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# A TRANSFERABLE VOTING SYSTEM INCLUDING INTENSITY OF PREFERENCE

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## INTRODUCTION

This paper describes a voting procedure with a number of interesting properties. Chief among these are the inclusion of intensity of preference in a non-controversial manner – i.e. in a way which avoids the difficulty of inter-personal comparison of utilities – and that in various limiting cases the procedure is equivalent to well-known voting systems such as simple majority, the single transferable vote, the single non-transferable vote, etc. The paper first describes the voting procedure, then looks at the properties mentioned, and finally shows that the procedure offers a partial solution to the problem of determining which voting procedure to use in some decision situation.

## I. THE PROCEDURE

Any voting procedure consists of two parts – that of vote casting, and that of vote counting. In this case the vote casting procedure for the elector is to assign weights to the different candidates to indicate the order and strength of his preferences between them. It is a basic assumption that strength of preference is transitive, e.g. that if a voter thinks that he prefers A twice as much as B, and B three times as much as C, then he prefers A six times as much as C and can express his preferences by

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assigning weights to A, B, and C in the ratio 6:3:1.

The vote counting procedure begins by normalising all the weights $w_{ij}$ which the $i$th voter gives to the $j$th candidate, so that

$$\sum_{j=1}^{j=c} w_{ij} = 1, \quad \text{all } i$$

$c$ being the number of candidates. This is the key, as we shall see later, to the avoidance of troubles due to inter-personal comparison of utilities, since it ensures that as far as possible each voter has an equal say in the voting procedure.

The count proceeds by summing all the weights for all the candidates, i.e. calculating

$$W_j = \sum_{i=1}^{i=v} w_{ij} \qquad \text{for all } j,$$

$v$ being the number of voters. Thereafter the count proceeds much in the same way as in the single transferable vote, as modified by the proposals in two earlier papers [1,2]. An STV-type quota is calculated according to the formula

$$\left[ q = \frac{W}{s+1} + 1 \right]$$

where $s$ is the number of seats to be filled and $W = \sum_{j=1}^{j=c} W_j$ is the total vote, and the brackets indicate that the integral part is to be taken. $q$ is the minimum number such that, if $s$ candidates have that number, any other candidate must have less than that number.

(In practice it is likely that working will be to fractions of votes - say three decimal places, in which case the "+1" in the formula for $q$ is replaced by "+0.001", or equivalently the weights $w_{ij}$ are normalised to sum 1000 for each voter and the formula for $q$ is unaltered.)

The count may proceed by one of two steps. If no $W_j$ exceeds $q$, i.e. no candidate has reached the quota, then the candidate with lowest $W_j$, say candidate $x$, is eliminated. All the $w_{ij}$ are then renormalised with all $w_{ix}$

made equal to zero. The principle adopted is that if a candidate is eliminated the count proceeds as if that candidate had never stood; the assumption is that the elimination of a candidate does not alter the voter's relative preferences between the remaining candidates. (It is of course quite possible to take issue with this assumption.)

If, however, a candidate, say $y$, has $W_y$ greater than $q$, another renormalisation takes place so that $W_y$ is reduced to $q$. This means that all $w_{iy}$ are reduced by the factor $q/W_y$, and all $w_{ij}$, $j \neq y$, are increased by the factor $(1 + w_{iy}q/W_y)/(1 - w_{iy})$. By this means the weights allocated by each voter $i$ are adjusted in a quite natural way, so that those supporting $y$ give him no more support than is necessary to ensure his election.

Counting continues by the application of one or other of these rules until the requisite number $s$ of candidates are elected. Once elected and allocated the quota $q$ the weights for that candidate are of course not included in the recalculation. This makes the procedure somewhat simpler than in the modified form of STV described in [1]. However, if all of a voter's choices - i.e. those candidates he has allotted a positive weight - are eliminated, the quota $q$ has to be recalculated as in [2] so that this undistributable vote is not included; similarly, when all a voter's choices have been elected and allotted recalculated weights, the residue is non-distributable and also must be subtracted from $W$. Recalculation of the quota does of course imply recalculation of the weights of elected candidates, and an iterative procedure as described in [2] can be used to obtain the new $q$ and $w_{iy}$ to any desired accuracy.

## II. INTENSITY OF PREFERENCE

When expressed crudely in the form "It is of more benefit to me to have A rather than B than it is for you to have B rather than A", inter-personal comparison of utilities are patently invidious. Nevertheless in actual voting situations intensity of preference is often taken into account. If A and B want to go to a museum when C wants to go to the funfair, the collective choice is frequently the funfair, without any sense of dictatorship or lack of democracy, simply because all know that C's preference is much the most intense. Lest this be regarded as too trivial an example, it is often the case in committee that the collective choice for chairman is X, even though a majority prefer Y, simply because a substantial minority strongly object to Y. Any theory of voting which does not allow for intensity of preference is certainly incomplete, and any voting system which does not permit its expression cannot be wholly satisfactory.

