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Erratum

A KINETIC EQUATION FOR GRANULAR MEDIA

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Abstract. In this short note we correct a conceptual error in the heuristic derivation of a kinetic equation used for the description of a one-dimensional granular medium in the so called quasi-elastic limit, presented by the same authors in reference [1]. The equation we derived is however correct so that, the rigorous analysis on this equation, which constituted the main purpose of that paper, remains unchanged.

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1. THE KINETIC EQUATION

We refer to [1] and use the same notation.

Let us start from the basic master (Liouville) equation describing the evolution of a probability density μ^N associated to the dynamics of N inelastic point particles in the line. In equation (1.4) of [1] appears the factor $\delta(x_i - x_j)$ because of the strictly local interaction. In order to avoid mistakes in the correct interpretation of this δ , we mollify the interaction replacing equation (1.3) by its regularized version:

$$\dot{x}_i = v_i, \quad \dot{v}_i = \alpha \sum_{j=1}^N \delta_\eta(x_i - x_j)(v_j - v_i)|v_j - v_i|, \quad (1)$$

where δ_η is an approximation of δ as the parameter $\eta \rightarrow 0$. A simple calculation on the two particle scattering problem shows that $\alpha = \alpha(\varepsilon) = -\frac{1}{2} \log(1 - \varepsilon)$ so that $\alpha \approx \varepsilon$ only in the limit $\varepsilon \rightarrow 0$. As a consequence of Eq. (1) we have the following master equation for the regularized problem:

$$\left(\partial_t + \sum_{i=1}^N v_i \partial_{x_i}\right) \mu^N(x_1, v_1, \dots, x_N, v_N) = -\alpha \sum_{i \neq j} \delta_\eta(x_i - x_j) \partial_{v_i} [\phi(v_j - v_i) \mu(x_1, v_1, \dots, x_N, v_N)] \quad (2)$$

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and the following hierarchy for the the j -particle distribution functions:

$$\begin{aligned} (\partial_t + \sum_{i=1}^j v_i \partial_{x_i}) f_j^N(x_1, v_1, \dots, x_j, v_j) = & -\alpha \sum_{i,k=1, i \neq k}^j \delta_\eta(x_i - x_k) \phi(v_k - v_i) \partial_{v_i} f_j^N(x_1, v_1, \dots, x_j, v_j) \\ & - \alpha(N-j) \sum_{i=1}^j \partial_{v_i} \int dv_{j+1} \int dx_{j+1} \delta_\eta(x_i - x_{j+1}) \phi(v_{j+1} - v_i) f_{j+1}^N(x_1, v_1, \dots, x_{j+1}, v_{j+1}) \end{aligned} \quad (3)$$

Since the regularized dynamics converges to the true dynamics pathwise, we should perform the limit $\eta \rightarrow 0$ for fixed N and ε . Here arises our error in [1] which is twofold. From one side $\alpha \neq \varepsilon$. From the other the limit $\eta \rightarrow 0$ is not innocent and it does not give equation (1.6) as asserted in [1]: we did confusion between the notion of δ as a distribution in time and as a limit of regularized versions δ_η .

We note also that, interchanging the limit $\eta \rightarrow 0$ with the quasi-elastic limit $N \rightarrow \infty$, $\varepsilon \rightarrow 0$, $N\varepsilon \rightarrow \lambda$, we would obtain equation (1.7) of [1] which is actually correct.

In order to derive the evolution equation for the j -particle distribution, instead of studying the limit $\eta \rightarrow 0$ (which is possible but involved) it is more natural to look directly at the true dynamics, considering the interaction as a boundary term, in the same spirit of the derivation of the BBKGY hierarchy for hard spheres (see *e.g.* Ref. [2]). We shortly outline the argument.

Let $\mu_0^N = \mu_0^N(x_1, v_1, \dots, x_N, v_N)$ be a probability density for the system at time 0. We assume μ_0^N continuous and symmetric in the exchange of particles. The time evolved probability density is defined as:

$$\mu^N(x_1, v_1, \dots, x_N, v_N, t) = \mu_0^N(T^{-t}(x_1, v_1, \dots, x_N, v_N)) J_\varepsilon^{-n}, \quad (4)$$

where T^t is the flow in the phase space generated by the dynamics, n is the number of collisions delivered by the phase point $(x_1, v_1, \dots, x_N, v_N)$ during the backward dynamics up to the time t and $J_\varepsilon = (1 - 2\varepsilon)^2$ denotes the Jacobian of the transformation induced in the phase space by a single collision.

Note that T^t is ambiguously defined on the manifold $\{x_i = x_j \mid \text{for some } i \neq j\}$. Indeed in this case we do not know whether the velocities v_i and v_j have to be understood as incoming or outgoing. However such a manifold has zero measure so that this ambiguity is irrelevant in the definition of $\mu^N(\cdot, t)$.

