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FICTITIOUS DOMAIN/MIXED FINITE ELEMENT APPROACH FOR A CLASS OF OPTIMAL SHAPE DESIGN PROBLEMS (*)

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Abstract — *A fictitious domain method is applied to shape optimization problems and we use a dualization of the Dirichlet boundary condition on the variable part of the boundary An approximation by means of mixed finite elements is presented and convergence results are established*

Key words shape optimization, fictitious domain, mixed finite elements

Résumé — *On présente une méthode de domaines fictifs pour la résolution numérique de problèmes d'optimisation de forme Cette approche est basée sur la dualization de la condition de Dirichlet sur la partie de frontière On étudie l'approximation par la méthode des éléments finis mixtes et on analyse la convergence de la méthode*

1. INTRODUCTION

Fictitious domain methods or domain imbedding methods have recently become a subject of increasing interest, see for instance [1] The obvious reason is that they allow to obtain numerical solutions to problems of complicated geometry by operating on a simple geometry domain containing the complicated one The use of fictitious domain methods in shape optimization is particularly attractive because of the special features of the problem. In the classical approach to shape optimization, i.e. when a boundary variation technique is used, one has to create a new triangulation, update all data (stiffness matrix, load vector, etc) and solve the resulting algebraic system to get the solution of the state problem for each iterative design. As a result the

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whole computational process may be not to effective. We refer to the text books [2, 3, 4] for a description of this classical approach to shape optimization. In [5] (see also [9]) a new approach based on a combination of the fictitious domain approach and optimal control was proposed in the case when the state is given by a homogeneous Dirichlet boundary value problem. The original optimal shape design problem was formally rewritten as a new one which uses as state problem again a homogeneous Dirichlet problem but posed on a a fixed domain and with the control entering only in the right hand side. The advantage of this method is obvious : the triangulation and consequently the stiffness matrix is constant during iterations and can, thus, be factorized once and for all. In order to treat the state constraint that arises as a means of getting equivalence between the two formulations, the penalty method was used in [5]. Here we describe an alternative approach. The main difference being that instead of the distributed controls in [5] we use controls concentrated on curves only. These controls can be interpreted as Lagrangian multipliers by means of which we satisfy the state constraint. The same approach for solution of the state problem (not the full shape optimization problem) was recently proposed in [6]. (We did not know of this work when obtaining the results of the present paper.) The method presented in this paper has two advantages as compared to the method described in [5] :

(i) for some cost functionals we avoid the evaluation of right hand sides over non-standard elements, created by cutting classical elements (triangles, e.g.) by segments ;

(ii) when the penalty method is used, the problem of finding a good strategy between the penalty parameter ε and the mesh size $h > 0$ is a delicate task in general. Here the boundary conditions are satisfied in a weak finite dimensional sense, namely by means of a mixed finite element method. In this case, the corresponding state constraint (satisfied in this weak sense), can be penalized without any problem. Consequently, we may let $\varepsilon \to 0+$ even when the mesh size $h > 0$ is fixed, which is impossible when we discretize the penalized problem.

The paper is organized as follows : in Section 2 we give a classical formulation, see [4], of the optimal shape design problem. In Section 3 the fictitious domain formulation is given and shown to be equivalent to the classical formulation. In Section 3 we give a mixed finite element approximation of the fictitious domain formulation and a convergence result is given in Theorem 4.2 which may be considered a main result of the paper. In the remaining Sections 5, 6 and 7 we, consecutively, give a matrix formulation, suggest a solution method based on an augmented Lagrangian concept and, finally, give some sensitivity formulas.

2. SETTING OF THE PROBLEM

Let $\Omega(\alpha)$ be a bounded domain in $\mathbf{R}^2$, the shape of which is given by

$$\Omega(\alpha) = \{(x_1, x_2) \in \mathbf{R}^2 | x_1 \in (0, \alpha(x_2)), x_2 \in (0, 1)\},$$

where α is a non-negative function, describing the variable part $\Gamma(\alpha)$ of $\partial\Omega(\alpha)$, with

$$\Gamma(\alpha) = \{(x_1, x_2) \in \mathbf{R}^2 | x_1 = \alpha(x_2), x_2 \in (0, 1)\}$$

(*see fig. 1*).

