqrt{t} \supset \frac{p}{(p+1)\sqrt{p+2}}$$

$$e^{t+\frac{1}{4}}\left[erf\left(t+\frac{1}{2}\right) - erf \frac{1}{2}\right] \supset \frac{e^{\frac{p^2}{4}}}{p+1}\, erfc \frac{p}{2}$$

$$a\sqrt{\frac{t}{\pi}}\, e^{-\frac{a^2}{4t}} + \left(t + \frac{a^2}{2}\right) erf \frac{a}{2\sqrt{t}} - \frac{a^2}{2} \supset \frac{1 - e^{-a\sqrt{p}}}{p}$$

$$e^{\frac{t}{a^2}}\left[1 - erf \frac{\sqrt{t}}{a}\right] \supset \frac{a\sqrt{p}}{1 + a\sqrt{p}}$$

$$e^{a^2 t}\, erfc\, a\sqrt{t} \supset \frac{\sqrt{p}}{a + \sqrt{p}}$$

$$2\sqrt{\frac{t}{\pi}}\, e^{-\frac{a^2}{4t}} - a\, erfc \frac{a}{2\sqrt{t}} \supset \frac{e^{-a\sqrt{p}}}{\sqrt{p}}$$

$$\frac{e^{-t}}{\sqrt{\pi t}} + erf \sqrt{t} \supset \sqrt{p+1}$$

$$\frac{1}{\sqrt{\pi t}} - e^{t}\, erfc \sqrt{t} \supset \frac{p}{\sqrt{p}+1}$$

$$e^{-at}\, erf \sqrt{(b-a)t} \supset \sqrt{b-a}\, \frac{p}{(p+a)\sqrt{p+b}}$$

$$\frac{e^{-at}}{\sqrt{\pi t}} + \sqrt{a-b}\, e^{-bt}\, erf \sqrt{(a-b)t} \supset \frac{p\sqrt{p+a}}{p+b}$$

$$1 - e^{\frac{a^2}{b^2}t}\, erfc \frac{a}{b}\sqrt{t} \supset \frac{1}{a + b\sqrt{p}}$$

$$e^{-t}\, erfc \frac{1}{2\sqrt{t}} \supset \frac{p}{p+1}\, e^{-\sqrt{p+1}}$$

7. — Fonctions de Bessel (1).

$$J_0(at) \supset \frac{p}{r}$$

$$J_\nu(at) \supset \frac{pa^\nu}{rP^\nu} \qquad R(\nu) > -1$$

$$e^{-bt} J_\nu(at) \supset \frac{pa^\nu}{\sqrt{(p+b)^2+a^2}\,\left[p+b+\sqrt{(p+b)^2+a^2}\right]^\nu} \qquad R(\nu) > -1$$

$$t^\nu J_\nu(t) \supset \frac{2^\nu}{\sqrt{\pi}}\Gamma\left(\nu+\frac{1}{2}\right)\frac{p}{q^{2\nu+1}} \qquad R(\nu) > -\frac{1}{2}$$

$$t^{\nu+1} J_\nu(t) \supset \frac{2^{\nu+1}}{\sqrt{\pi}}\Gamma\left(\nu+\frac{3}{2}\right)\frac{p^2}{q^{2\nu+3}} \qquad R(\nu) > -1$$

$$t^{\frac{\nu}{2}} J_\nu(2\sqrt{at}) \supset a^{\frac{\nu}{2}} p^{-\nu} e^{-\frac{a}{p}} \qquad R(\nu) > -\frac{1}{2}$$

$$J_0(2\sqrt{at}) \supset e^{-\frac{a}{p}}$$

$$\sqrt{t}\; J_{\frac{1}{2}}(t) \supset \sqrt{\frac{2}{\pi}}\frac{p}{q^2}$$

$$\sqrt{t}\, J_{-\frac{1}{2}}(t) \supset \sqrt{\frac{2}{\pi}}\frac{p^2}{q^2}$$

$$J_{\frac{1}{2}}(t) \supset \frac{p}{q\sqrt{Q}}$$

$$J_{-\frac{1}{2}}(t) \supset \frac{p\sqrt{Q}}{q}$$

$$t^{-1} J_\nu(at) \supset \frac{a^\nu p}{\nu P^\nu} \qquad R(\nu) > 0$$

$$t\, J_\nu(t) \supset \frac{p(p+\nu q)}{q^3 Q^\nu} \qquad R(\nu) > -2$$

$$t^\nu J_m(t) \supset (-1)^m \Gamma(\nu - m + 1)\frac{p}{q^{\nu+1}} P_\nu^m\left(\frac{p}{q}\right) \qquad (m > 0) \qquad R(\nu + m) > -1$$

$$t^\nu J_\mu(t) \supset \frac{\Gamma(\mu+\nu+1)}{2^\nu \Gamma(\mu+1)} p^{-\mu-\nu}\, {}_2F_1\left(\frac{\mu+\nu+1}{2}, \frac{\mu+\nu+2}{2}, \mu+1, -\frac{1}{p^2}\right) \qquad R(\mu+\nu) > 0$$

$$t^2 J_0(t) \supset \frac{p(2p^2-1)}{q^5}$$

$$\frac{J_\nu(t)}{t^2} \supset \frac{p(2\nu q - p)}{2\nu(4\nu^2-1)Q^{2\nu}} \qquad R(\nu) > 1$$

(1) Pour tout ce qui concerne les fonctions du genre Bessel, le lecteur voudra bien ne pas oublier les abréviations indiquées à la page 3.

$$J_0^2(t) \supset \frac{pk}{\pi} E\left(k, \frac{\pi}{2}\right) \qquad \left(k = \frac{2}{\sqrt{p^2+4}}\right)$$

$$J_\nu^2(t) \supset \frac{pk^{2\nu+1}}{\pi} \int_0^{\frac{\pi}{2}} \frac{\cos^{2\nu}\theta \, d\theta}{\sqrt{1-k^2\sin^2\theta}\left(\frac{pk}{2} + \sqrt{1-k^2\sin^2\theta}\right)^{2\nu}} \quad \left\{ \begin{array}{l} \left(k = \frac{2}{\sqrt{p^2+4}}, k^2 < 1\right) \\ R(\nu) > -\frac{1}{2} \end{array} \right.$$

$$J_\mu(t)\, J_\nu(t) \supset \frac{pk^{\mu+\nu+1}}{\pi} \int_0^{\frac{\pi}{2}} \frac{\cos(\mu-\nu)\theta \cos^{\mu+\nu}\theta \, d\theta}{\sqrt{1-k^2\sin^2\theta}\left(\frac{pk}{2} + \sqrt{1-k^2\sin^2\theta}\right)^{\mu+\nu}} \quad \left\{ \begin{array}{l} \left(k = \frac{2}{\sqrt{p^2+4}}\right) \\ R(\mu+\nu) > -1 \end{array} \right.$$

$$J_{\nu+\frac{1}{2}}\left(\frac{t^2}{2}\right) \supset \frac{\Gamma(\nu+1)}{\sqrt{\pi}} p\, D_{-\nu-1}\left(p e^{\frac{\pi i}{4}}\right) D_{-\nu-1}\left(p e^{-\frac{\pi i}{4}}\right) \qquad R(\nu) > -1$$

$$J_\nu(2\sqrt{t}) \supset \frac{1}{2}\sqrt{\frac{\pi}{p}}\, e^{-\frac{1}{2p}}\left[I_{\frac{\nu-1}{2}}\left(\frac{1}{2p}\right) - I_{\frac{\nu+1}{2}}\left(\frac{1}{2p}\right)\right]$$

$$t^\nu J_\mu(2\sqrt{t}) \supset \frac{\Gamma\left(\frac{\mu}{2}+\nu+1\right)}{\Gamma(\mu+1)\, p^{\frac{\mu}{2}+\nu}}\, {}_1F_1\left(\frac{\mu}{2}+\nu+1; \mu+1; -\frac{1}{p}\right)$$

$$t^{n+\frac{\alpha}{2}} J_\alpha(2\sqrt{t}) \supset \frac{n!\, e^{-\frac{1}{p}}}{p^{n+\alpha}} L_n^\alpha\left(\frac{1}{p}\right) \qquad (\alpha > -1,\ n > 0)$$

$$\frac{J_0(a\sqrt{t})}{\sqrt{t}} \supset \sqrt{\pi p}\, e^{-\frac{a^2}{8p}} I_0\left(\frac{a^2}{8p}\right)$$

$$t^n J_0(2\sqrt{t}) \supset \frac{e^{-\frac{1}{p}}}{p^n} L_n\left(\frac{1}{p}\right)$$

$$\frac{J_{2\nu}(\sqrt{8t})}{\sqrt{t}} \supset \sqrt{\pi p}\, e^{-\frac{1}{p}} I_\nu\left(\frac{1}{p}\right) \qquad R(\nu) > -\frac{1}{2}$$

$$\sum_{n=0}^{\infty} J_{2\nu+2n+1}(4\sqrt{t}) \supset \sqrt{\frac{\pi}{p}}\, e^{-\frac{2}{p}} I_\nu\left(\frac{2}{p}\right) \qquad R(\nu) > -\frac{3}{2}$$

$$J_\nu^2(2a\sqrt{t}) \supset e^{-\frac{2a^2}{p}} I_\nu\left(\frac{2a^2}{p}\right) \qquad R(\nu) > -1$$

$$J_\nu(2a\sqrt{t})\, J_\nu(2b\sqrt{t}) \supset e^{-\frac{a^2+b^2}{p}} I_\nu\left(\frac{2ab}{p}\right) \qquad R(\nu) > -1$$

$$J_\nu(2a\sqrt{t})\, I_\nu(2b\sqrt{t}) \supset e^{-\frac{a^2-b^2}{p}} J_\nu\left(\frac{2ab}{p}\right) \qquad R(\nu) > -1$$

$$\frac{J_\nu^2(2\sqrt{t})}{t} \supset \frac{p}{\nu} e^{-\frac{2}{p}}\left[I_\nu\left(\frac{2}{p}\right) + 2\sum_{s=1}^{\infty} I_{\nu+s}\left(\frac{2}{p}\right)\right] \qquad R(\nu) > 0$$

$$t^{\frac{\nu}{2}}[J_\nu(2\sqrt{t}) - I_\nu(2\sqrt{t})] \supset 2p^{-\nu}\,\mathrm{sh}\frac{1}{p} \qquad R(\nu) > -1$$

$$t^{\frac{\nu}{2}}[J_\nu(2\sqrt{t}) + I_\nu(2\sqrt{t})[\supset 2p^{-\nu}\,\mathrm{ch}\frac{1}{p} \qquad R(\nu) > -1$$

$$\frac{J_{2n}(2a\sqrt{t})}{\sqrt{t}} \supset \sqrt{\pi p}\, e^{-\frac{a^2}{2p}}\, I_n\left(\frac{a^2}{2p}\right)$$

$$J_0(a\,\mathrm{sh}\,t) \supset p\,K_{\frac{p}{2}}\left(\frac{a}{2}\right) I_{\frac{p}{2}}\left(\frac{a}{2}\right)$$

$$\frac{J_{-\frac{1}{3}}\left(\frac{2}{3\sqrt{3t}}\right)}{\sqrt{3t}} \supset p^{\frac{1}{3}}\, s_1\left(-p^{\frac{1}{3}}\right)$$

$$\frac{J_{\frac{1}{3}}\left(\frac{2}{3\sqrt{3t}}\right)}{\sqrt{3t}} \supset p^{\frac{1}{3}}\, s_3\left(-p^{\frac{1}{3}}\right)$$

$$J_0(ay) \supset \frac{p\,e^{-br}}{r} = \sqrt{\frac{2b}{\pi}}\,\frac{p\,K_{\frac{1}{2}}(br)}{\sqrt{r}}$$

$$\frac{J_1(ay)}{y} \supset \frac{p}{ab}[e^{-bp} - e^{-br}] \qquad \star$$

$$\frac{t}{y} J_1(ay) \supset \frac{p}{a}\left[e^{-pb} - \frac{p}{r}e^{-br}\right]$$

$$y\,J_1(ay) \supset \frac{ap}{r^2}e^{-br}\left(b + \frac{1}{r}\right)$$

$$t\,J_0(ay) \supset \frac{p^2}{r^2}e^{-br}\left(b + \frac{1}{r}\right)$$

$$\left(\frac{t-b}{t+b}\right)^{\frac{\nu}{2}} J_\nu(ay) \supset \frac{p}{r}e^{-br}\frac{a^\nu}{P^\nu} \qquad R(\nu) > -1$$

$$J_\nu(ay) \supset \frac{1}{\sqrt{\pi}}\sum_{m=0}^{\infty}\frac{(-1)^m (ab)^{\nu+2m}\, b\,\Gamma\left(\frac{\nu}{2}+m+1\right)}{m!\,\Gamma(\nu+m+1)(2p)^{\frac{\nu}{2}+m-\frac{1}{2}}}K_{\frac{\nu}{2}+\frac{1}{2}+m}(pb)$$

$$J_0\left(a\sqrt{t^2+2bt}\right) \supset \frac{p}{r}e^{b(p-r)}$$

$$\frac{t^{\frac{\nu}{2}} J_\nu\left(a\sqrt{t^2+2bt}\right)}{(t+2b)^{\frac{\nu}{2}}} \supset \frac{p}{r}e^{b(p-r)}\frac{a^\nu}{P^\nu} \qquad R(\nu) > -1$$

$$\int_0^t \mathrm{J}_0(t)\,dt \supset \frac{1}{q}$$

$$\int_0^t \mathrm{J}_\nu(at)\,dt \supset \frac{a^\nu}{r\mathrm{P}^\nu} \qquad \mathrm{R}(\nu) > -1$$

$$\int_0^t t^\nu \mathrm{J}_\nu(t)\,dt \supset \frac{2^\nu \Gamma\left(\nu + \frac{1}{2}\right)}{\sqrt{\pi}\, q^{2\nu+1}} \qquad \mathrm{R}(\nu) > -\frac{1}{2}$$

$$\int_0^t \frac{\mathrm{J}_\nu^2(2\sqrt{\lambda})\,d\lambda}{\lambda\sqrt{t-\lambda}} \supset \frac{\sqrt{\pi p}}{\nu} e^{-\frac{2}{p}} \left[\mathrm{I}_\nu\left(\frac{2}{p}\right) + 2\sum_{s=1}^{\infty} \mathrm{I}_{\nu+s}\left(\frac{2}{p}\right)\right]$$

$$\int_0^t \frac{\mathrm{J}_\nu(2\sqrt{\lambda})\,d\lambda}{\sqrt{t-\lambda}} \supset \sqrt{\frac{\pi}{p}}\, e^{-\frac{2}{p}} \mathrm{I}_\nu\left(\frac{2}{p}\right) \qquad \mathrm{R}(\nu) > -\frac{3}{2}$$

$$\int_b^t \frac{\mathrm{J}_1(ay)}{y}\,dt \supset \frac{e^{-bp} - e^{-br}}{ab}$$

$$\int_b^t \frac{\mathrm{J}_1(ay)}{y}\,t\,dt \supset \frac{1}{a}\left(e^{-bp} - \frac{p}{r} e^{-br}\right)$$

$$\int_t^\infty \frac{\mathrm{J}_1(ay)}{y}\,t\,dt \supset \frac{p}{ar} e^{-br}$$

$$\int_t^\infty \frac{\mathrm{J}_1(ay)}{y}\,dt \supset \frac{e^{-br} - e^{-b(p+a)}}{ab}$$

$$\int_t^\infty e^{-ct} \frac{\mathrm{J}_1(ay)}{y}\,dt \supset \frac{e^{-b\sqrt{(p+c)^2+a^2}} - e^{-b(p+\sqrt{a^2+c^2})}}{ab}$$

$$\frac{1}{\sqrt{t}}\int_0^\infty e^{-\frac{x^2}{4b^2t}} \mathrm{J}_\nu(2\sqrt{x})\, x^{\frac{\nu}{2}}\,dx \supset b^{\nu+1}\sqrt{\pi}\, p^{-\frac{\nu}{2}} e^{-\frac{b}{\sqrt{p}}}$$

$$\frac{e^{-\frac{b^2}{4t}}}{\sqrt{t}} - \frac{b}{\sqrt{t}}\int_b^\infty e^{-\frac{x^2}{4t}} \frac{\mathrm{J}_1(\sqrt{x^2-b^2})}{\sqrt{x^2-b^2}}\,dx \supset \sqrt{\pi p}\, e^{-b\sqrt{p+1}}$$

$$t^{\frac{\nu}{2}} \mathrm{J}_\nu(2\sqrt{bt}) - bt^{\frac{\nu}{2}}\int_b^\infty \frac{\mathrm{J}_\nu(2\sqrt{tx})\,\mathrm{J}_1(\sqrt{x^2-b^2})\,dx}{\sqrt{x^2-b^2}} \supset b^{\frac{\nu}{2}} p^{-\nu} e^{-\frac{\sqrt{p^2+1}}{bp}}$$

$$\mathrm{J}_n^{(k)}(t) \supset 2^k \frac{p(p^2+1)^{\frac{k-1}{2}}}{(p+\sqrt{p^2+1})^n}$$