The present system is based on two principles: that the only person who can gauge the intensity of his preferences is the voter himself; and that as far as possible each voter should contribute equally in the choice of those elected. In a multi-vacancy election ($s > 1$) there is more than just a single choice involved, and so it makes sense to allow a voter to express his preference intensities by contributing all his voting power to the choice of one candidate, or to share this power between the choices of different candidates. Of course, it is possible to regard an $s$-vacancy election as a single choice from the ${}^{n}C_{s}$ possible combinations of $s$ candidates out of $n$ elected, but this view invalidates the assumption that elimination of a candidate does not alter the voter's relative preferences. This is because each combination is independent; a voter may rank candidates individually A, B, C, D in that order, but rate them in pairs AB, BD, CD, BC, ......... since he thinks A will only work satisfactorily in combination with B. This kind of multiple election is essentially the election of a team of $s$ people, rather than $s$ individuals. STV, and the present system, is concerned with choosing a set of $s$ independent individuals from a larger set of $c$ candidates. An STV vote is a vote for one individual (the first choice) and only subsidiarily and in special circumstances for lower choices. The present system enables the voter to have a say in all the $s$ choices if he wishes, but his share in the whole decision process remains the same, up to the wastage involved in non-transferable votes or those given to unelected candidates who remain when the $s$ winners have been chosen.

## III. EQUIVALENCE TO OTHER VOTING SYSTEMS

### 3.1. STV

Let $1 > \varepsilon > 0$. Let the voters order their choices $1 - \varepsilon$, $\varepsilon - \varepsilon^2$, $\varepsilon^2 - \varepsilon^3$, ......... $\varepsilon^{c-2} - \varepsilon^{c-1}$, $\varepsilon^{c-1}$. Then the closer $\varepsilon$ is to 0 the closer the actual voting process becomes equivalent to STV. For example, suppose there are 5 candidates and $\varepsilon = 0.01$. A voter's choice will be in the proportions 0.99, 0.0099, 0.000099, 0.00000099, 0.00000001, counting 99% for his first choice. If his first choice is eliminated, the four lower votes remain, and total 0.01. These have to be renormalised to add up to 1, and so are multiplied by 100 to give 0.99, 0.0099, 0.000099, 0.000001. A similar argument applies to votes transferred from elected candidates.

### 3.2. Single non-transferable vote

This, trivially, is when the voter gives 1 to his first choice and 0 to all the others.

### 3.3. Simple majority with multiple vote

Here the voter gives $1/s$ to each of $s$ candidates, or perhaps $1/k$ to each of $k$ candidates, $k < s$. These are special cases of giving $\alpha$ to $k$ candidates and $\beta$ to $c - k$ candidates, where $\alpha k + \beta(c - k) = 1$ giving a weighting between a more preferred and a less preferred group.

3.4. Cumulative vote

In this case the voter gives $\alpha_1$, $\alpha_2$, ....... $\alpha_k$ to k candidates respectively, such that $\sum_{i=1}^{k} \alpha_i = 1$. For an exact analogy to the cumulative vote each $\alpha_i$ must be a multiple of 1/s.

## IV. THE CHOICE OF VOTING PROCEDURE

Such a voting system would require a more than usual sophistication on the part of the voter. This being so, one can consider a further sophistication. The choice of voting procedure is one of immense importance in the democratic process, and no system is wholly stable wherein a substantial minority is dissatisfied with the voting procedure in current use. The required consensus may either be achieved through ignorance or habit, or by general agreement that a system is fair even though another may be advantageous to many, perhaps even a majority. In situations where there is awareness of and controversy about the different properties of voting systems, the present system offers a possible way out of deadlock. For, if most voters use the system in one of the ways described in the last section, then the election will be largely determined according to that voting procedure. Looking at it from the point of view of parties, each party can urge the voters to use the method they favour of filling in the ballot forms. However, it is a weakness in this area that voting systems are so often argued about in terms of fairness to parties or candidates, seldom in terms of fairness to voters. The present system, whose main fault is its complexity, has the virtue of that fault in that each voter can specify as precisely as he wishes the way his vote is to be counted, without this being imposed by others on him or on others by him.

Most voting systems allow some such flexibility; the virtue of this system is the much greater precision with which the sophisticated voter can specify his wishes, without his being able by strategic voting to exercise more influence on the final result than is implied by his actual possession of a vote.

## V. CONCLUDING REMARKS

Despite the scope for manipulation which the system offers, it is clearly derived from and shares the principles of the STV system, particularly with the concept of the quota and the transferability of votes above the quota. One of the chief objections to STV is that it does not guarantee the election of a Condorcet winner, e.g. when one candidate is everyone's second choice. While the present system does not guarantee the election of such a candidate (this is obvious, since as shown earlier the system can approach arbitrarily closely to STV), it does render it more likely, and will ensure it provided that the weights given to the candidate are large enough, i.e. if the candidate is considered a good enough substitute for their first choice by a sufficient number of electors. The price that one has to pay for this improvement to STV is the greater complexity, particularly for the voter.

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