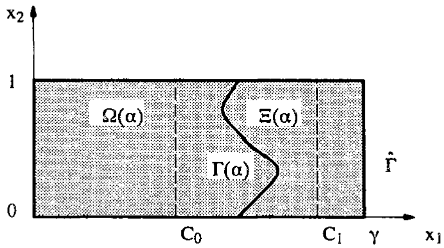

Figure 1. — The domain $\Omega(\alpha)$ of the original problem and the fictitious domain $\Xi(\alpha)$.

Next we shall suppose that the function α belongs to the set U_{ad}, where

$$U_{ad} = \{\alpha \in C^{0,1}([0, 1]) | 0 < C_0 \leqslant \alpha(x_2) \leqslant C_1, \quad \forall x_2 \in [0, 1],$$
$$|\alpha'(x_2)| \leqslant C_2 \quad \text{a.e. in } (0, 1), \text{meas } \Omega(\alpha) = C_3\}.$$

That is, U_{ad} contains all lipschitz continuous functions, which are uniformly bounded, uniformly lipschitz continuous and preserve the area of $\Omega(\alpha)$. The constants C_0, C_1, C_2 and C_3 are chosen in such a way that $U_{ad} \neq \Phi$.

On each $\Omega(\alpha)$, $\alpha \in U_{ad}$, we shall consider the homogeneous Dirichlet boundary value problem

$$(\mathscr{P}(\alpha)) \quad \begin{cases} -\Delta u(\alpha) = f & \text{in} \quad \Omega(\alpha) \\ u(\alpha) = 0 & \text{on} \quad \partial\Omega(\alpha) \end{cases}$$

or in weak form

$$(\mathscr{P}(\alpha)) \quad \begin{cases} \text{find } u(\alpha) \in V(\alpha) \text{ such that} \\ (\nabla u(\alpha), \nabla\varphi)_{0,\Omega(\alpha)} = (f, \varphi)_{0,\Omega(\alpha)} \quad \forall \varphi \in V(\alpha), \end{cases}$$

where $V(\alpha) = H_0^1(\Omega(\alpha))$ and the symbol $(\,.\,,\,.\,)_{0,\Omega(\alpha)}$ stands for the $L^2(\Omega(\alpha))$ scalar product. Let $\hat{\Omega}$ be a domain containing $\Omega(\alpha)$ for all $\alpha \in U_{ad}$ and such that the shape of $\hat{\Omega}$ is simple ($\hat{\Omega} = (0, 2\,C_1) \times (0, 1)$, e.g.) and $f \in L^2(\hat{\Omega})$.

Finally, let $I:(\alpha, y) \to \mathbf{R}^1$, $\alpha \in U_{ad}$, $y \in V(\alpha)$, be a *cost functional* and define the following optimal shape design problem

$$(\mathbf{P}) \quad \begin{cases} \text{find } \alpha^* \in U_{ad} \text{ such that} \\ I(\alpha^*, u(\alpha^*)) \leqslant I(\alpha, u(\alpha)) \quad \forall \alpha \in U_{ad} \end{cases}$$

with $u(\alpha) \in V(\alpha)$ being the solution of $(\mathscr{P}(\alpha))$.

In order to guarantee the existence of at least one solution of (**P**), the following lower continuity of I will hold

$$(\mathbf{A}) \quad \left.\begin{array}{c} \alpha_n \rightrightarrows \alpha \text{ (uniformly) in } [0, 1],\ \alpha_n, \alpha \in U_{ad} \\ \hat{y}_n \to \hat{y} \quad \text{in } H_0^1(\hat{\Omega}) \end{array}\right\} \Rightarrow$$

$$\Rightarrow \liminf I(\alpha_n, \hat{y}_n|_{\Omega_n}) \geqslant I(\alpha, \hat{y}|_{\Omega(\alpha)}),$$

where $\Omega_n = \Omega(\alpha_n)$.