Note also that if t is a collision instant involving the i and j particle, for the phase point $(\bar{X}, \bar{V}) = (\bar{x}_1, \bar{v}_1, \dots, \bar{x}_N, \bar{v}_N)$ in the forward dynamics, then

$$\lim_{\tau \rightarrow t^+} \mu^N(T^\tau(\bar{X}, \bar{V}), \tau) = J_\varepsilon^{-1} \lim_{\tau \rightarrow t^-} \mu^N(T^\tau(\bar{X}, \bar{V}), \tau),$$

provided that μ_0^N is continuous. This expression can be rewritten in terms of

$$(x_1, v_1, \dots, x_i, v_i, \dots, x_i, v_j, \dots, x_N, v_N) = \lim_{\tau \rightarrow t^-} T^\tau(\bar{X}, \bar{V})$$

and

$$(x_1, v_1, \dots, x_i, v'_i, \dots, x_i, v'_j, \dots, x_N, v_N) = \lim_{\tau \rightarrow t^+} T^\tau(\bar{X}, \bar{V}),$$

(where v'_i, v'_j are the outgoing velocities and v_i, v_j are the incoming ones) as

$$\lim_{\tau \rightarrow t^+} \mu^N(x_1, v_1, \dots, x_i, v'_i, \dots, x_i, v'_j, \dots, x_N, v_N, \tau) = J_\varepsilon^{-1} \lim_{\tau \rightarrow t^-} \mu^N(x_1, v_1, \dots, x_i, v_i, \dots, x_i, v_j, \dots, x_N, v_N, \tau).$$

Now we want to derive an equation for the j -particle distribution functions.

We first note that if $(x_1, v_1, \dots, x_N, v_N)$ is not in the collision manifold, then:

$$\partial_t \mu^N(x_1, v_1, \dots, x_N, v_N, t) + \sum_{k=1}^N v_k \partial_{x_k} \mu^N(x_1, v_1, \dots, x_N, v_N, t) = 0. \tag{5}$$

Our next step is to integrate over the last $N - j$ variables and this generates boundary terms which give rise to the collision operator. To compute these terms explicitly we consider the simple case in which $N = 2$ and $j = 1$. An easy calculation shows that:

$$\partial_t f_1(x_1, v_1, t) + v_1 \partial_{x_1} f_1(x_1, v_1, t) = \int dv_2 (v_2 - v_1) \{ \mu^2(x_1, v_1, x_1^-, v_2, t) - \mu^2(x_1, v_1, x_1^+, v_2, t) \}, \tag{6}$$

where $\mu^2(x_1, v_1, x_1^\pm, v_2, t)$ denotes the right and left limit for $x \rightarrow x_1$ respectively. We note that for the configuration point $(x_1 - \delta, x_1)$ for a positive small δ , the velocities v_2, v_1 are incoming or outgoing if $v_1 > v_2$ or $v_1 < v_2$ respectively. Taking into account this fact we readily arrive to the following equation:

$$\partial_t f_1(x_1, v_1, t) + v_1 \partial_{x_1} f_1(x_1, v_1, t) = \int dv_2 |v_2 - v_1| \{ J_\epsilon^{-1} \mu^2(x_1, v_1^*, x_1, v_2^*, t) - \mu^2(x_1, v_1, x_1, v_2, t) \}, \tag{7}$$

where v_1^* and v_2^* denote the precollisional pair

$$v_1^* = v_1 + \frac{\epsilon}{1 - 2\epsilon} (v_1 - v_2) \quad v_2^* = v_2 - \frac{\epsilon}{1 - 2\epsilon} (v_1 - v_2).$$

Note that, as in the case of the Boltzmann equation, we represent μ^2 in terms of the precollisional variables, so that the time t appearing in the right hand side of equation (7) is the left limit.

For the general case we easily deduce the following hierarchy of equations:

$$(\partial_t + \mathcal{L}_j) f_j^N(x_1, v_1, \dots, x_j, v_j) = (N - j) \sum_{k=1}^j \int dv_{j+1} |v_k - v_{j+1}| \cdot \tag{8}$$

$$\{ J_\epsilon^{-1} f_{j+1}^N(x_1, v_1, \dots, x_k, v_k^*, \dots, x_k, v_{j+1}^*) - f_{j+1}^N(x_1, v_1, \dots, x_k, v_k, \dots, x_k, v_{j+1}) \},$$

for $i = 1, n$. Here \mathcal{L}_j denotes the generator of the j -particle dynamics. equations (8) are the analogue of the BBGKY hierarchy for Hamiltonian systems.

The integral in the right end side of (8) is $O(\epsilon)$, so that we are lead to consider the scaling limit $\epsilon \rightarrow 0$, $N \rightarrow \infty$ in such a way that $N\epsilon \rightarrow \lambda$, where λ is a positive parameter. Using the Taylor formula and neglecting terms of $o(\epsilon)$, integrating by parts and performing the limit we arrive to the hierarchy of equations (1.7) of reference [1].

Finally, propagation of chaos implies, as usual, the kinetic equation (1.8) which is the object of investigation in [1].

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