8. — Fonctions de Bessel de seconde espèce.

$$\mathrm{Y}_0(at) \supset \frac{-2p}{\pi r}\log\frac{\mathrm{P}}{a}$$

$$\mathrm{Y}_\nu(at) \supset \frac{p\left[\left(\frac{a}{\mathrm{P}}\right)^{\nu}\cos\nu\pi - \left(\frac{\mathrm{P}}{a}\right)^{\nu}\right]}{r\sin\nu\pi} \qquad -1 < \mathrm{R}(\nu) < 1$$

$$\mathrm{Y}_{\frac{1}{2}}(t) \supset -\frac{p}{q}\sqrt{\mathrm{Q}}$$

$$\mathrm{Y}_{-\frac{1}{2}}(t) \supset \frac{p}{q\sqrt{\mathrm{Q}}}$$

$$\sqrt{t}\;\mathrm{Y}_{\frac{1}{2}}(t) \supset -\sqrt{\frac{2}{\pi}}\frac{p^2}{q^2}$$

$$\sqrt{t}\,\mathrm{Y}_{-\frac{1}{2}}(t) \supset \sqrt{\frac{2}{\pi}}\frac{p}{q^2}$$

$$\frac{\mathrm{Y}_0(a\sqrt{t})}{\sqrt{t}} \supset -\sqrt{\frac{p}{\pi}}\,e^{-\frac{a^2}{8p}}\mathrm{K}_0\left(\frac{a^2}{8p}\right)$$

$$t^{\mu-1}\,\mathrm{Y}_\nu(at) \supset \frac{a^\nu\,\Gamma(\mu+\nu)\,p\cot\nu\pi}{2^\nu\,\Gamma(\nu+1)\,r^{\mu+\nu}}\;{}_2\mathrm{F}_1\left[\frac{\mu+\nu}{2},\frac{1-\mu+\nu}{2};\nu+1;\frac{a^2}{r^2}\right]$$
$$-\frac{2^\nu\,\Gamma(\mu-\nu)\,p\,\mathrm{coséc}\,\nu\pi}{a^\nu\,\Gamma(1-\nu)\,r^{\mu-\nu}}\;{}_2\mathrm{F}_1\left[\frac{\mu-\nu}{2},\frac{1-\mu-\nu}{2};1-\nu;\frac{a^2}{r^2}\right]$$
$$\mathrm{R}(\mu) > |\,\mathrm{R}(\nu)\,|,\quad \mathrm{R}(p+ia) > 0,\quad \mathrm{R}(p-ia) > 0$$

$$\mathrm{Y}_{-n-\frac{1}{2}}(t) \supset \frac{(-1)^n p}{q\,\mathrm{Q}^{n+\frac{1}{2}}} \qquad (n \text{ entier} \geqq 0)$$

$$\mathrm{Y}_0(ay) \supset -\frac{2p\,e^{-br}}{\pi r}\log\frac{\mathrm{P}}{a}$$

$$t\,\mathrm{Y}_0(at) \supset \frac{2p}{\pi r^2}\left[1-\frac{p}{r}\log\frac{\mathrm{P}}{a}\right]$$

$$t\,\mathrm{Y}_1(at) \supset -\frac{2p}{\pi r^2}\left[\frac{p}{a}+\frac{a}{r}\log\frac{\mathrm{P}}{a}\right]$$

9. — Fonctions de Bessel de troisième espèce (F. de Hankel).

$$H_0^{(1)}(at) \supset \frac{p}{r}\left[1 - \frac{2i}{\pi}\log\frac{P}{a}\right]$$

$$H_0^{(2)}(at) \supset \frac{p}{r}\left[1 + \frac{2i}{\pi}\log\frac{P}{a}\right]$$

$$H_\nu^{(1)}(at) \supset \frac{pa^\nu}{rP^\nu}\left[1 + \frac{i}{\sin\nu\pi}\left\{\cos\nu\pi - \frac{P^{2\nu}}{a^{2\nu}}\right\}\right] \qquad -1 < R(\nu) < 1$$

$$H_\nu^{(2)}(at) \supset \frac{pa^\nu}{rP^\nu}\left[1 - \frac{i}{\sin\nu\pi}\left\{\cos\nu\pi - \frac{P^{2\nu}}{a^{2\nu}}\right\}\right] \qquad \text{»}$$

$$H_{\frac{1}{2}}^{(1)}(at) \supset \frac{p}{r}\sqrt{\frac{a}{P}}\left[1 - i\frac{P}{a}\right]$$

$$H_{\frac{1}{2}}^{(2)}(at) \supset \frac{p}{r}\sqrt{\frac{a}{P}}\left[1 + i\frac{P}{a}\right]$$

$$H_{-\frac{1}{2}}^{(1)}(at) \supset \frac{p}{r}\sqrt{\frac{P}{a}}\left[1 + i\frac{a}{P}\right]$$

$$H_{-\frac{1}{2}}^{(2)}(at) \supset \frac{p}{r}\sqrt{\frac{P}{a}}\left[1 - i\frac{a}{P}\right]$$

$$\sqrt{t}\,H_{\frac{1}{2}}^{(1)}(at) \supset \sqrt{\frac{2}{\pi}}\;\frac{p}{r^2\sqrt{a}}(a - ip)$$

$$\sqrt{t}\,H_{\frac{1}{2}}^{(2)}(at) \supset \sqrt{\frac{2}{\pi}}\;\frac{p}{r^2\sqrt{a}}(a + ip)$$

$$\sqrt{t}\,H_{-\frac{1}{2}}^{(1)}(at) \supset \sqrt{\frac{2}{\pi}}\;\frac{p}{r^2\sqrt{a}}(p + ia)$$

$$\sqrt{t}\,H_{-\frac{1}{2}}^{(2)}(at) \supset \sqrt{\frac{2}{\pi}}\;\frac{p}{r^2\sqrt{a}}(p - ia)$$

$$t\,H_0^{(1)}(at) \supset \frac{p}{r^2}\left[\frac{p}{r}\left\{1 - \frac{2i}{\pi}\log\frac{P}{a}\right\} + \frac{2i}{\pi}\right]$$

$$t\,H_0^{(2)}(at) \supset \frac{p}{r^2}\left[\frac{p}{r}\left\{1 + \frac{2i}{\pi}\log\frac{P}{a}\right\} - \frac{2i}{\pi}\right]$$

$$t\,H_1^{(1)}(at) \supset \frac{p}{r^2}\left[\frac{a}{r}\left\{1 - \frac{2i}{\pi}\log\frac{P}{a}\right\} - \frac{2ip}{\pi a}\right]$$

$$t\,H_1^{(2)}(at) \supset \frac{p}{r^2}\left[\frac{a}{r}\left\{1 + \frac{2i}{\pi}\log\frac{P}{a}\right\} + \frac{2ip}{\pi a}\right]$$

$$\frac{H_0^{(1)}(a\sqrt{t})}{\sqrt{t}} \supset \sqrt{\pi p}\, e^{-\frac{a^2}{8p}}\left[I_0\left(\frac{a^2}{8p}\right) - \frac{i}{\pi} K_0\left(\frac{a^2}{8p}\right)\right]$$

$$\frac{H_0^{(2)}(a\sqrt{t})}{\sqrt{t}} \supset \sqrt{\pi p}\, e^{-\frac{a^2}{8p}}\left[I_0\left(\frac{a^2}{8p}\right) + \frac{i}{\pi} K_0\left(\frac{a^2}{8p}\right)\right]$$

$$H_0^{(1)}(ay) \supset \frac{p\, e^{-br}}{r}\left[1 - \frac{2i}{\pi}\log\frac{P}{a}\right]$$

$$H_0^{(2)}(ay) \supset \frac{p\, e^{-br}}{r}\left[1 + \frac{2i}{\pi}\log\frac{P}{a}\right]$$

$$t\, H_0^{(1)}(ay) \supset \frac{p\, e^{-br}}{r^2}\left[p\left(b + \frac{1}{r}\right)\left(1 - \frac{2i}{\pi}\log\frac{P}{a}\right) + \frac{2i}{\pi}\right]$$

$$t\, H_0^{(2)}(ay) \supset \frac{p\, e^{-br}}{r^2}\left[p\left(b + \frac{1}{r}\right)\left(1 + \frac{2i}{\pi}\log\frac{P}{a}\right) - \frac{2i}{\pi}\right]$$

$$y\, H_0^{(1)}(ay) \supset \frac{p\, e^{-br}}{r^2}\left[a\left(b + \frac{1}{r}\right)\left(1 - \frac{2i}{\pi}\log\frac{P}{a}\right) - \frac{2ip}{\pi a}\right]$$

$$y\, H_0^{(2)}(ay) \supset \frac{p\, e^{-br}}{r^2}\left[a\left(b + \frac{1}{r}\right)\left(1 + \frac{2i}{\pi}\log\frac{P}{a}\right) + \frac{2ip}{\pi a}\right]$$

10. — Fonctions de Bessel d'argument imaginaire.

$$\mathrm{I}_0(at) \supset \frac{p}{s}$$

$$\mathrm{I}_\nu(at) \supset \frac{pa^\nu}{s\mathrm{R}^\nu} \qquad \mathrm{R}(\nu) > -1$$

$$e^{-bt}\,\mathrm{I}_\nu(at) \supset \frac{pa^\nu}{\sqrt{(p+b)^2-a^2}\,\left[p+b+\sqrt{(p+b)^2-a^2}\right]^\nu} \qquad \mathrm{R}(\nu) > -1$$

$$t^\nu\,\mathrm{I}_\nu(t) \supset \frac{2^\nu\,\Gamma\left(\nu+\frac{1}{2}\right)}{\sqrt{\pi}}\,\frac{p}{u^{2\nu+1}} \qquad \mathrm{R}(\nu) > -\frac{1}{2}$$

$$t^{\nu+1}\,\mathrm{I}_\nu(t) \supset \frac{2^{\nu+1}\,\Gamma\left(\nu+\frac{3}{2}\right)}{\sqrt{\pi}}\,\frac{p^2}{u^{2\nu+3}} \qquad \mathrm{R}(\nu) > -1$$

$$t^{\frac{\nu}{2}}\,\mathrm{I}_\nu(2\sqrt{t}) \supset p^{-\nu}\,e^{\frac{1}{p}} \qquad \mathrm{R}(\nu) > -1$$

$$\mathrm{I}_0(2\sqrt{t}) \supset e^{\frac{1}{p}}$$

$$\sqrt{t}\;\mathrm{I}_{\frac{1}{2}}(t) \supset \sqrt{\frac{2}{\pi}}\,\frac{p}{u^2}$$

$$\sqrt{t}\;\mathrm{I}_{-\frac{1}{2}}(t) \supset \sqrt{\frac{2}{\pi}}\,\frac{p^2}{u^2}$$

$$\mathrm{I}_{\frac{1}{2}}(t) \supset \frac{p}{u\sqrt{\mathrm{S}}}$$

$$\mathrm{I}_{-\frac{1}{2}}(t) \supset \frac{p\sqrt{\mathrm{S}}}{u}$$

$$\frac{\mathrm{I}_\nu(at)}{t} \supset \frac{pa^\nu}{\nu\,\mathrm{R}^\nu} \qquad \mathrm{R}(\nu) > 0$$

$$t\;\mathrm{I}_\nu(t) \supset \frac{p(p+\nu u)}{u^3\,\mathrm{S}^\nu} \qquad \mathrm{R}(\nu) > -2$$

$$t^\nu\mathrm{I}_\mu(t) \supset \frac{\Gamma(\mu+\nu+1)}{2^\mu\,\Gamma(\mu+1)}\,p^{-\mu-\nu}\,{}_2\mathrm{F}_1\left(\frac{\mu+\nu+1}{2},\,\frac{\mu+\nu+2}{2};\,\mu+1;\,\frac{1}{p^2}\right) \qquad \mathrm{R}(\mu+\nu) > 1$$

$$t^2\,\mathrm{I}_0(t) \supset \frac{p(2p^2+1)}{u^5}$$

$$\frac{\mathrm{I}_\nu(t)}{t^2} \supset \frac{p(2\nu u-p)}{2\nu(4\nu^2-1)\mathrm{S}^{2\nu}} \qquad \mathrm{R}(\nu) > 1$$

$$I_0^2(t) \supset \frac{2}{\pi}\,E\left(k, \frac{\pi}{2}\right) \qquad \left(k = \frac{2}{p}\right)$$

$$I_\nu^2(t) \supset \frac{p k^{2\nu+1}}{\pi} \int_0^{\frac{\pi}{2}} \frac{\sin^{2\nu}\theta \, d\theta}{\sqrt{1-k^2\sin^2\theta}\left(\frac{1}{2}+\sqrt{1-k^2\sin^2\theta}\right)^{2\nu}} \qquad \left(k = \frac{2}{p}\right) \qquad R(\nu) > -\frac{1}{2}$$

$$I_{\nu+\frac{1}{2}}\left(\frac{t^2}{2}\right) \supset \frac{(-1)^\nu}{\sqrt{\pi}}\,\Gamma(\nu+1)\,p\,D_{-\nu-1}(p)\,D_{-\nu-1}(-p) \qquad R(\nu) > -1$$