Then it holds.

THEOREM 2.1 : *Let* (**A**) *be satisfied. Then* (**P**) *has at least one solution.*

For the proof we refer to [4].

3. FICTITIOUS DOMAIN APPROACH

We start by giving some notations. Let $\hat{\Omega} = (0, \gamma) \times (0, 1)$, $\gamma > C_1$ be a fixed domain. Obviously, $\hat{\Omega} \supset \overline{\Omega(\alpha)}\ \forall \alpha \in U_{ad}$. Denote by $\Xi(\alpha) = \hat{\Omega} \backslash \overline{\Omega(\alpha)}$ and $\hat{\Gamma} = \{(x_1, x_2) \in \mathbf{R}^2 | x_1 = \gamma, x_2 \in (0, 1)\}$ (*see fig. 1*). It holds that $\overline{\Gamma(\alpha)} = \partial\Omega(\alpha) \cap \overline{\Xi(\alpha)}$. A superposed bar denotes the closure of a set.

Further, let

$$\hat{V} = H_0^1(\hat{\Omega}),$$

$$\hat{V}_1 = \{v \in H^1(\hat{\Omega}) | \hat{v} = 0 \text{ on } \partial\hat{\Omega} \backslash \overline{\hat{\Gamma}}\},$$

$$V_1(\alpha) = \{v \in H^1(\Omega(\alpha)) | v = 0 \text{ on } \partial\Omega(\alpha) \backslash \Gamma(\alpha)\}.$$

On $\Gamma(\alpha)$ and $\hat{\Gamma}$ we introduce the following spaces of traces :

$$H^{1/2}(\Gamma(\alpha)) = \{w: \Gamma(\alpha) \to \mathbf{R}^1 | \exists v \in V_1(\alpha): v = w \text{ on } \Gamma(\alpha)\}$$

$$H^{1/2}(\hat{\Gamma}) = \{\hat{w}: \hat{\Gamma} \to \mathbf{R}^1 | \exists \hat{v} \in V_1 : \hat{v} = \hat{w} \text{ on } \hat{\Gamma}\} .$$

These spaces will be equipped with norms $\| w \|_{1/2, \Gamma(\alpha)}$ and $\| \hat{w} \|_{1/2}$ defined as follows :

$$\| w \|_{1/2, \Gamma(\alpha)} = \inf_{\substack{v \in V_1(\alpha) \\ v = w \text{ on } \Gamma(\alpha)}} |v|_{1, \Omega(\alpha)}$$

and

$$\| \hat{w} \|_{1/2,} = \inf_{\substack{\hat{v} \in \hat{V}_1 \\ \hat{v} = \hat{w} \text{ on } \hat{\Gamma}}} |\hat{v}|_{1, \hat{\Omega}} ,$$

where $|\cdot|_{1, \Omega(\alpha)}$ and $|\cdot|_{1, \hat{\Omega}}$ denote seminorms in corresponding spaces, which are in fact norms because of Friedrich's inequality.

Let $v \in V_1(\alpha)$ and define the function $\hat{v}_\alpha : \hat{\Gamma} \to \mathbf{R}^1$ by the relation

$$\hat{v}_\alpha(\gamma, \hat{x}_2) = v(\alpha(\hat{x}_2), \hat{x}_2) , \quad \hat{x}_2 \in (0, 1) . \tag{3.1}$$

Then it holds.

LEMMA 3.1 : *There exist constants* $c_1, c_2 > 0$ *which do not depend on* $\alpha \in U_{ad}$ *and* $v \in V_1(\alpha)$ *such that*

$$c_1 \| v \|_{1/2, \Gamma(\alpha)} \leq \| \hat{v}_\alpha \|_{1/2} \leq c_2 \| v \|_{1/2, \Gamma(\alpha)} , \tag{3.2}$$

where the relation between $v \in V_1(\alpha)$ *and* $\hat{v}$ *is given by* (3.1).

Proof : Let $F : \Omega(\alpha) \to \hat{\Omega}$ be the mapping, defined by

$$F(x_1, x_2) = (\hat{x}_1, \hat{x}_2), \quad \hat{x}_1 = \frac{\gamma x_1}{\alpha(x_2)}, \quad \hat{x}_2 = x_2 , \quad (x_1, x_2) \in \Omega(\alpha) .$$

With any $v \in V_1(\alpha)$ we associate the function $\Pi v \in \hat{V}_1$, where

$$\Pi v(\hat{x}_1, \hat{x}_2) = v\left(\alpha(\hat{x}_2) \frac{\hat{x}_1}{\gamma}, \hat{x}_2\right) , \quad (\hat{x}_1, \hat{x}_2) \in \hat{\Omega} .$$

Then $\Pi v|_{\hat{\Gamma}} = \hat{v}_\alpha$ and consequently, $\hat{v}_\alpha \in H^{1/2}(\hat{\Gamma})$. A direct calculation shows that there exist constants c_1, c_2 not depending on $\alpha \in U_{ad}$ and $v \in V_1(\alpha)$ such that

$$c_1 |v|_{1,\Omega(\alpha)} \leq |\Pi v|_{1,\hat{\Omega}} \leq c_2 |v|_{1,\Omega(\alpha)} .$$

From this and the definition of norms in $H^{1/2}(\Gamma(\alpha))$ and $H^{1/2}(\hat{\Gamma})$, the assertion follows. ∎

By the symbol $\langle\ ,\ \rangle$ we denote the duality paring between $H^{-1/2}(\hat{\Gamma})$ and $H^{1/2}(\hat{\Gamma})$. Let

$$\hat{V}_0(\alpha) = \{v \in H_0^1(\hat{\Omega}) | v|_{\Gamma(\alpha)} = 0\} .$$

Then it holds.

LEMMA 3.2 : *Let $\alpha \in U_{ad}$ and $\hat{v} \in \hat{V}$ be given. Then the function*

$$\sup_{\mu \in H^{-1/2}(\hat{\Gamma})} \langle \mu, \hat{v}_\alpha \rangle \tag{3.3}$$

is the indicator function of the space $\hat{V}_0(\alpha)$.

Proof: If $\hat{v} \in H_0^1(\hat{\Omega})$, then the function $\hat{v}_\alpha \equiv \hat{v}\,(\alpha(\hat{x}_2), \hat{x}_2)$ belongs to $H^{1/2}(\hat{\Gamma})$. The function (3.3) equals to zero iff $\hat{v}\,|_{\Gamma(\alpha)} = 0$, otherwise it takes the infinite value. ∎

Let $\alpha \in U_{ad}$ be fixed and define the problem

$$(\hat{\mathcal{P}}(\alpha)) \quad \begin{cases} \text{find } \hat{u} \in \hat{V}, \lambda \in H^{-1/2}(\hat{\Gamma}) \text{ such that} \\ (\nabla \hat{u}, \nabla \hat{\varphi})_{0,\hat{\Omega}} = (f, \hat{\varphi})_{0,\hat{\Omega}} + \langle \lambda, \hat{\varphi}_\alpha \rangle \quad \forall \hat{\varphi} \in \hat{V} \\ \langle \mu, \hat{u}_\alpha \rangle = 0 \quad \forall \mu \in H^{-1/2}(\hat{\Gamma}) . \end{cases}$$

LEMMA 3.3 : *Let $\hat{u} \in \hat{V}$ be a solution of $(\hat{\mathcal{P}}(\alpha))$. Then $u \equiv \hat{u}|_{\Omega(\alpha)}$ solves the homogeneous Dirichlet boundary value problem on $\Omega(\alpha)$.*

Proof: The function $u \equiv \hat{u}|_{\Omega(\alpha)}$ belongs to $H_0^1(\Omega(\alpha))$ as follows from the definition of $(\hat{\mathcal{P}}(\alpha))$ and Lemma 3.2. Restricting ourselves to functions $\hat{\varphi} \in \hat{V}$ such that supp $\hat{\varphi} \subset \Omega(\alpha)$ in $(\hat{\mathcal{P}}(\alpha))$, we arrive at the assertion. ∎