$$I_\nu(2\sqrt{t}) \supset \frac{1}{2}\sqrt{\frac{\pi}{p}}\,e^{\frac{1}{2p}}\left[I_{\frac{\nu-1}{2}}\left(\frac{1}{2p}\right) - I_{\frac{\nu+1}{2}}\left(\frac{1}{2p}\right)\right]$$

$$\frac{I_{2\nu}(\sqrt{8t})}{\sqrt{t}} \supset \sqrt{\pi p}\,e^{\frac{1}{p}}\,I_\nu\left(\frac{1}{p}\right) \qquad R(\nu) > -\frac{1}{2}$$

$$\sum_{n=0}^{\infty} I_{2\nu+2n+1}(4\sqrt{t}) \supset \sqrt{\frac{\pi}{p}}\,e^{\frac{2}{p}}\,I_\nu\left(\frac{2}{p}\right) \qquad R(\nu) > -\frac{3}{2}$$

$$I_\nu^2(2\sqrt{t}) \supset e^{\frac{2}{p}}\,I_\nu\left(\frac{2}{p}\right) \qquad R(\nu) > -1$$

$$I_\nu(2a\sqrt{t})\,I_\nu(2b\sqrt{t}) \supset e^{\frac{a^2+b^2}{p}}\,I_\nu\left(\frac{2ab}{p}\right) \qquad R(\nu) > -1$$

$$I_\nu(2a\sqrt{t})\,J_\nu(2b\sqrt{t}) \supset e^{\frac{a^2-b^2}{p}}\,J_\nu\left(\frac{2ab}{p}\right) \qquad R(\nu) > -1$$

$$\frac{I_\nu^2(2\sqrt{t})}{t} \supset \frac{p}{\nu}\,e^{\frac{2}{p}}\left[I_\nu\left(\frac{2}{p}\right) + 2\sum_{r=1}^{\infty} I_{\nu+r}\left(\frac{2}{p}\right)\right] \qquad R(\nu) > 0$$

$$\frac{I_{\nu+\frac{1}{2}}(t)}{\sqrt{t}} \supset \sqrt{\frac{2}{\pi}p}\,Q_\nu(p)$$

$$\frac{I_{2\nu}(\sqrt{8t}) + J_{2\nu}(\sqrt{8t})}{\sqrt{t}} \supset 2\sqrt{\pi p}\,I_\nu\left(\frac{1}{p}\right)\operatorname{ch}\frac{1}{p} \qquad R(\nu) > -\frac{1}{2}$$

$$\frac{I_{2\nu}(\sqrt{8t}) - J_{2\nu}(\sqrt{8t})}{\sqrt{t}} \supset 2\sqrt{\pi p}\,I_\nu\left(\frac{1}{p}\right)\operatorname{sh}\frac{1}{p} \qquad R(\nu) > -\frac{1}{2}$$

$$I_\nu^2(2\sqrt{t}) + J_\nu^2(2\sqrt{t}) \supset 2\,I_\nu\left(\frac{2}{p}\right)\operatorname{ch}\frac{2}{p} \qquad R(\nu) > -1$$

$$I_\nu^2(2\sqrt{t}) - J_\nu^2(2\sqrt{t}) \supset 2\,I_\nu\left(\frac{2}{p}\right)\operatorname{sh}\frac{2}{p} \qquad R(\nu) > -1$$

$$\frac{I_\nu^2(2\sqrt{t}) + J_\nu^2(2\sqrt{t})}{t} \supset \frac{2p}{\nu}\operatorname{ch}\frac{1}{p}\left[I_\nu\left(\frac{2}{p}\right) + 2\sum_{r=1}^{\infty} I_{\nu+r}\left(\frac{2}{p}\right)\right] \qquad R(\nu) > 0$$

$$\frac{I_\nu^2(2\sqrt{t}) - J_\nu^2(2\sqrt{t})}{t} \supset \frac{2p}{\nu}\operatorname{sh}\frac{1}{p}\left[I_\nu\left(\frac{2}{p}\right) + 2\sum_{r=1}^{\infty} I_{\nu+r}\left(\frac{2}{p}\right)\right] \qquad R(\nu) > 0$$

$$\mathrm{I}_\nu(2\sqrt{t})\,\mathrm{I}_\nu(2\sqrt{at}) + \mathrm{J}_\nu(2\sqrt{t})\,\mathrm{J}_\nu(2\sqrt{at}) \supset 2\,\mathrm{I}_\nu\left(\frac{2a}{p}\right)\mathrm{ch}\,\frac{a^2+1}{p} \qquad \mathrm{R}(\nu) > -1$$

$$\mathrm{I}_\nu(2\sqrt{t})\,\mathrm{I}_\nu(2\sqrt{at}) - \mathrm{J}_\nu(2\sqrt{t})\,\mathrm{J}_\nu(2\sqrt{at}) \supset 2\,\mathrm{I}_\nu\left(\frac{2a}{p}\right)\mathrm{sh}\,\frac{a^2+1}{p} \qquad \mathrm{R}(\nu) > -1$$

$$e^{-t^2}\,\mathrm{I}_0(t^2) \supset \frac{p}{\sqrt{8\pi}}\,e^{\frac{p^2}{16}}\,\mathrm{K}_0\left(\frac{p^2}{16}\right)$$

$$e^{-at}\,\mathrm{I}_\nu(at) \supset \frac{2^\nu a^\nu p}{\sqrt{p(p+2a)}\,(\sqrt{p}+\sqrt{p+2a})^{2\nu}} \qquad \mathrm{R}(\nu) > -1$$

$$e^{-(a+b)t}\,\mathrm{I}_0[(a-b)t] \supset \frac{p}{\sqrt{(p+2a)(p+2b)}}$$

$$e^{-at}[(1+2at)\,\mathrm{I}_0(at) + 2at\,\mathrm{I}_1(at)] \supset \sqrt{1+\frac{2a}{p}}$$

$$\left(\frac{t}{a-b}\right)^{\nu-\frac{1}{2}} e^{-\frac{a+b}{2}t}\,\mathrm{I}_{\nu-\frac{1}{2}}\left[\frac{a-b}{2}t\right] \supset \frac{\Gamma(\nu)p}{\sqrt{\pi}\,(p+a)^\nu(p+b)^\nu} \qquad \mathrm{R}(\nu) > -\frac{1}{2}$$

$$e^{-\frac{at}{2}}\left[\mathrm{I}_{\nu-1}\left(\frac{at}{2}\right) - 2\,\mathrm{I}_\nu\left(\frac{at}{2}\right) + \mathrm{I}_{\nu+1}\left(\frac{at}{2}\right)\right] \supset \frac{4a^{\nu-1}p\sqrt{p}}{\sqrt{p+a}[\sqrt{p}+\sqrt{p+a}]^{2\nu}} \qquad \mathrm{R}(\nu) > -1$$

$$\frac{e^{-\frac{at}{2}}\,\mathrm{I}_\nu\left(\frac{at}{2}\right)}{t} \supset \frac{\nu p[\sqrt{p+a}-\sqrt{p}]^\nu}{\sqrt{p+a}+\sqrt{p}}$$

$$e^{-at}\,\mathrm{I}_0(\beta t) + (\alpha-\beta)\int_0^t e^{-\alpha t}\,\mathrm{I}_0(\beta t)\,dt \supset \sqrt{\frac{p+2b}{p+2a}} \qquad (\alpha = a+b,\ \beta = a-b)$$

$$t\,\mathrm{I}_0(ay) \supset \frac{p^2}{s^2}\,e^{-bs}\left(b+\frac{1}{s}\right)$$

$$y\,\mathrm{I}_1(ay) \supset \frac{ap}{s}\,e^{-bs}\left(b+\frac{1}{s}\right)$$

$$\frac{t\,\mathrm{I}_1(ay)}{y} \supset \frac{p}{a}\left[\frac{p}{s}\,e^{-bs} - e^{-bp}\right] \qquad \star$$

$$\left(\frac{t-b}{t+b}\right)^{\frac{\nu}{2}}\mathrm{I}_\nu(ay) \supset \frac{p}{s}\,e^{-bs}\,\frac{a^\nu}{\mathrm{R}^\nu} \qquad (t > b) \qquad \mathrm{R}(\nu) > -1$$

$$\int_0^t \mathrm{I}_0(at)\,dt \supset \frac{1}{s}$$

$$\int_0^t t^\nu\,\mathrm{I}_\nu(t)\,dt \supset \frac{2^\nu\,\Gamma\left(\nu+\frac{1}{2}\right)}{\sqrt{\pi}\,u^{2\nu+1}} \qquad \mathrm{R}(\nu) > -\frac{1}{2}$$

$$\int_b^t \frac{I_1(ay)\,dt}{y} \supset \frac{e^{-bs}-e^{-bp}}{ab}$$

$$\int_b^t \frac{I_1(ay)}{y}\,t\,dt \supset \frac{1}{a}\left(\frac{p}{s}\,e^{-bs}-e^{-bp}\right)$$

$$\int_t^\infty e^{-at}\,\frac{I_1(ay)}{y}\,dt \supset \frac{e^{-bp}-e^{-b\sqrt{p^2+2ap}}}{ab}$$

$$\int_t^\infty e^{-\alpha t}\,\frac{I_1(\beta\sqrt{t^2-\gamma^2})}{\sqrt{t^2-\gamma^2}}\,dt \supset \frac{e^{-\gamma(p+2\sqrt{ab})}-e^{-\gamma\sqrt{(p+2a)(p+2b)}}}{\beta\gamma} \quad \begin{pmatrix}\alpha=a+b\\ \beta=a-b\end{pmatrix}$$

$$e^{-\alpha t} \quad I_0(\beta\sqrt{t^2-\gamma^2}) \quad \supset \frac{p\,e^{-\gamma\sqrt{(p+2a)(p+2b)}}}{\sqrt{(p+2a)(p+2b)}}$$

$$e^{-\alpha(t+\gamma)}\,I_0(\beta\sqrt{t^2+2\gamma t}) \supset \frac{p\,e^{\gamma[p-\sqrt{(p+2a)(p+2b)}]}}{\sqrt{(p+2a)(p+2b)}}$$

$$e^{-\alpha t}\,I_0(\beta\sqrt{t^2-\gamma^2})+2b\int_\gamma^t e^{-\alpha t}\,I_0(\beta\sqrt{t^2-\gamma^2})\,dt \supset \sqrt{\frac{p+2b}{p+2a}}\,e^{-\gamma\sqrt{(p+2a)(p+2b)}}$$

$$\int_\gamma^t e^{-\alpha t}\,\frac{I_1(\beta\sqrt{t^2-\gamma^2})}{\sqrt{t^2-\gamma^2}}\,dt \supset \frac{e^{-\gamma\sqrt{(p+2a)(p+2b)}}-e^{-\gamma(p+\alpha)}}{\beta\gamma}$$

$$\int_b^t e^{-at}\,\frac{I_1(ay)}{y}\,dt \supset \frac{e^{-b\sqrt{p^2+2ap}}-e^{-b(p+a)}}{ab}$$

$$I_0(ay) \supset \frac{p\,e^{-bs}}{s}$$

$$\frac{I_1(ay)}{y} \supset \frac{p}{ab}\,(e^{-bs}-e^{-bp}) \qquad \bigstar$$

11. — Fonctions K de Bessel.

$$\mathrm{K}_0(at) \supset \frac{p}{s}\log\frac{\mathrm{R}}{a}$$

$$\mathrm{K}_\nu(at) \supset \frac{\pi p}{2s\sin\nu\pi}\left[\left(\frac{\mathrm{R}}{a}\right)^\nu-\left(\frac{a}{\mathrm{R}}\right)^\nu\right] \qquad -1<\mathrm{R}(\nu)<1$$

$$\mathrm{K}_{\pm\frac{1}{2}}(t) \supset \frac{\pi p}{\sqrt{2(p+1)}}$$

$$t^{\frac{1}{2}}\mathrm{K}_{\pm\frac{1}{2}}(t) \supset \frac{p\sqrt{\pi}}{(p+1)\sqrt{2}}$$

$$\int_t^\infty \mathrm{K}_0(at)\,dt \supset \frac{\pi}{2a}-\frac{1}{s}\log\frac{\mathrm{R}}{a}$$

$$\mathrm{K}_1(a\sqrt{t}) \supset \frac{a}{8}\sqrt{\frac{\pi}{p}}\,e^{\frac{a^2}{8p}}\left[\mathrm{K}_1\left(\frac{a^2}{8p}\right)-\mathrm{K}_0\left(\frac{a^2}{8p}\right)\right]$$

$$t\,\mathrm{K}_0(at) \supset \frac{p}{s^2}\left[\frac{p}{s}\log\frac{\mathrm{R}}{a}-1\right]$$

$$t\,\mathrm{K}_1(at) \supset \frac{p}{s^2}\left[\frac{p}{a}-\frac{a}{s}\log\frac{\mathrm{R}}{a}\right]$$

$$t^{-\frac{1}{2}}\mathrm{K}_\nu(a\sqrt{t}) \supset \frac{\sqrt{\pi p}}{2\cos\frac{\nu\pi}{2}}\,e^{\frac{a^2}{8p}}\,\mathrm{K}_{\frac{\nu}{2}}\left(\frac{a^2}{8p}\right) \qquad \mathrm{R}(\nu)>-\frac{1}{4}.$$

$$\frac{1}{\sqrt{4\pi t}}\,e^{-\frac{a^2}{8t}}\,\mathrm{K}_\nu\left(\frac{a^2}{8t}\right) \supset \sqrt{p}\,\mathrm{K}_{2\nu}(a\sqrt{p})$$

$$\mathrm{K}_0(ay) \supset \frac{p}{s}\log\frac{\mathrm{R}}{a}\,e^{-bs}$$

$$t\,\mathrm{K}_0(ay) \supset -\frac{p}{s^2}e^{-bs}\left[1+p\log\frac{\mathrm{R}}{a}\left\{b-\frac{1}{s}\right\}\right]$$

$$y\,\mathrm{K}_1(ay) \supset -\frac{p}{s^2}e^{-bs}\left[\frac{p}{a}+a\log\frac{\mathrm{R}}{a}\left\{b+\frac{1}{s}\right\}\right]$$

$$\left(\frac{t-b}{t+b}\right)^{\frac{\nu}{2}}\mathrm{K}_\nu(ay) \supset \frac{\pi p\,e^{-bs}}{2s\sin\nu\pi}\left[\left(\frac{\mathrm{R}}{a}\right)^\nu-\left(\frac{a}{\mathrm{R}}\right)^\nu\right] \qquad -1<\mathrm{R}(\nu)<1$$

$$\sqrt{\frac{2}{\pi t}}\,e^{-\frac{1}{t}}\,\mathrm{K}_\nu\left(\frac{1}{t}\right) \supset \sqrt{8p}\,\mathrm{K}_{2\nu}(\sqrt{8p})$$

$$\frac{\mathrm{K}_0(a\sqrt{t})+\frac{\pi}{2}\mathrm{Y}_0(a\sqrt{t})}{\sqrt{t}} \supset \sqrt{\pi p}\,\mathrm{sh}\,\frac{a^2}{8p}\,\mathrm{K}_0\left(\frac{a^2}{8p}\right)$$

$$\frac{\mathrm{K}_0(a\sqrt{t})-\frac{\pi}{2}\mathrm{Y}_0(a\sqrt{t})}{\sqrt{t}} \supset \sqrt{\pi p}\,\mathrm{ch}\,\frac{a^2}{8p}\,\mathrm{K}_0\left(\frac{a^2}{8p}\right)$$

12. — Fonctions de Kelvin (*ber* et *bei*).

$$ber\,t \supset \frac{p\left(\sqrt{p^4+1}+p^2\right)^{\frac{1}{2}}}{\sqrt{2(p^4+1)}}$$

$$bei\,t \supset \frac{p\left(\sqrt{p^4+1}-p^2\right)^{\frac{1}{2}}}{\sqrt{2(p^4+1)}}$$

$$ber_\nu t + i\,bei_\nu t \supset i^{\frac{3\nu}{2}} \frac{p}{T^\nu \varpi} \qquad R(\nu) > -1$$

$$ber(2\sqrt{t}) \supset \cos\frac{1}{p}$$

$$bei(2\sqrt{t}) \supset \sin\frac{1}{p}$$

$$t^{\frac{\nu}{2}}\,ber_\nu \sqrt{t} \supset (2p)^{-\nu} \cos\left(\frac{1}{4p} + \frac{3\nu\pi}{4}\right) \qquad R(\nu) > -1$$

$$t^{\frac{\nu}{2}}\,bei_\nu \sqrt{t} \supset (2p)^{-\nu} \sin\left(\frac{1}{4p} + \frac{3\nu\pi}{4}\right) \qquad R(\nu) > -1$$

$$\frac{ber_\nu t + i\,bei_\nu t}{t} \supset i^{\frac{3\nu}{2}} \frac{p}{\nu T^\nu} \qquad R(\nu) > 0$$

$$\sqrt{t}\left(ber_\nu 2\sqrt{t}\,bei'_\nu 2\sqrt{t} - bei_\nu 2\sqrt{t}\,ber'_\nu 2\sqrt{t}\right) \supset \frac{1}{p} I_\nu\left(\frac{2}{p}\right) \qquad R(\nu) > -2$$

$$ber^2_\nu 2\sqrt{t} + bei^2_\nu 2\sqrt{t} \supset I_\nu\left(\frac{2}{p}\right) \qquad R(\nu) > -1$$

$$2t^{-\frac{1}{2}}\left(ber_\nu 2\sqrt{t}\,ber'_\nu 2\sqrt{t} + bei_\nu 2\sqrt{t}\,bei'_\nu 2\sqrt{t}\right) \supset p\;I_\nu\left(\frac{2}{p}\right) \qquad R(\nu) > 0$$

$$ber'^2_\nu 2\sqrt{t} + bei'^2_\nu 2\sqrt{t} \supset p^2 I_\nu\left(\frac{2}{p}\right) \qquad R(\nu) > 0$$

$$ber^2_n(-2\sqrt{t}) + bei^2_n(-2\sqrt{t}) \supset (-1)^{\frac{n}{2}} J_n\left(\frac{2}{p}\right)$$

$$ber\,ay + i\,bei\,ay \supset p\,\frac{e^{-b\rho}}{\rho}$$

$$\left(\frac{t-b}{t+b}\right)^{\frac{\nu}{2}}\left[ber_\nu ay + i\,bei_\nu ay\right] \supset \frac{a^\nu p\, e^{-b\rho+\frac{3}{4}\nu\pi i}}{\rho\, T^\nu} \qquad R(\nu) > -1$$

$$t(ber\,ay + i\,bei\,ay) \supset \frac{p^2}{\rho^2} e^{-b\rho}\left(b + \frac{1}{\rho}\right)$$

$$\frac{ber_1 ay + i\,bei_1 ay}{y} \supset \frac{p\,e^{-\frac{3}{4}\pi i}}{ab}\left(e^{-bp} - e^{-bv}\right) \qquad \star$$

$$y(ber_1 ay + i\,bei_1 ay) \supset \frac{pa\,e^{-bv+\frac{3}{4}\pi i}}{v^2}\left(b + \frac{1}{v}\right)$$

$$t\,\frac{ber_1 ay + i\,bei_1 ay}{y} \supset \frac{p\,e^{-\frac{3}{4}\pi i}}{a}\left(e^{-bp} - p\,\frac{e^{-bv}}{v}\right)$$

$$\frac{1}{\sqrt{t}}\int_0^\infty e^{-\frac{x^2}{16t}}\,ber(2\sqrt{x})\,dx \supset 2\sqrt{\pi}\cos\left(\frac{2}{\sqrt{p}}\right)$$

13. — Fonctions *ker* et *kei*.

$$\mathrm{ker}\,t + i\,\mathrm{kei}\,t \supset \frac{p}{\varpi}\log\frac{T}{\sqrt{i}}$$

$$\mathrm{ker}_\nu t + i\,\mathrm{kei}_\nu t \supset \frac{\pi p[T^\nu - i^\nu T^{-\nu}]}{2 i^{\frac{3\nu}{2}}\varpi\sin\nu\pi} \qquad -1 < R(\nu) < 1$$

$$\mathrm{ker}\,ay + i\,\mathrm{kei}\,ay \supset \frac{p}{\rho}e^{-b\rho}\log\frac{T}{a\sqrt{i}}$$

$$\left(\frac{t-b}{t+b}\right)^{\frac{\nu}{2}}[\mathrm{ker}_\nu ay + i\,\mathrm{kei}_\nu ay] \supset \frac{\pi p e^{-b\rho-\frac{\nu\pi i}{2}}}{2\rho\sin\nu\pi}\left[\left(\frac{T}{a\sqrt{i}}\right)^\nu - \left(\frac{a\sqrt{i}}{T}\right)^\nu\right] \qquad -1 < R(\nu) < 1$$

$$t(\mathrm{ker}\,ay + i\,\mathrm{kei}\,ay) \supset -\frac{p\,e^{-b\rho}}{\rho^2}\left[1 + p\log\frac{T}{a\sqrt{i}}\left\{b - \frac{1}{\rho}\right\}\right]$$

$$y(\mathrm{ker}_1 y + i\,\mathrm{kei}_1 y) \supset -\frac{p\,e^{-b\rho-\frac{\pi i}{4}}}{\rho}\left[\frac{p}{a\sqrt{i}} + a\sqrt{i}\log\frac{T}{a\sqrt{i}}\left\{b + \frac{1}{\rho}\right\}\right]$$

$$t(\mathrm{ker}\,at + i\,\mathrm{kei}\,at) \supset \frac{p}{\rho^2}\left[\frac{p}{\rho}\log\frac{T}{a\sqrt{i}} - 1\right]$$

$$t(\mathrm{ker}_1 at + i\,\mathrm{kei}_1 at) \supset -\frac{p\,e^{-\frac{\pi i}{4}}}{\rho^2}\left[\frac{p}{a\sqrt{i}} + \frac{a\sqrt{i}}{\rho}\log\frac{T}{a\sqrt{i}}\right]$$

14. — Fonctions de Bessel intégrales.

$$\mathrm{J}i_0(t) \supset \arg \operatorname{sh} p = \log \mathrm{Q}$$

$$\mathrm{J}i_\nu(t) \supset \frac{1}{\nu}[1-(q-p)^\nu] \qquad \mathrm{R}(\nu) > 0$$

$$\mathrm{J}i_0(2\sqrt{t}) \supset \frac{1}{2}\mathrm{E}i\left(-\frac{1}{p}\right)$$

$$\mathrm{Y}i_0(t) \supset -\frac{1}{\pi}(\log \mathrm{Q})^2$$

$$\mathrm{Y}i_\nu(t) \supset \frac{1+\cos\nu\pi - \mathrm{Q}^\nu - \mathrm{Q}^{-\nu}\cos\nu\pi}{\nu\sin\nu\pi} \qquad -1 < \mathrm{R}(\nu) < 1$$

$$\mathrm{K}i_0(t) \supset \frac{(\arg \operatorname{ch} p)^2}{2} + \frac{\pi^2}{8}$$

$$\mathrm{K}i_\nu(t) \supset \frac{\pi\left[\mathrm{S}^\nu + \mathrm{S}^{-\nu} - 2\cos\frac{\nu\pi}{2}\right]}{2\nu\sin\nu\pi} \qquad -1 < \mathrm{R}(\nu) < 1$$

15. — Fonctions de Struve.

$$\mathbf{H}_0(at) \supset \frac{2p}{\pi r}\operatorname{arg\,sh}\frac{a}{p} = \frac{2p}{\pi r}\log\frac{a+r}{p}$$

$$\mathbf{H}_1(t) \supset \frac{2}{\pi}\left(1 - \frac{p^2}{q}\log \mathrm{U}\right)$$

$$\mathbf{H}_2(t) \supset \frac{2}{\pi}\left(-2p + \frac{1}{3p} + \frac{2p^3+p}{q}\log \mathrm{U}\right)$$

$$\mathbf{H}_3(t) \supset \frac{2}{\pi}\left[4p^2 + \frac{1}{3} + \frac{2}{15p^2} - \frac{4p^4+3p^2}{q}\log \mathrm{U}\right]$$

$$\frac{\mathbf{H}_1(t)}{t} \supset \frac{2}{\pi}(-p + pq\log \mathrm{U})$$

$$\frac{\mathbf{H}_2(t)}{t} \supset \frac{2}{\pi}\left(p^2 + \frac{1}{3} - pq\log \mathrm{U}\right)$$

$$\frac{\mathbf{H}_3(t)}{t} \supset \frac{2}{\pi}\left[-\frac{4p^3}{3} - \frac{7p}{9} + \frac{1}{15p} + \frac{pq(4p^2+1)}{3}\log \mathrm{U}\right]$$

$$\mathbf{H}_0(ti\sqrt{i}) = \mathit{ster}\,t + i\,\mathit{stei}\,t \supset \frac{2}{\pi}\frac{p}{w}\log\frac{i\sqrt{i}+w}{p}$$

$$\mathbf{H}_{\frac{1}{2}}(t) \supset \sqrt{2p} - \frac{p}{q}\sqrt{\mathrm{Q}}$$

$$\mathbf{H}_{-\frac{1}{2}}(t) \supset \frac{p}{q\sqrt{\mathrm{Q}}}$$

$$\sqrt{t}\,\mathbf{H}_{\frac{1}{2}}(t) \supset \frac{\sqrt{2}}{q^2\sqrt{\pi}}$$

$$\sqrt{t}\,\mathbf{H}_{-\frac{1}{2}}(t) \supset \sqrt{\frac{2}{\pi}}\,\frac{p}{q^2}$$

$$\frac{\mathbf{H}_{\frac{1}{2}}(t)}{\sqrt{t}} \supset \sqrt{\frac{2}{\pi}}\,p\log\frac{q}{p}$$

$$\mathbf{H}_{-n-\frac{1}{2}}(t) \supset (-1)^n\frac{p}{q\mathrm{Q}^{n+\frac{1}{2}}} \qquad (n \text{ entier} \geqq 0)$$

$$\sqrt{t}\,\mathbf{H}_{\frac{3}{2}}(t) \supset \sqrt{\frac{2}{\pi}}\left[\frac{1}{2p} - \frac{p}{q^2} + p\log\frac{q}{p}\right]$$

$$t\sqrt{t}\,\mathbf{H}_{\frac{3}{2}}(t) \supset \sqrt{\frac{2}{\pi}}\,\frac{3p^2+1}{p^2q^4}$$

$$\frac{\mathbf{H}_{-\frac{1}{2}}(t)}{\sqrt{t}} \supset \sqrt{\frac{2}{\pi}}\, p \left(\frac{\pi}{2} - \operatorname{arc\,tg} p\right) = \sqrt{\frac{2}{\pi}}\, p \operatorname{arc\,tg} \frac{1}{p}$$

$$\sqrt{t}\, \mathbf{H}_{-\frac{3}{2}}(t) \supset \sqrt{\frac{2}{\pi}} \left(\frac{p^2}{q^2} - p \operatorname{arc\,tg} \frac{1}{p}\right)$$

$$\frac{\mathbf{H}_{\frac{3}{2}}(t)}{\sqrt{t}} \supset \sqrt{\frac{2}{\pi}} \left(\frac{1}{2} - p^2 \log \frac{q}{p}\right)$$

$$\frac{\mathbf{H}_\nu(t)}{\sqrt{t}} \supset \sqrt{\frac{2}{\pi}}\, p \int_p^q \frac{dx}{(p+x)^{\nu+\frac{1}{2}} \sqrt{q^2 - x^2}} \qquad R(\nu) > -\frac{3}{2}$$

$$= \sqrt{\frac{2}{\pi}}\, p \int_0^{\frac{\pi}{2}} \frac{\sin\theta \, d\theta}{\sqrt{p^2 + \sin^2\theta}\left(p + \sqrt{p^2 + \sin^2\theta}\right)^{\nu+\frac{1}{2}}}$$

$$\frac{\pi t}{2} \left[\mathrm{J}_1(t)\, \mathbf{H}_0(t) - \mathrm{J}_0(t)\, \mathbf{H}_1(t)\right] \supset \frac{1}{q^3}$$

$$\frac{\pi}{2}\, t \left[\mathrm{J}_\nu(t)\, \mathbf{H}'_\nu(t) - \mathrm{J}'_\nu(t)\, \mathbf{H}_\nu(t)\right] \supset \frac{1}{q^{2\nu+1}} \qquad R(\nu) > -\frac{1}{2}$$

$$\sqrt{\frac{2t}{\pi}} \int_0^{\frac{\pi}{2}} \mathbf{H}_\nu(t \sin\theta) \sin^{1-\nu}\theta \, d\theta \supset (2p)^{\frac{1}{2}-\nu} - \frac{p\left[p + \sqrt{p^2+1}\right]^{\frac{1}{2}-\nu}}{\sqrt{p^2+1}}$$

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16. — Fonctions de Struve d'argument imaginaire.