LEMMA 3.4 : *Problem $(\hat{\mathcal{P}}(\alpha))$ has a solution for any $\alpha \in U_{ad}$.*

Proof: Let u_1 and u_2 be the solutions of the homogeneous Dirichlet boundary value problem on $\Omega(\alpha)$ and $\Xi(\alpha)$, respectively :

$$\begin{cases} -\Delta u_1 = f \text{ in } \Omega(\alpha) \\ u_1 = 0 \text{ on } \partial\Omega(\alpha) \end{cases} \qquad \begin{cases} -\Delta u_2 = f \text{ in } \Xi(\alpha) \\ u_2 = 0 \text{ on } \partial\Xi(\alpha) \end{cases}$$

and define the function

$$\hat{u} = \begin{cases} u_1 & \text{in} \quad \Omega(\alpha) \\ u_2 & \text{in} \quad \Xi(\alpha) . \end{cases}$$

Then by applying the Green's formula, one has

$$\int_{\hat{\Omega}} \nabla \hat{u} \, \nabla \hat{\varphi} \, dx = \int_{\hat{\Omega}} f \hat{\varphi} \, dx + \left\langle \frac{\partial u_1}{\partial n}, \hat{\varphi} \right\rangle_\alpha + \left\langle \frac{\partial u_2}{\partial n}, \hat{\varphi} \right\rangle_\alpha \qquad \forall \hat{\varphi} \in \hat{V},$$

where $\langle \ , \ \rangle_\alpha$ denotes the duality paring between $H^{-1/2}(\Gamma(\alpha))$ and $H^{1/2}(\Gamma(\alpha))$. The mapping

$$\hat{\varphi} \mapsto \left\langle \frac{\partial u_1}{\partial n}, \hat{\varphi} \right\rangle_\alpha + \left\langle \frac{\partial u_2}{\partial n}, \hat{\varphi} \right\rangle_\alpha$$

defines the linear continuous functional on $H^{1/2}(\hat{\Gamma})$ as follows from Lemma 3.1. Hence there exists an element $\lambda \in H^{-1/2}(\hat{\Gamma})$ such that

$$\langle \lambda, \hat{\varphi}_\alpha \rangle = \left\langle \frac{\partial u_1}{\partial n}, \hat{\varphi} \right\rangle_\alpha + \left\langle \frac{\partial u_2}{\partial n}, \hat{\varphi} \right\rangle_\alpha .$$

■

Finally define the problem

$$(\hat{\mathbf{P}}) \quad \begin{cases} \text{find } \alpha^* \in U_{ad} \text{ such that} \\ I(\alpha^*, \hat{u}(\alpha^*)|_{\Omega(\alpha^*)}) \leq I(\alpha, \hat{u}(\alpha)|_{\Omega(\alpha)}) \ \forall \alpha \in U_{ad} \end{cases}$$

with $\hat{u}(\alpha)$ being a part of the solution of $(\hat{\mathcal{P}}(\alpha))$.

From Lemmas 3.3 and 3.4 we immediately obtain.

THEOREM 3.1 : *The problems* (**P**) *and* $(\hat{\mathbf{P}})$ *are equivalent.*

4. MIXED FINITE ELEMENT APPROXIMATION OF $(\hat{\mathbf{P}})$

In this section we describe the approximation of problem $(\hat{\mathbf{P}})$, which will be based on the mixed variational formulation $(\hat{\mathcal{P}}(\alpha))$.