$$\mathbf{L}_0(t) \supset \frac{2p}{\pi u}\operatorname{arc\,sin}\frac{1}{p}$$

$$\mathbf{L}_1(t) \supset \frac{2}{\pi}\left[-1+\frac{p^2}{u}\operatorname{arc\,sin}\frac{1}{p}\right]$$

$$\mathbf{L}_2(t) \supset \frac{2}{\pi}\left[-2p-\frac{1}{3p}+\frac{2p^3-p}{u}\operatorname{arc\,sin}\frac{1}{p}\right]$$

$$\mathbf{L}_3(t) \supset \frac{2}{\pi}\left[-4p^2+\frac{1}{3}-\frac{2}{15p^2}+\frac{4p^4-3p^2}{u}\operatorname{arc\,sin}\frac{1}{p}\right]$$

$$\frac{\mathbf{L}_1(t)}{t} \supset \frac{2}{\pi}\left[p-pu\operatorname{arc\,sin}\frac{1}{p}\right]$$

$$\frac{\mathbf{L}_2(t)}{t} \supset \frac{2}{\pi}\left[p^2-\frac{1}{3}-p^2u\operatorname{arc\,sin}\frac{1}{p}\right]$$

$$\frac{\mathbf{L}_3(t)}{t} \supset \frac{2}{\pi}\left[\frac{4p^3}{3}-\frac{7p}{9}-\frac{1}{15p}-\frac{pu(4p^2-1)}{3}\operatorname{arc\,sin}\frac{1}{p}\right]$$

$$\mathbf{L}_{\frac{1}{2}}(t) \supset \frac{p\sqrt{S}}{u}-\sqrt{2p}$$

$$\mathbf{L}_{-\frac{1}{2}}(t) \supset \frac{p}{u\sqrt{S}}$$

$$\sqrt{t}\,\mathbf{L}_{\frac{1}{2}}(t) \supset \frac{\sqrt{2}}{u^2\sqrt{\pi}}$$

$$\sqrt{t}\,\mathbf{L}_{-\frac{1}{2}}(t) \supset \frac{p\sqrt{2}}{u^2\sqrt{\pi}}$$

$$\mathbf{L}_{-n-\frac{1}{2}}(t) \supset \frac{p}{u\,S^{n+\frac{1}{2}}}$$

$$\frac{\mathbf{L}_{\frac{1}{2}}(t)}{\sqrt{t}} \supset \sqrt{\frac{2}{\pi}}\,p\log\frac{u}{p}$$

$$\sqrt{t}\,\mathbf{L}_{\frac{3}{2}}(t) \supset \sqrt{\frac{2}{\pi}}\left[\frac{p}{u^2}-p\log\frac{u}{p}-\frac{1}{2p}\right]$$

$$\frac{\mathbf{L}_{\frac{3}{2}}(t)}{\sqrt{t}} \supset \sqrt{\frac{2}{\pi}}\left[p^2\log\frac{u}{p}-\frac{1}{2}\right]$$

$$t\sqrt{t}\,\mathbf{L}_{\frac{3}{2}}(t) \supset \sqrt{\frac{2}{\pi}}\,\frac{3p^2-1}{p^2u^4}$$

$$\sqrt{t}\,\mathbf{L}_{-\frac{1}{2}}(t) \supset \sqrt{\frac{2}{\pi}}\,p \arg\coth p$$

$$\sqrt{t}\,\mathbf{L}_{-\frac{3}{2}}(t) \supset \sqrt{\frac{2}{\pi}}\left[\frac{p^2}{u^2} - p \arg\coth p\right]$$

$$\frac{\mathbf{L}_\nu(t)}{\sqrt{t}} \supset \sqrt{\frac{2}{\pi}}\,p\int_p^q \frac{dx}{(p+x)^{\nu+\frac{1}{2}}(x^2-u^2)} \qquad \mathrm{R}(\nu) > -\frac{3}{2}$$

$$= \sqrt{\frac{2}{\pi}}\,p\int_0^{\frac{\pi}{2}} \frac{\sin\theta\, d\theta}{\sqrt{p^2-\sin^2\theta}\left(p+\sqrt{p^2-\sin^2\theta}\right)^{\nu+\frac{1}{2}}}$$

$$\frac{\pi}{2}\,t[\mathrm{I}_0(at)\,\mathbf{L}_1(at) - \mathrm{I}_1(at)\,\mathbf{L}_0(at)] \supset -\frac{a^3}{s^3}$$

$$\frac{\pi}{2}\,t[\mathrm{I}_\nu(t)\quad \mathbf{L}'_\nu(t) \quad - \mathrm{I}'_\nu(t) \quad \mathbf{L}_\nu(t)\quad] \supset \frac{1}{u^{2\nu+1}} \qquad \mathrm{R}(\nu) > -\frac{1}{2}$$

$$\sqrt{\frac{2t}{\pi}}\int_0^{\frac{\pi}{2}} \mathbf{L}_\nu(t\sin\theta)\sin^{1-\nu}\theta\, d\theta \supset \frac{p\left[p+\sqrt{p^2-1}\right]^{\frac{1}{2}-\nu}}{\sqrt{p^2-1}} - (2p)^{\frac{1}{2}-\nu}$$

17. — Fonctions de Bessel du troisième ordre.

$$t^{-\frac{m+n}{3}} J_{m,n}\left(3t^{\frac{1}{3}}\right) \supset (-1)^n p^{\frac{m+n}{2}} J_{n-m}\left(\frac{2}{\sqrt{p}}\right)$$

$$t^{\frac{2m-n}{3}} J_{m,n}\left(3t^{\frac{1}{3}}\right) \supset p^{\frac{n}{2}-m} J_n\left(\frac{2}{\sqrt{p}}\right)$$

$$t^{\frac{2n-m}{3}} J_{m,n}\left(3t^{\frac{1}{3}}\right) \supset p^{\frac{m}{2}-n} J_m\left(\frac{2}{\sqrt{p}}\right)$$

$$-\frac{m+n}{3} t^{-1-\frac{m+n}{3}} J_{m,n}\left(3t^{\frac{1}{3}}\right)$$

$$+t^{-\frac{m+n+2}{3}} J'_{m,n}\left(3t^{\frac{1}{3}}\right) \supset (-1)^n p^{\frac{m+n+2}{2}} J_{n-m}\left(\frac{2}{\sqrt{p}}\right)$$

$$(t-a)^{\frac{2m-n}{3}} J_{m.n}\left[3(t-a)^{\frac{1}{3}}\right] \supset e^{-ap} p^{\frac{n}{2}-m} J_n\left(\frac{2}{\sqrt{p}}\right) \qquad (t > a)$$

$$t^{\frac{1}{6}} J_{0,-\frac{1}{2}}\left(3t^{\frac{1}{3}}\right) \supset p^{-\frac{1}{4}} J_{-\frac{1}{2}}\left(\frac{2}{\sqrt{p}}\right)$$

$$\frac{3^{\mu+\nu}\,\Gamma(\mu+1)\,\Gamma(\nu+1)}{\Gamma(\mu+\nu+\alpha+1)} t^{\alpha} J_{\mu,\nu}(t) \supset p^{-(\mu+\nu+\alpha)}\, {}_3F_2\left[\frac{\mu+\nu+\alpha+1}{3}, \frac{\mu+\nu+\alpha+2}{3}, \frac{\mu+\nu+\alpha+3}{3}; \mu+1, \nu+1; -\frac{1}{p^3}\right]$$

$$R(\mu+\nu+\alpha) > -1$$

$$\frac{2\sqrt{\pi t}}{3} J_{\frac{1}{6},-\frac{1}{6}}(t) \supset \frac{p}{\sqrt{p^3+1}}$$

$$\frac{2\sqrt{\pi t}}{3} J_{-\frac{1}{6},-\frac{5}{6}}(t) \supset \frac{p^2}{\sqrt{p^3+1}}$$

$$\frac{2\sqrt{\pi t}}{3} J_{-\frac{5}{6},-\frac{7}{6}}(t) \supset \frac{p^3}{\sqrt{p^3+1}} \qquad \star$$

$$\Gamma\left(\frac{2}{3}\right)\left(\frac{t}{3}\right)^{\frac{1}{3}} J_{0,-\frac{1}{3}}(t) \supset \left(\frac{p}{p^3+1}\right)^{\frac{1}{3}}$$

$$\frac{1}{3}\sqrt{\frac{\pi}{t}} J_{\frac{1}{6},-\frac{1}{6}}\left(\frac{-t}{\sqrt[3]{4}}\right) \supset p\int_p^{\infty} \frac{dp}{\sqrt{4p^3-1}}$$

$$\frac{1}{3\sqrt[3]{4}}\sqrt{\frac{\pi}{t}} J_{-\frac{1}{6},-\frac{5}{6}}\left(\frac{-t}{\sqrt[3]{4}}\right) \supset -p\int_p^{\infty} \frac{p\,dp}{\sqrt{4p^3-1}} \qquad \star$$

$$J_{2n,n}\left(3t^{\frac{2}{3}}\right) \supset \frac{1}{\Gamma(n+1)\,p^{2n+1}}\, {}_1F_1\left(n+\frac{1}{2}; 2n+1; -\frac{4}{p^2}\right)$$

$$= e^{-\frac{2}{p^2}} I_n\left(\frac{2}{p^2}\right)$$

18. — Fonctions hypergéométriques confluentes M.

$$M_{\mu,\nu}(t) \supset \frac{\Gamma\left(\nu+\frac{3}{2}\right)}{\left(p+\frac{1}{2}\right)^{\nu+\frac{3}{2}}}\, p \quad {}_2F_1\left(\nu+\frac{3}{2},\ -\mu+\nu+\frac{1}{2};\ 2\nu+1;\ \frac{1}{p+\frac{1}{2}}\right)$$

$$R(\nu) > -\frac{3}{2}$$

$$= \frac{\Gamma\left(\nu+\frac{3}{2}\right)}{\left(p+\frac{1}{2}\right)^{\mu+\nu+\frac{1}{2}}}\, p\left(p-\frac{1}{2}\right)^{\mu-1} \quad {}_2F_1\left(\nu-\frac{1}{2},\ \mu+\nu+\frac{1}{2};\ 2\nu+1;\ \frac{1}{p+\frac{1}{2}}\right)$$

$$= \frac{\Gamma\left(\nu+\frac{3}{2}\right)}{\left(p-\frac{1}{2}\right)^{\nu+\frac{3}{2}}}\, p \quad {}_2F_1\left(\nu+\frac{3}{2},\ \mu+\nu+\frac{1}{2};\ 2\nu+1;\ \frac{1}{\frac{1}{2}-p}\right)$$

$$= \frac{\Gamma\left(\nu+\frac{3}{2}\right)}{\left(p+\frac{1}{2}\right)^{\mu+1}}\, p\left(p-\frac{1}{2}\right)^{\mu-\nu-\frac{1}{2}} \quad {}_2F_1\left(\nu-\frac{1}{2},\ -\mu+\nu+\frac{1}{2};\ 2\nu+1;\ \frac{1}{\frac{1}{2}-p}\right)$$

$$t^{\alpha} M_{\mu,\nu}(t) \supset \frac{\Gamma\left(\alpha+\nu+\frac{3}{2}\right)}{\left(p+\frac{1}{2}\right)^{\alpha+\nu+\frac{3}{2}}}\, p \quad {}_2F_1\left(\alpha+\nu+\frac{3}{2},\ -\mu+\nu+\frac{1}{2};\ 2\nu+1;\ \frac{1}{p+\frac{1}{2}}\right)$$

$$= \frac{\Gamma\left(\alpha+\nu+\frac{3}{2}\right)}{\left(p+\frac{1}{2}\right)^{\mu+\nu+\frac{1}{2}}}\, p\left(p-\frac{1}{2}\right)^{\mu-\alpha-1} \quad {}_2F_1\left(\nu-\alpha-\frac{1}{2},\ \mu+\nu+\frac{1}{2};\ 2\nu+1;\ \frac{1}{p+\frac{1}{2}}\right)$$

$$= \frac{\Gamma\left(\alpha-\nu+\frac{3}{2}\right)}{\left(p-\frac{1}{2}\right)^{\alpha+\nu+\frac{3}{2}}}\, p \quad {}_2F_1\left(\alpha+\nu+\frac{3}{2},\ \mu+\nu+\frac{1}{2};\ 2\nu+1;\ \frac{1}{\frac{1}{2}-p}\right)$$

$$= \frac{\Gamma\left(\alpha+\nu+\frac{3}{2}\right)}{\left(p+\frac{1}{2}\right)^{\mu+\alpha+1}}\, p\left(p-\frac{1}{2}\right)^{\mu-\nu+\frac{1}{2}} \quad {}_2F_1\left(\nu-\alpha-\frac{1}{2},\ -\mu+\nu+\frac{1}{2};\ 2\nu+1;\ \frac{1}{\frac{1}{2}-p}\right)$$

$$R(\alpha+\nu) > -\frac{3}{2}$$

$$t^{\alpha}\,M_{-\mu,\nu}\,(-t) \supset (-1)^{\alpha+2\nu+2}\,\Gamma\left(\alpha+\nu+\frac{3}{2}\right) p\left(\frac{1}{2}-p\right)^{-\alpha-\nu-\frac{3}{2}} {}_2F_1\left(\alpha+\nu+\frac{3}{2},\ \mu+\nu+\frac{1}{2};\ 2\nu+1;\ \frac{1}{\frac{1}{2}-p}\right)$$

$$t^{\alpha}\,M_{-\mu,-\nu}(-t) \supset (-1)^{\alpha-2\nu+2}\,\Gamma\left(\alpha-\nu+\frac{3}{2}\right) p\left(\frac{1}{2}-p\right)^{-\alpha+\nu-\frac{3}{2}} {}_2F_1\left(\alpha-\nu+\frac{3}{2},\ \mu-\nu+\frac{1}{2};\ 1-2\nu;\ \frac{1}{\frac{1}{2}-p}\right)$$

$$t^{\alpha}\,e^{\frac{t}{2}}\,M_{\mu,\nu}(t) \supset \frac{\Gamma\left(\alpha+\nu+\frac{3}{2}\right)}{p^{\alpha+\nu+\frac{1}{2}}}\ {}_2F_1\left(\alpha+\nu+\frac{3}{2},\ -\mu+\nu+\frac{1}{2};\ 2\nu+1;\ \frac{1}{p}\right) \qquad R(\alpha+\nu)>-\frac{3}{2}$$

$$t^{\alpha}\,e^{-\frac{t}{2}}M_{\mu,\nu}(t) \supset \frac{\Gamma\left(\alpha+\nu+\frac{3}{2}\right)}{(p+1)^{\alpha+\nu+\frac{1}{2}}}\,p\ {}_2F_1\left(\alpha+\nu+\frac{3}{2},\ -\mu+\nu+\frac{1}{2};\ 2\nu+1;\ \frac{1}{p+1}\right)$$

$$= \frac{\Gamma\left(\alpha+\nu+\frac{3}{2}\right)}{p^{\alpha+\nu+\frac{1}{2}}}\ {}_2F_1\left(\alpha+\nu+\frac{3}{2},\quad \mu+\nu+\frac{1}{2};\ 2\nu+1;\ -\frac{1}{p}\right)$$

$$t^{\nu-\frac{1}{2}}\,e^{\frac{t}{2}}\ M_{n+\nu+\frac{1}{2},\,\nu}(t) \supset \frac{\Gamma(2\nu+1)}{p^{2\nu}}\left(1-\frac{1}{p}\right)^{n} \qquad R(\nu)>-\frac{1}{2}$$

$$t^{-\frac{3}{4}}\ e^{-\frac{t}{2}}\,M_{\mu,-\frac{1}{4}}\quad(t) \supset \frac{\sqrt{\pi}\,p^{\mu+\frac{1}{4}}}{(p+1)^{\mu+\frac{1}{4}}}$$

$$t^{-\frac{1}{4}}\ e^{-\frac{t}{2}}\,M_{\mu,\frac{1}{4}}\quad(t) \supset \frac{\sqrt{\pi}\,p^{\mu+\frac{1}{4}}}{2(p+1)^{\mu+\frac{3}{4}}}$$

$$t^{-\frac{5}{4}}\ e^{\frac{t}{2}}\ M_{-\frac{1}{4},\frac{n}{2}+\frac{1}{4}}\,(t) \supset \frac{2^{n+1}}{\sqrt{\pi}}\ \frac{\Gamma\left(\frac{n+1}{2}\right)\Gamma\left(n+\frac{3}{2}\right)}{\Gamma(n+1)}\,p\,Q_n(\sqrt{p})$$

$$\frac{t^{\nu-\frac{1}{2}}\,e^{\frac{t}{2}+1}}{\Gamma(2\nu+1)}\sum_{r=0}^{\infty}(-1)^{r}\frac{M_{r+\nu+\frac{1}{2},\,\nu}(t)}{r!} \supset \frac{e^{\frac{1}{p}}}{p^{2\nu}} \qquad R(\nu)>-\frac{1}{2}$$

$$\frac{t^{\nu-\frac{1}{2}}\,e^{\frac{t}{2}+1}}{\Gamma(2\nu+1)}\sum_{r=0}^{\infty}\qquad\frac{M_{r+\nu+\frac{1}{2},\,\nu}(t)}{r!} \supset \frac{e^{-\frac{1}{p}}}{p^{2\nu}} \qquad R(\nu)>-\frac{1}{2}$$