Let $\{\hat{\mathcal{T}}_h\}$ be a *regular* family of triangulations of $\hat{\Omega}$. With any $\hat{\mathcal{T}}_h$, the space $\hat{V}_h$ of all piecewise linear functions over $\hat{\mathcal{T}}_h$ and vanishing on $\partial\hat{\Omega}$ will be associated :

$$\hat{V}_h = \{\hat{v}_h \in C(\bar{\hat{\Omega}}) | \hat{v}_h|_T \in P_1(T) \quad \forall T \in \hat{\mathcal{T}}_h , \hat{v}_h = 0 \text{ on } \partial\hat{\Omega}\} .$$

Let $D_H : 0 \equiv a_0 < a_1 < \cdots < a_{m(H)} = 1$ be a partition of [0,1], the norm of which will be denoted by H. With any D_H, the following sets will be associated :

$$U_{ad}^H = \{\alpha_H \in C([0,1]) \mid \alpha_H|_{\overline{a_{i-1}a_i}} \in P_1(\overline{a_{i-1}a_i}), i = 1, \dots, m\} \cap U_{ad}$$

$$\Lambda_H = \{\mu_H \in L^2((0,1)) \mid \mu_H|_{\overline{a_{i-1}a_i}} \in P_0(\overline{a_{i-1}a_i}), i = 1, \dots, m\} .$$

The sets U_{ad}^H and Λ_H will be the approximations of U_{ad} and $H^{-1/2}(\hat{\Gamma})$, respectively.

For $\alpha_H \in U_{ad}^H$ and $\mu_H \in \Lambda_H$ we denote

$$\langle \mu_H, \hat{v}_{h\alpha_H} \rangle \equiv \int_0^1 \mu_H \hat{v}_h(\alpha_H(x_2), x_2)\, dx_2, \quad \hat{v}_h \in \hat{V}_h .$$

Let $\alpha_H \in U_{ad}^H$ be given and define the problem

$$(\hat{\mathcal{P}}(\alpha_H))_h \quad \begin{cases} \text{find } \hat{u}_h \in \hat{V}_h, \lambda_H \in \Lambda_H \text{ such that} \\ (\nabla \hat{u}_h, \nabla \hat{\varphi}_h)_{0,\hat{\Omega}} = (f, \hat{\varphi}_h)_{0,\hat{\Omega}} + \langle \lambda_H, \hat{\varphi}_{h\alpha_H} \rangle \quad \forall \hat{\varphi}_h \in \hat{V}_h \\ \langle \mu_H, \hat{u}_{h\alpha_H} \rangle = 0 \quad \forall \mu_H \in \Lambda_H . \end{cases}$$

Remark 4.1 : (Interpretation of $((\hat{\mathcal{P}}(\alpha_H))_h)$ Denote by

$$\hat{V}_{0h}(\alpha_H) = \left\{ \hat{v}_h \in \hat{V}_h \mid q_i(\hat{v}_h) \equiv \int_{\alpha_H^i} \hat{v}_h\, ds = 0 , \quad \alpha_H^i = \alpha_H|_{\overline{a_{i-1}a_i}} \right\} .$$

That is, $\hat{V}_{0h}(\alpha_h)$ contains those functions from $\hat{V}_h$, the integral mean value of which on each segment α_H^i equals zero.

Let $\alpha_H \in U_{ad}^H$ be given. Then it holds

LEMMA 4.1 : *Let $(\hat{u}_h, \lambda_H) \in \hat{V}_h \times \Lambda_H$ be a solution of $(\hat{\mathcal{P}}(\alpha_H))_h$. Then $\hat{u}_h \in \hat{V}_{0h}(\alpha_H)$, it is uniquely determined and it solves the problem*

$$(\hat{\mathcal{P}}(\alpha_H))_h \quad (\nabla \hat{u}_h, \nabla \hat{\varphi}_h)_{0,\hat{\Omega}} = (f, \hat{\varphi}_h)_{0,\hat{\Omega}} \quad \forall \hat{\varphi}_h \in \hat{V}_{0h}(\alpha_H) .$$