19. — Fonction hypergéométrique confluente de Whittaker.

$$t^{\alpha} W_{\mu,\nu}(t) \supset \frac{\Gamma\left(\alpha+\nu+\frac{3}{2}\right)\Gamma\left(\alpha-\nu+\frac{3}{2}\right)}{\Gamma(\alpha-\mu+2)} p\, {}_2F_1\left(\alpha+\nu+\frac{3}{2}, \alpha-\nu+\frac{3}{2}; \alpha-\mu+2; \frac{1}{2}-p\right) \quad \star$$

$$\frac{W_{0,\nu+\frac{1}{2}}(2t)}{2t} \supset -\frac{\pi}{2\sin\nu\pi} p\, P_{\nu}(p) \quad \star$$

$$\frac{e^{-\frac{t}{2}}}{\sqrt{t}} W_{-\frac{1}{2},0}(t) \supset \log(p+1)$$

$$t^{\nu-\frac{1}{2}} e^{\frac{t}{2}} W_{m+\nu+\frac{1}{2},\nu}(t) \supset \Gamma(2\nu+m+1)(1-p)^{m} p^{-2\nu-m} \qquad R(\nu) > -\frac{1}{2}$$

$$t^{-\nu-\frac{1}{2}} e^{\frac{t}{2}} W_{\mu,\nu}(t) \supset \Gamma\left(\mu-\nu+\frac{1}{2}\right) p^{\nu-\mu+\frac{1}{2}} (1-p)^{\mu+\nu-\frac{1}{2}}$$ si $\mu-\nu+\frac{1}{2}$ n'est pas un entier négatif

$$\left(\frac{t}{a}\right)^{-\nu-\frac{1}{2}} e^{-\frac{t}{2}} W_{\mu,\nu}\left(\frac{t}{a}\right) \supset (-1)^{\mu+\nu-\frac{1}{2}} \Gamma\left(\mu-\nu+\frac{1}{2}\right)(pa+1)^{\nu-\mu-1}(pa)^{\mu+\nu+\frac{1}{2}}$$

$$= (-1)^{\mu+\nu+\frac{1}{2}} \Gamma\left(\mu-\nu+\frac{1}{2}\right) (pa)^{\mu+\nu-\frac{1}{2}} [a(p+1)]^{\nu-\mu-\frac{1}{2}}$$

$$t^{\alpha} e^{-\frac{t}{2}} W_{\mu,\nu}(t) \supset (-1)^{\mu-\alpha-1} \frac{\Gamma\left(\mu+\nu+\frac{1}{2}\right)\Gamma\left(\mu-\nu+\frac{1}{2}\right)}{\Gamma(\mu-\alpha)} p^{\mu-\alpha}\, {}_2F_1\left(\mu+\nu+\frac{1}{2}, \mu-\nu+\frac{1}{2}; \mu-\alpha; -p\right) \quad \star$$

$$= (-1)^{\mu-\alpha-1} \Gamma\left(\mu+\nu+\frac{1}{2}\right)\Gamma\left(\mu-\nu+\frac{1}{2}\right) p^{\mu-\alpha}(p+1)^{-\mu-\nu+\frac{1}{2}}$$

$$\times\, {}_2F_1\left(\mu+\nu+\frac{1}{2}, \nu-\alpha-\frac{1}{2}; \mu-\alpha; \frac{p}{p+1}\right)$$

$$= \frac{\Gamma\left(\alpha+\nu+\frac{3}{2}\right)\Gamma\left(\alpha-\nu+\frac{3}{2}\right)}{\Gamma(\alpha-\mu+2)} p\, {}_2F_1\left(\alpha+\nu+\frac{3}{2}, \alpha-\nu+\frac{3}{2}; \alpha-\mu+2; -p\right) \quad \star$$

$$t^{-\frac{m}{2}} e^{t} W_{n-\frac{m}{2},\frac{1-m}{2}}(2t) \supset 2^{1-\frac{m}{2}} \Gamma(1+n-m)(2-p)^{n-1} p^{m-n}$$

$$t^{\alpha} e^{\frac{t}{2}} W_{\mu,\nu}(t) \supset \frac{\Gamma\left(\mu+\nu+\frac{1}{2}\right)\Gamma\left(\mu-\nu+\frac{1}{2}\right)}{\Gamma(\mu-\alpha)} p^{\frac{1}{2}-\mu-\nu} (1-p)^{\mu-\alpha-1}\, {}_2F_1\left(\mu+\nu+\frac{1}{2}, \nu-\alpha-\frac{1}{2}; \mu-\alpha; 1-\frac{1}{p}\right)$$

si $\mu-\alpha>0$

$$t^{-\frac{\nu+1}{2}} e^{\frac{t}{2}} W_{\frac{\nu}{2}+\frac{1}{2}+\mu,\frac{\nu}{2}}(t) \supset \Gamma(\mu+1)(1-p)^{\nu-\mu} p^{-\nu-\mu}\, {}_2F_1\left(\mu+\nu+1, \mu, \mu+\nu+1; 1-\frac{1}{p}\right)$$

$$= \Gamma(\mu+1)(-1)^{\mu+\nu} p^{\nu}\left(1-\frac{1}{p}\right)^{\mu+\nu}$$

$$\frac{a^{2\mu}}{t^{\mu}} e^{-\frac{a^2}{8t}} W_{\mu,\nu}\left(\frac{a^2}{4t}\right) \supset \frac{a^{2\mu+1}}{p^{\frac{2\mu-1}{2}}} K_{2\nu}(a\sqrt{p})$$

20. — Fonctions de Weber.

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$$D_{2n+1}(2\sqrt{t}) \supset (-1)^n \sqrt{\pi}\,\frac{(2n+1)!}{2^n\, n!}\,\frac{p(p-1)^n}{(p+1)^{n+\frac{3}{2}}}$$

$$\frac{D_{2n}(2\sqrt{t})}{\sqrt{t}} \supset \sqrt{\pi}\,\frac{(2n)!}{2^n\, n!}\,\frac{p(1-p)^n}{(1+p)^{n+\frac{1}{2}}}$$

$$e^{\frac{t}{2}}\, D_{2n+1}(\sqrt{2t}) \supset \sqrt{\frac{\pi}{2}}\,\frac{(2n+1)!}{2^n\, n!\,\sqrt{p}}\left(\frac{1}{p}-1\right)^n \qquad (n \geqq 0)$$

$$(2t)^{-\frac{\nu}{2}}\, e^{-\frac{a^2}{8t}}\, D_{\nu-1}\left(\frac{a}{\sqrt{2t}}\right) \supset \sqrt{\frac{\pi}{2}}\, p^{\frac{\nu}{2}}\, e^{-a\sqrt{p}}$$

$$(2t)^{\frac{\nu-1}{2}}\, e^{-\frac{t}{2}}\, D_{-\nu}(\sqrt{2t}) \supset \sqrt{\frac{\pi}{2}}\,\frac{(\sqrt{p+1}-1)^\nu}{p^{\nu-1}\sqrt{p+1}}$$

$$(2t)^{\frac{\nu-1}{2}}\, e^{-\frac{t}{2}}\, D_{-\nu-2}(\sqrt{2t}) \supset \sqrt{\frac{\pi}{2}}\,\frac{(\sqrt{p+1}-1)^{\nu+1}}{(\nu+1)p^\nu} \qquad R(\nu) > -1$$

$$(2t)^{\frac{\mu+\nu}{2}}\, e^{-\frac{t}{2}}\, D_{-(\mu+\nu+1)}(\sqrt{2t}) \supset \sqrt{\frac{\pi}{2}}\,\frac{(\sqrt{p+1}-1)^{\mu+\nu+1}}{p^{\mu+\nu}\sqrt{p+1}}$$

$$D^2_{-(n+1)}(i\sqrt{2t}) - D^2_{-(n+1)}(-i\sqrt{2t}) \supset \frac{2\pi}{i\, n!}\sqrt{p}\,\frac{(p-1)^n}{(p+1)^{n+1}}$$

$$\frac{D_n(2\sqrt{at})\, D_n(2\sqrt{bt})}{\sqrt{t}} \supset \frac{n!\,\sqrt{\pi}\, p}{\sqrt{p+a+b}}\left(\frac{a+b-p}{a+b+p}\right)^{\frac{n}{2}} P_n\left[2\sqrt{\frac{ab}{(a+b)^2-p^2}}\right]$$

$$e^{\frac{a+b}{2}t}\,\frac{D_{2n}(\sqrt{2at})\, D_{2m}(\sqrt{2bt})}{\sqrt{t}} \supset \frac{(-1)^{m+n}\sqrt{\pi}\,(2n+2m)!\,(p-a)^n(p-b)^m}{2^{n+m}(n+m)!\, p^{n+m-\frac{1}{2}}}$$

$$\times\, {}_2F_1\left[-n, -m; -n-m+\frac{1}{2}; \frac{p(p-a-b)}{(p-a)(p-b)}\right]$$

$$e^{\frac{a+b}{2}t}\,\frac{D_{2n+1}(\sqrt{2at})\, D_{2m+1}(\sqrt{2bt})}{\sqrt{abt}} \supset \frac{(-1)^{m+n}\sqrt{\pi}\,(2n+2m+2)!\,(p-a)^n(p-b)^m}{2^{n+m+1}(n+m+1)!\, p^{n+m+\frac{1}{2}}}$$

$$\times\, {}_2F_1\left[-n, -m; -n-m-\frac{1}{2}; \frac{p(p-a-b)}{(p-a)(p-b)}\right]$$

$$\frac{D_n(2\sqrt{t}\cos\alpha)\,D_n(2\sqrt{t}\sin\alpha)}{2^n\,n!\,\sqrt{\pi t}} \supset \sum_{s=0}^{m}\frac{(-1)^s(2n-2s)!}{s!\,(n-s)!\,(n-2s)!}\,\frac{p}{\sqrt{p+1}}\left(\frac{1-p}{1+p}\right)^{\frac{n}{2}}\frac{(\sin 2\alpha)^{n-2s}}{(1-p^2)^{\frac{n}{2}-s}}$$

$$\left(m=\frac{n}{2}\ \text{ou}\ \frac{n-1}{2}\right)$$

$$e^{\frac{t}{2}}\frac{D_n^2(\sqrt{t})}{\sqrt{t}} \supset \frac{\sqrt{\pi}}{2^{2n}}\sum_{s=0}^{n}\frac{n!\,(2s)!\,(2n-2s)!}{s!\,s!\,(n-s)!}\sum_{r=0}^{n-s}\frac{(-1)^r}{r!\,(n-s-r)!}p^{-n+s+r+\frac{1}{2}}$$

$$\frac{e^{\frac{t}{2}}}{\sqrt{t}}\sum_{n=0}^{\infty}\frac{2^n}{(2n)!}\quad D_{2n}(\sqrt{2t}) \supset e^{\frac{1}{p}-1}\sqrt{\pi p}$$

$$\sum_{n=0}^{\infty}\frac{2^n\,n!}{(2n+1)!}D_{2n+1}(2\sqrt{t}) \supset \frac{\sqrt{\pi}}{2\sqrt{p+1}}$$

$$\sum_{n=0}^{\infty}\frac{2^n\,n!}{(2n)!}\quad\frac{D_{2n}(2\sqrt{t})}{\sqrt{t}} \supset \frac{\sqrt{\pi}}{2}\sqrt{p+1}$$

$$e^{-\frac{t^2}{4}}\sum_{s=0}^{n}(-1)^{n-s}\frac{n!}{s!\,[(n-s)!]^2}t^{n-s}D_{n-s}(t) \supset p^{n+1}e^{\frac{p^2}{4}}D_{-n-1}(p)$$

$$\int_0^t\frac{D_n(\sqrt{2x})\,D_n(i\sqrt{2x})\,D_m[\sqrt{2(t-x)}]\,D_m[i\sqrt{2(t-x)}]}{\sqrt{x(t-x)}}dx \supset (-1)^{\frac{m+n}{2}}\pi\,m!\,n!\,P_m\left(\frac{1}{p}\right)P_n\left(\frac{1}{p}\right)$$

21. — Séries hypergéométriques.

$$ {}_rF_s(\alpha_1, \ldots, \alpha_r; \gamma_1, \ldots, \gamma_s; t) \supset {}_{r+1}F_s\left(\alpha, \ldots, \alpha_r, 1; \gamma_1, \ldots, \gamma_s; \frac{1}{p}\right) $$

$$ {}_rF_s(\alpha_1, \ldots, \alpha_r; \gamma_1, \ldots, \gamma_{s-1}, 1; t) \supset {}_rF_{s-1}\left(\alpha_1, \ldots, \alpha_r; \gamma_1, \ldots, \gamma_{s-1}; \frac{1}{p}\right) $$

$$ J_0(2i\sqrt{t}) = {}_0F_1(1; t) \supset {}_0F_0\left(\frac{1}{p}\right) = e^{\frac{1}{p}} $$

$$ {}_0F_1(n+1; t) \supset {}_1F_1\left(1; n+1; \frac{1}{p}\right) \qquad (n \geqq 1) $$ *

$$ {}_1F_1(\alpha; 1; t) \supset {}_1F_0\left(\alpha; \frac{1}{p}\right) = \left(1 - \frac{1}{p}\right)^{-\alpha} $$

$$ {}_2F_1(\alpha, \beta; 1; t) \supset {}_2F_0\left(\alpha, \beta; \frac{1}{p}\right) $$

$$ {}_rF_s(\alpha_1, \ldots, \alpha_r; \gamma_1, \ldots, \gamma_s; t^2) \supset {}_{r+2}F_s\left(\alpha_1, \ldots, \alpha_r, 1, \frac{1}{2}; \gamma_1, \ldots, \gamma_s; \frac{4}{p^2}\right) $$

$$ J_0(2it) = {}_0F_1(1; t^2) \supset {}_1F_0\left(\frac{1}{2}; \frac{4}{p^2}\right) = \frac{p}{\sqrt{p^2 - 4}} $$

$$ t^\nu {}_rF_s(\alpha_1, \ldots, \alpha_r; \gamma_1, \ldots, \gamma_s; t^n) \supset p^{-\nu}\Gamma(\nu+1)_{r+n}F_s\left(\alpha_1, \ldots, \alpha_r, \frac{\nu+1}{n}, \ldots, \frac{\nu+n}{n}; \gamma_1, \ldots, \gamma_s; \frac{n^n}{p^n}\right) $$

$$ {}_0F_n\left(\frac{1}{n}, \frac{2}{n}, \ldots, \frac{n-1}{n}; 1; \frac{t^n}{n^n}\right) \supset e^{p^{-n}} $$

$$ \frac{1}{\sqrt{\pi t}} {}_{2r}F_{2s}\left(\frac{\alpha_1}{2}, \frac{\alpha_1+1}{2}, \ldots, \frac{\alpha_r}{2}, \frac{\alpha_r+1}{2}; \frac{\gamma_1}{2}, \frac{\gamma_1+1}{2}, \ldots; -\frac{a^2 2^{r-s-2}}{t}\right) $$
$$ \supset \sqrt{p}\, {}_rF_s(\alpha_1, \ldots, \alpha_r; \gamma_1, \ldots, \gamma_s; -a\sqrt{p}) $$

22. — Polynomes de type hypergéométrique (Hermite, Legendre, Abel, Laguerre, etc.).

$$\mathrm{H}\,e_n(t) \supset \frac{n!}{p^n}\sum_{s=0}^{n}\frac{(-1)^s}{s!}\left(\frac{p^2}{2}\right)^s$$

$$\mathrm{H}\,e_{2n+1}(\sqrt{t}) \supset \frac{(2n+1)!}{2^{n+1}\,n!}\sqrt{\frac{\pi}{p}}\left(\frac{1}{2p}-1\right)^n$$

$$\frac{1}{2^n\sqrt{\pi}\,t^{\frac{n+1}{2}}}e^{-\frac{a^2}{4t}}\,\mathrm{H}\,e_n\left(\frac{a}{2\sqrt{t}}\right) \supset p^{\frac{n+1}{2}}\,e^{-a\sqrt{p}}$$

$$\sum_{n=0}^{\infty}\frac{(-1)^n\,\mathrm{H}\,e_{2n+1}(\sqrt{t})}{(2n+1)!} \supset \sqrt{\frac{\pi}{p}}\,e^{1-\frac{1}{p}}$$

$$\left.\begin{aligned}
\mathrm{P}_0(\theta) &\supset 1\\
\mathrm{P}_1(\theta) &\supset \frac{p^2}{p^2+1}\\
\mathrm{P}_2(\theta) &\supset \frac{p^2+1}{p^2+2^2}\\
\mathrm{P}_3(\theta) &\supset \frac{p^2(p^2+2^2)}{(p^2+1^2)(p^2+3^2)}
\end{aligned}\right\}\quad(\theta = \cos t)$$

$$\mathrm{P}_n(1-t) \supset p^{n+1}\left(1+\frac{1}{2}\frac{d}{dp}\right)^n\frac{1}{p^{n+1}}$$

$$\Phi_m(t) \supset \left(\frac{p-1}{p}\right)^m = \left(1-\frac{1}{p}\right)^m$$

$$\Phi'_m(t) \supset p\left[\left(1-\frac{1}{p}\right)^{m-1}\right]$$

$$t\,\Phi'_m(t) \supset -\frac{m}{p}\left(1-\frac{1}{p}\right)^{m-1}$$

$$\frac{d}{dm}\Phi_m(t) \supset \left(1-\frac{1}{p}\right)^m\log\left(1-\frac{1}{p}\right)$$

$$e^{-\frac{t}{2}}\,t^m\,\mathrm{T}_m^n(t) \supset \frac{p\left(\frac{1}{2}-p\right)^n}{n!\left(\frac{1}{2}+p\right)^{m+n+1}}$$

$$t^m\,\mathrm{T}_m^n(t) \supset \frac{(1-p)^n}{n!\,p^{m+n}}$$

$$L_n(t) \supset \left(1-\frac{1}{p}\right)^n$$

$$t^n L_n(t) \supset \frac{n!}{p^n} P_n\left(1-\frac{2}{p}\right)$$

$$e^{-t} L_n(t) \supset \frac{p^{n+1}}{(p+1)^{n+1}}$$

$$e^{-\frac{t}{2}} L_n(t) \supset \frac{2p}{2p+1}\left(\frac{2p-1}{2p+1}\right)^n$$

$$\sum_{n=0}^{\infty} \frac{L_n(t)}{n!} = e\, J_0(2\sqrt{t}) \supset e^{1-\frac{1}{p}}$$

$$e^{-t}\sum_{n=0}^{\infty}(-1)^n L_{2n}(2t) \supset \frac{p^2+p}{2(p^2+1)}$$

$$t^{\alpha} L_n^{\alpha}(t) \supset \frac{\Gamma(n+\alpha+1)}{n!}\frac{(p-1)^n}{p^{n+\alpha}} \qquad R(\alpha) > -1$$

$$t^{\beta} L_n^{\alpha}(t) \supset \sum_{s=0}^{n} \frac{\Gamma(\alpha-\beta+s)\,\Gamma(\beta+1+n-s)}{\Gamma(\alpha-\beta)\,s!\,(n-s)!}\frac{(p-1)^{n-s}}{p^{n-s+\beta}} \qquad R(\beta) > -1$$

$$t^{\alpha}\sum_{n=0}^{\infty}\frac{L_n^{\alpha}(t)}{\Gamma(\alpha+n+1)} \supset \frac{e^{1-\frac{1}{p}}}{p^{\alpha}} \qquad R(\alpha) > -1$$

$$\sum_{n=0}^{\infty}\frac{(-1)^n\, n!\, t^{\alpha} L_n^{\alpha}(t)}{\Gamma(\alpha+n+1)} \supset \frac{1}{2p^{\alpha-1}\left(p-\frac{1}{2}\right)} \qquad R(\alpha) > -1$$

$$t^{\alpha} L_n^{\alpha}(at)\, L_m^{\alpha}(bt) \supset \frac{\Gamma(n+m+\alpha+1)}{n!\,m!}\frac{(p-a)^n(p-b)^m}{p^{n+m+\alpha}}\, {}_2F_1\left[-m,-n;-n-m-\alpha;\frac{p(p-a-b)}{(p-a)(p-b)}\right]$$

$$R(\alpha) > -1$$

$$e^{-t} t^{\alpha} L_n^{\alpha}(t) \supset \frac{\Gamma(n+\alpha+1)}{n!} p^{n+1}(p+1)^{-n-\alpha-1} \qquad R(\alpha) > -1$$

$$e^{-t} t^{\alpha}\sum\frac{(-1)^n(2n)!\,L_{2n}(2t)}{\Gamma(2n+\alpha+1)} \supset \frac{p}{2(p+1)^{\alpha-1}(p^2+1)} \qquad R(\alpha) > -1$$

$$e^{-t} t^{\alpha} L_n^{\alpha}(t)\, L_m^{\alpha}(t) \supset \frac{\Gamma(n+\alpha+1)\,\Gamma(m+\alpha+1)p^{n+m+1}}{n!\,m!\,\Gamma(\alpha+1)(p+1)^{n+m+\alpha+1}}\, {}_2F_1\left(-m,-n;\alpha+1;\frac{1}{p^2}\right)$$

$$R(\alpha) > -1$$

$$L_n(at)\, L_n(bt) \supset \frac{(p-a-b)^n}{p^n} P_n\left[\frac{p^2-(a+b)p+2ab}{p(p-a-b)}\right]$$

$$e^{-t} t^{2\alpha}[L_n^{\alpha}(t)]^2 \supset \frac{\Gamma(n+\alpha+1)}{2^{2n}\,n!}\sum_{s=0}^{n}\frac{\Gamma(2s+2\alpha+1)(2n-2s)!}{s!\,\Gamma(s+\alpha+1)[(n-s)!]^2}\frac{p(p-1)^{2s}}{(p+1)^{2s+2\alpha+1}} \qquad R(\alpha) > -\frac{1}{2}$$

$$e^{-\frac{t(a+b)}{2}} t^{2\alpha} \mathrm{L}_n^\alpha(at)\, \mathrm{L}_n^\alpha(bt) \supset \frac{2^{2\alpha}\Gamma(n+\alpha+1)\Gamma\left(\alpha+\frac{1}{2}\right)\Gamma(\alpha+1)}{n!\sqrt{\pi}}\, p \sum_{s=0}^{\infty} (-1)^s \frac{c_1^{n-s}}{a_1^{2\alpha-n+s+1}\Gamma(\alpha-s+1)r!}\, \mathrm{C}_{n-s}^{\alpha+\frac{1}{2}}\left(\frac{b_1^2}{a_1 c_1}\right)$$

$$a_1^2 = p^2 + p(a+b) + \frac{(a+b)^2}{4}$$

$$b_1^2 = p^2 - \frac{(a+b)^2}{4}$$

$$c_1^2 = p^2 - p(a+b) + \frac{(a+b)^2}{4}$$

$$t^{2\alpha} \mathrm{L}_n^\alpha(at)\, \mathrm{L}_n^\alpha(bt) \supset \frac{2^{2\alpha}\Gamma(n+\alpha+1)\Gamma\left(\alpha+\frac{1}{2}\right)\Gamma(\alpha+1)}{n!\sqrt{\pi}} \sum_{s=0}^{\infty} (-1)^s \frac{c_2^{n-s}}{s!\,\Gamma(\alpha-r+1)p^{2\alpha-n+s+1}}\, \mathrm{C}_{n-s}^{\alpha+\frac{1}{2}}\left(\frac{b_2^2}{pc_2}\right)$$

$$b_2^2 = p^2 - (a+b)p + 2ab$$

$$c_2^2 = [p-a-b]^2$$

$$(-i)^n \mathrm{T}_n(it) \supset p\, \mathrm{O}_n(p)$$

$$\frac{2(-i)^{n-1}\mathrm{U}_n(it)}{\sqrt{1+t^2}} \supset p\, \mathrm{S}_n(p)$$

$$\frac{1}{t^{\frac{n}{2}+1}} \int_0^\infty e^{-\frac{x^2}{4t}} \mathrm{H}e_n\left(\frac{x}{2\sqrt{t}}\right) \mathrm{J}_\nu(2\sqrt{bx})\, x^{\frac{\nu}{2}}\, dx \supset 2^n \sqrt{\pi}\, b^{\frac{\nu}{2}} p^{\frac{n-\nu}{2}}\, e^{-\frac{b}{\sqrt{p}}}$$

$$t^{\frac{\nu}{2}} \int_0^\infty e^{-\frac{b^2}{4x}} \mathrm{H}e_n\left(\frac{b}{2\sqrt{x}}\right) \mathrm{J}_\nu(2\sqrt{tx})\, x^{-\frac{n+\nu+1}{2}}\, dx \supset 2^n \sqrt{\pi}\; p^{-\frac{n}{2}-\nu+1}\, e^{-\frac{b}{\sqrt{p}}}$$

$$t^{\frac{\nu}{2}} \int_0^\infty e^{-\beta x}\, \mathrm{J}_\nu(2\sqrt{tx})\, \mathrm{L}_n^\alpha(x)\, x^{\alpha-\frac{\nu}{2}}\, dx \supset \frac{\Gamma(n+\alpha+1)p^{1-\nu+\alpha}\{1+(\beta-1)p]^n}{n!\,[1+\beta p]^{n+\alpha-1}}$$

$$\mathrm{R}(\nu) > -1$$

23. — Fonctions discontinues.

$$|\sin t| \supset \frac{p}{p^2+1}\,\frac{1+e^{-\pi p}}{1-e^{-\pi p}} \qquad (0<t<\infty)$$

$$\sin\omega t \supset \frac{\omega p}{p^2+\omega^2}\left(1-e^{-\frac{2n\pi p}{\omega}}\right) \qquad \left(0<t<\frac{2\pi n}{\omega}\right)$$

$$\cos\omega t \supset \frac{p^2}{p^2+\omega^2}\left(1-e^{-\frac{2n\pi p}{\omega}}\right) \qquad \left(0<t<\frac{2\pi n}{\omega}\right)$$

$$\left.\begin{array}{l} \sin(\omega t+\alpha) \supset \dfrac{p}{p^2+\omega^2}\left(1-e^{-\frac{2n\pi p}{\omega}}\right)(\omega\cos\alpha+p\sin\alpha) \\ \cos(\omega t+\alpha) \supset \dfrac{p}{p^2+\omega^2}\left(1-e^{-\frac{2n\pi p}{\omega}}\right)(p\cos\alpha-\omega\sin\alpha) \end{array}\right\} \qquad \left(0<t<\frac{2\pi n}{\omega}\right)$$

$$\left.\begin{array}{ll} \sin t & (0<t<\tau) \\ 0 & (\tau<t<\infty) \end{array}\right\} \supset \frac{p}{\sqrt{p^2+1}}\left[\frac{1}{\sqrt{p^2+1}}-e^{-p\tau}\cos(\tau+\operatorname{arc\,tg} p)\right]$$

$$\left.\begin{array}{ll} t\sin\omega t & \left(0<t<\dfrac{6\pi}{\omega}\right) \\ 0 & \left(\dfrac{6\pi}{\omega}<t<\infty\right) \end{array}\right\} \supset \frac{p^2}{(p^2+\omega^2)^2}\left[2\omega\left(1-e^{-\frac{6\pi p}{\omega}}\right)-\frac{6\pi(p^2+\omega^2)}{p}e^{-\frac{6\pi p}{\omega}}\right]$$

$$(n \text{ entier} > 0) \quad \left.\begin{array}{ll} t^n & (0<t<h) \\ 0 & (h<t<\infty) \end{array}\right\} \supset \frac{n!}{p^n}-e^{-ph}\left[h^n+\frac{nh^{n-1}}{p}+\frac{n(n-1)h^{n-2}}{p^2}+\ldots+\frac{n!}{p^n}\right]$$

$$\left.\begin{array}{ll} \dfrac{A_1 t}{h} & (0\leqq t\leqq h) \\ A_1 & (h\leqq t<\infty) \end{array}\right\} \supset \frac{A_1}{ph}(1-e^{-ph})$$

$$\left.\begin{array}{ll} e^{-at}\sin\omega t & \left(0<t<\dfrac{2n\pi}{\omega}\right) \\ 0 & \left(\dfrac{2n\pi}{\omega}<t<\infty\right) \end{array}\right\} \supset \frac{\omega p}{(p+a)^2+\omega^2}\left[1-e^{-\frac{2\pi n}{\omega}(p+a)}\right]$$

$$\left.\begin{array}{ll} [a^2-(t-a)^2]^{\nu-\frac{1}{2}} & (0<t<2a) \\ 0 & (2a<t<\infty) \end{array}\right\} \supset \sqrt{\pi}\,\Gamma\left(\nu+\frac{1}{2}\right)(2a)^\nu p^{1-\nu}e^{-ap}\,I_\nu(ap) \qquad R(\nu)>-\frac{1}{2}$$

$$\left.\begin{array}{ll} \dfrac{A_1 t}{h} & (0\leqq t\leqq h) \\ \dfrac{A_1(2h-t)}{h} & (h\leqq t\leqq 2h) \\ 0 & (2h\leqq t<\infty) \end{array}\right\} \supset A_1\frac{(1-e^{-ph})^2}{ph}$$

$$\left.\begin{array}{ll} A_1 t^2 & (0 < t < h) \\ A_1 h^2 & (h < t < bh) \\ 0 & (bh < t < \infty) \\ & (b > 1) \end{array}\right\} \supset A_1\left[\frac{2}{p^2} - 2e^{-ph}\frac{h}{p}\left(1+\frac{h}{p}\right) - h^2 e^{-pbh}\right]$$

$$\left.\begin{array}{ll} A_1 \sin\omega t & \left(0 < t < \frac{\pi}{\omega}\right) \\ 0 & \left(\frac{\pi}{\omega} < t < \frac{2\pi}{\omega}\right) \\ A_1 \sin\omega t & \left(\frac{2\pi}{\omega} < t < \frac{3\pi}{\omega}\right) \\ \dots\dots & \dots\dots\dots\dots \end{array}\right\} \supset \frac{A_1 \omega p}{(p^2+\omega^2)\left(1 - e^{-p\frac{\pi}{\omega}}\right)}$$

$$\left.\begin{array}{ll} \sin\omega t & \left(0 < t < \frac{2n\pi}{\omega}\right) \\ 0 & \left(\frac{2n\pi}{\omega} < t < \frac{4n\pi}{\omega}\right) \\ \sin\omega t & \left(\frac{4n\pi}{\omega} < t < \frac{6n\pi}{\omega}\right) \\ \dots\dots & \dots\dots\dots\dots \end{array}\right\} \supset \frac{\omega p}{(p^2+\omega^2)\left(1 + e^{-\frac{2n\pi p}{\omega}}\right)}$$

$$\left.\begin{array}{ll} A_1 & (0 < t < h) \\ 0 & (h < t < 2h) \\ \dots & \dots\dots\dots\dots \end{array}\right\} \supset \frac{A_1}{1+e^{-ph}}$$

$$\left.\begin{array}{ll} A_1 & (0 < t < h) \\ -A_1 & (h < t < 2h) \\ \dots\dots & \dots\dots\dots\dots \end{array}\right\} \supset \frac{A_1(1-e^{-ph})}{(1+e^{-ph})} = 2A_1 \operatorname{th}\frac{ph}{2}$$

$$\left.\begin{array}{ll} A_1 & (0 < t < h) \\ 2A_1 & (h < t < 2h) \\ \dots & \dots\dots\dots\dots \\ nA_1 & [(n-1)h < t < nh) \\ \dots & \dots\dots\dots\dots \end{array}\right\} \supset \frac{A_1}{2}\left(1 + \coth\frac{ph}{2}\right)$$

$$\left.\begin{array}{ll} t\sin\omega t & \left(0 < t < \frac{2n\pi}{\omega}\right) \\ 0 & \left(\frac{2n\pi}{\omega} < t < \frac{4n\pi}{\omega}\right) \\ & \dots\dots\dots\dots \end{array}\right\} \supset \frac{p^2}{(p^2+\omega^2)^2}\left\{\frac{2\omega}{1+e^{-\frac{2n\pi p}{\omega}}} - \frac{2n\pi(p^2+\omega^2)e^{-\frac{2n\pi p}{\omega}}}{p\left[1-e^{-\frac{4n\pi p}{\omega}}\right]}\right\}$$

$\frac{A_1 t}{2h}$ $(0 < t < 2h)$ valeur répétée dans chaque intervalle de $2h$ $\quad \supset A_1\left[\frac{1}{2ph} - \frac{e^{-2ph}}{1-e^{-2ph}}\right]$

$A_1(t-h)^3$ $(0 < t < 2h)$ valeur répétée dans chaque intervalle de $2h$ $\quad \supset A_1\left[\frac{6}{p^3} - \frac{\left(\frac{6h}{p^2} - \frac{3h^2}{p} + h^3\right)}{(1-e^{-2ph})}\right]$

$\sqrt{2at-t^2}$ $(0 < t < 2a)$ valeur répétée dans chaque intervalle de $2a$ $\quad \supset \pi a \frac{I_1(ap)}{2 \operatorname{sh} ap}$

$$\left.\begin{matrix} f(t) & (0 < t < h) \\ 0 & (h < t < \infty) \end{matrix}\right\} \supset \sum_{s=0}^{n=1} p^{-s} f^{(s)}(0) - e^{-ph}\sum_{s=0}^{n-1} p^{-s} f^{(s)}(h) + p^{-n+1}\int_0^h e^{-pt} f^{(n)}(t)\,dt$$
$$= \varphi_{0,h}(p) = \varphi_{0,\infty}(p) - \varphi_{h,\infty}(p) = p\int_0^h e^{-pt} f(t)\,dt$$

$$\left.\begin{matrix} f(t) & (h < t < \infty) \\ 0 & (0 < t < h) \end{matrix}\right\} \supset e^{-ph}\sum_{s=0}^{n-1} p^{-s} f^{(s)}(h) + p^{-n+1}\int_h^\infty e^{-pt} f^{(n)}(t)\,dt$$
$$= \varphi_{h,\infty}(p) = \varphi_{0,\infty}(p) - \varphi_{0,h}(p) = p\int_h^\infty e^{-pt} f(t)\,dt$$

$$\left.\begin{matrix} f(t) & (h_1 < t < h_2) \\ 0 & (0 < t < h_1) \\ 0 & (h_2 < t < \infty) \end{matrix}\right\} \supset \sum_{s=0}^{n-1} p^{-s}\left[e^{-ph_1} f^{(s)}(h_1) - e^{-ph_2} f^{(s)}(h_2)\right] + p^{-n+1}\int_{h_1}^{h_2} e^{-pt} f^{(n)}(t)\,dt$$
$$= \varphi_{h_1,h_2}(p) = p\int_{h_1}^{h_2} e^{-pt} f(t)\,dt.$$

$$\left.\begin{matrix} A_1 \quad (0 < t < h) \\ \text{avec } \int_0^h f(t)\,dt = B_1 \end{matrix}\right\} \supset B_1\left(p - \frac{p^2 h}{2!} + \frac{p^3 h^2}{3!} - \dots\right) \qquad \star$$

et, si $h \to 0$, $f(t) \supset B_1 p$ $\star$

Si $\varphi_{0,h}(p) = p\int_0^h e^{-pt} f(t)\,dt$, et si la valeur est répétée dans chaque intervalle de h à l'infini, donc $\varphi_{0,\infty}(p) = \frac{\varphi_{0,h}(p)}{(1-e^{-ph})}$. La série de Fourier, pour la fonction répétée, est fournie par l'intégrale $\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} e^{zt}\frac{\varphi_{0,h}(z)\,dz}{z(1-e^{-zh})}$, si elle est convergente.

D. — CALCUL SYMBOLIQUE A DEUX VARIABLES.

On définit la correspondance à deux variables

$$f(x, y) \supset\supset \varphi(p, q),$$

où x et y sont les variables indépendantes, p et q les variables paramétriques correspondantes, par l'intégrale [1]

$$\varphi(p, q) = pq \int_0^\infty \int_0^\infty e^{-px-qy} f(x, y)\, dx\, dy$$

Règles.

Si $f_1(x) \supset \varphi_1(p), f_2(x) \supset \varphi_2(p)$,

$$f(x, y) = f_1(x) f_2(y) \supset\supset \varphi(p, q) = \varphi_1(p)\varphi_2(q)$$

$$f(ax, by) \supset\supset \varphi\left(\frac{p}{a}, \frac{q}{b}\right)$$

$$\frac{\partial f(x, y)}{\partial x} \supset\supset p\,\varphi(p, q), \qquad \text{si } f(0, y) = 0$$

$$\frac{\partial f(x, y)}{\partial y} \supset\supset q\,\varphi(p, q), \qquad \text{si } f(x, 0) = 0$$

$$x\frac{\partial f(x, y)}{\partial x} \supset\supset -p\frac{\partial \varphi(p, q)}{\partial p}$$

$$y\frac{\partial f(x, y)}{\partial y} \supset\supset -q\frac{\partial \varphi(p, q)}{\partial q}$$

$$\int_0^x f(\lambda, y)\, d\lambda \supset\supset \frac{1}{p}\varphi(p, q)$$

$$\int_0^y f(x, \rho)\, d\rho \supset\supset \frac{1}{q}\varphi(p, q)$$

$$\int_0^x \int_0^y f(\lambda, \rho)\, d\lambda\, d\rho \supset\supset \frac{1}{pq}\varphi(p, q)$$

$$\int_x^\infty \frac{f(\lambda, y)}{\lambda}\, d\lambda \supset\supset \int_0^p \frac{\varphi(\theta, q)}{\theta}\, d\theta$$

$$\int_y^\infty \frac{f(x, \rho)}{\rho}\, d\rho \supset\supset \int_0^q \frac{\varphi(p, \sigma)}{\sigma}\, d\sigma$$

---

(1) Il est évident que q n'a plus ici la même signification que lorsqu'il s'agissait des fonctions de Bessel et de même pour y.

$$\int_x^\infty \int_y^\infty f(\lambda, \rho) \frac{d\lambda \, d\rho}{\lambda\rho} \supset\supset \int_0^p \int_0^q \varphi(\theta, \sigma) \frac{d\theta \, d\sigma}{\theta\sigma}$$

$$e^{-ax-by} f(x, y) \supset\supset \frac{pq}{(p+a)(q+b)} f[p+a, q+b]$$

$$\frac{1}{\pi\sqrt{xy}} \int_0^\infty \int_0^\infty e^{-\frac{\theta^2}{4x}-\frac{\rho^2}{4y}} f(\theta, \rho) \, d\theta \, d\rho \supset\supset \varphi(\sqrt{p}, \sqrt{q})$$

$$\int_0^\infty \int_0^\infty J_0(2\sqrt{\theta x}) \, J_0(2\sqrt{\rho y}) \, f(\theta, \rho) \, d\theta \, d\rho \supset\supset pq \, \varphi\left(\frac{1}{p}, \frac{1}{q}\right)$$

Si $f_1(x, y) \supset\supset \varphi_1(p, q)$ et $f_2(x, y) \supset\supset \varphi_2(p, q)$, on a

$$\int_0^x \int_0^y f_1(x-\lambda, y-\rho) f_2(\lambda, \rho) \, d\lambda \, d\rho = \int_0^x \int_0^y f_1(\lambda, \rho) f_2(x-\lambda, y-\rho) \, d\lambda \, d\rho \supset\supset \frac{1}{pq} \varphi_1(p, q) \varphi_2(p, q)$$

Si $f(x) \supset \varphi(p)$, on a

$$f(x+y) \supset\supset \frac{q\,\varphi(p) - p\,\varphi(q)}{q-p}$$

$$\frac{1}{\pi\sqrt{xy}} \int_0^\infty e^{-\frac{\theta^2}{4}\left(\frac{1}{x}+\frac{1}{y}\right)} f(\theta) \, d\theta \supset\supset \frac{\sqrt{p}}{\sqrt{p}+\sqrt{q}} \varphi(\sqrt{p}+\sqrt{q})$$

$$\int_0^\infty J_0(2\sqrt{\theta x}) \, J_0(2\sqrt{\theta y}) \, f(\theta) \, d\theta \supset\supset \frac{pq}{p+q} \varphi\left(\frac{1}{p}+\frac{1}{q}\right)$$

Si $f(x) = \sum_{m=0}^{\infty} a_m x^m$, on a

$$f(xy) \supset\supset \sum a_m \frac{m!\, m!}{(pq)^m}$$

Correspondances.

$$\frac{x^m y^n}{m!\, n!} \supset\supset p^{-m} q^{-n} \qquad (m, n \geqq 0)$$

$$(x+y)^m \supset\supset \frac{m!}{(pq)^m} \, \frac{q^{m+1} - p^{m+1}}{q-p}$$

$$\frac{1}{\sqrt{x+y}} \supset\supset \frac{\sqrt{\pi pq}}{\sqrt{p}+\sqrt{q}}$$

$$\frac{\sqrt{xy}}{x+y} \supset\supset \frac{\pi\sqrt{pq}}{2(\sqrt{p}+\sqrt{q})^2}$$

$$e^{xy} \supset\supset pq \, e^{-pq} \, \mathrm{E}i(pq)$$

$$e^{\frac{xy}{x+y}} \frac{\sqrt{xy}}{x+y} \supset\supset \frac{\pi}{2}\left(1+p+q+2\sqrt{pq}\right)^{-\frac{3}{2}}$$

$$\log xy \supset\supset -2\gamma - \log pq$$

$$\log(x+y) \supset\supset \frac{p\log q - q\log p}{q-p} - \gamma$$

$$\sin(x+y) \supset\supset \frac{pq(p+q)}{(p^2+1)(q^2+1)}$$

$$\cos(x+y) \supset\supset \frac{pq(pq-1)}{(p^2+1)(q^2+1)}$$

$$\left.\begin{array}{l}
\dfrac{e^{x+y}+e^{jx+j^2y}+e^{j^2x+jy}}{3} \supset\supset pq\,\dfrac{p^2q^2+pq+1}{(p^3-1)(q^3-1)} \\[2ex]
\dfrac{e^{x+y}+j\,e^{jx+j^2y}+j^2\,e^{j^2x+jy}}{3} \supset\supset pq\,\dfrac{p^2q+q^2+p}{(p^3-1)(q^3-1)} \\[2ex]
\dfrac{e^{x+y}+j^2\,e^{jx+j^2y}+j\,e^{j^2x+jy}}{3} \supset\supset pq\,\dfrac{q^2p+p^2+q}{(p^3-1)(q^3-1)}
\end{array}\right\} \begin{array}{c}\text{(cosinus d'Appell)}\\ (j^3=1)\end{array}$$

$$\mathrm{E}i(xy) \supset\supset -\gamma - \log p - \log q + e^{-pq}\,\mathrm{E}i(pq)$$

$$\mathrm{J}_0(2\sqrt{x+y}) \supset\supset \frac{q\,e^{-\frac{1}{p}} - p\,e^{-\frac{1}{q}}}{q-p}$$

$$\mathrm{J}_0(2i\sqrt{xy}) \supset\supset \frac{pq}{pq-1}$$

$$e^{x+y}\,\mathrm{J}_0(2i\sqrt{xy}) \supset\supset \left(1-\frac{1}{p}-\frac{1}{q}\right)^{-1}$$

$$ber(2\sqrt{xy}) \supset\supset \frac{p^2q^2}{p^2q^2+1}$$

$$bei(2\sqrt{xy}) \supset\supset \frac{pq}{p^2q^2+1}$$

$$\mathrm{J}_{0,0}(-3\sqrt[3]{xy}) \supset\supset e^{\frac{1}{pq}}$$

$$2\,\mathrm{J}i_0(2\sqrt{xy}) \supset\supset \log(pq+1)$$

$$\frac{1}{\sqrt{\pi xy}}\,\mathrm{J}_0\left[\frac{i}{2}(xy)^{-\frac{1}{2}}\right] \supset\supset \sqrt{pq}\,\mathrm{J}_0\left[2i(pq)^{\frac{1}{4}}\right]$$

$${}_r\mathrm{F}_s(\alpha_1,\ldots,\alpha_r;\ \gamma_1,\ldots,\gamma_s;\ xy) \supset\supset {}_{r+2}\mathrm{F}_s\left(\alpha_1,\ldots,\alpha_r,1,1;\ \gamma_1,\ldots,\gamma_s;\ \frac{1}{pq}\right)$$

E. — RÉFÉRENCES.

Cette liste, très limitative, ne contient que les Mémoires d'où ont été tirées les diverses formules classées ci-dessus. On trouvera dans l'ouvrage (27) une bibliographie très étendue sur le calcul symbolique, contenant plus de 200 références.

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TABLE DES MATIÈRES.