Moreover, there exists a constant $c > 0$, which depends solely on $\|f\|_{0,\hat{\Omega}}$ and such that

$$(4.1) \qquad \|\hat{u}_h\|_{1,\hat{\Omega}} \leq c .$$

Proof : We have

$$\int_{\alpha_H} \mu_H \hat{u}_h\, ds = \int_0^1 \mu_H \hat{u}_h(\alpha_H(x_2), x_2) \sqrt{1 + (\alpha_H')^2}\, dx_2 = 0$$

as $\mu_H \sqrt{1 + (\alpha_H')^2} \in \Lambda_H$ and consequently $\hat{u}_h \in \hat{V}_{0h}(\alpha_H)$. Restricting ourselves to functions $\hat{\varphi}_h \in \hat{V}_{0h}(\alpha_H)$, the problem $(\hat{\mathcal{P}}(\alpha_H))_h$ transforms into $(\widehat{\mathcal{P}}_0(\alpha_H))_h$. Finally the uniqueness of $\hat{u}_h$ as well as (4.1) follow from $(\widehat{\mathcal{P}}_0(\alpha_H))_h$. ∎

Remark 4.2 : The function $u_h \equiv \hat{u}_h|_{\Omega(\alpha_H)}$ is the approximation of the homogeneous Dirichlet boundary value problem on $\Omega(\alpha_H)$ with the Dirichlet boundary condition on $\Gamma(\alpha_H)$ satisfied in a weak sense.

Remark 4.3 : An alternative construction of Λ_H is possible : let $a_{i+1/2}$ denote the mid-point of the segment $\overline{a_i a_{i+1}}$. Then we set

$$\Lambda_H = \{\mu_H \in L^2((0,1)) | \mu_H|_{\overline{a_{i-1} a_{i+1/2}}} \in P_0(\overline{a_{i-1/2} a_{i+1/2}}), i = 1, ..., m\} .$$

The problem $(\hat{\mathcal{P}}(\alpha_H))_h$ is defined in the same way as before. Then

$$\hat{V}_{0h}(\alpha_H) = \left\{ \hat{v}_h \in \hat{V}_h | q_i(\hat{v}_h) \equiv \int_{a_{i-1/2}}^{a_{i+1/2}} \hat{v}_h \, dx_2 = 0, \quad \forall i = 1, ..., m-1 \right\} ,$$

That is, $\hat{V}_{0h}(\alpha_H)$ contains those functions from $\hat{V}_h$, the integral mean value of $\hat{v}_h$ on any $\overline{a_{i-1/2} a_{i+1/2}}$ is equal to zero.

In order to guarantee the existence and uniqueness of a solution of $(\hat{\mathcal{P}}(\alpha_H))_h$ we assume that the implication

(4.2) $$\mu_H \in \Lambda_H \;\&\; \langle \mu_H, \hat{v}_{h\alpha_H} \rangle = 0 \quad \forall \hat{v}_h \in \hat{V}_h \Rightarrow \mu_H \equiv 0$$

holds.

Remark 4.4 : If condition (4.2) holds, then there exists a constant $\beta > 0$, depending generally on h, H and such that

(4.3) $$\sup_{\hat{v}_h \in \hat{V}_h} \frac{\langle \mu_H, \hat{v}_{h\alpha_H} \rangle}{\| \hat{v}_h \|_{1,\hat{\Omega}}} \geq \beta \| \mu_H \|_{0,\Gamma(\alpha)} .$$

Let us point out that we *do not need* to satisfy the Babuška-Brezzi condition, i.e. the case when the constant β, appearing in (4.3) is independent on h, H and the norm $\| \cdot \|_{0,\Gamma(\alpha)}$ is replaced by $\| \cdot \|_{-1/2,\Gamma(\alpha)}$.

Using classical results from mixed finite element methods one has.

LEMMA 4.2 : *Let* (4.2) *be satisfied. Then* $(\hat{\mathcal{P}}(\alpha_H))_h$ *has a unique solution.*

Remark 4.5 : The condition (4.2) is satisfied, if the ratio h/H is sufficiently small, i.e. the triangulation $\hat{\mathcal{T}}_h$, used for the construction of $\hat{V}_h$ is finer than the partition D_H, characterizing Λ_H.

The approximation of problem $(\hat{\mathbf{P}})$ now reads as follows

$$(\hat{\mathbf{P}})_h \quad \begin{cases} \text{find } \alpha_H^* \in U_{ad}^H \text{ such that} \\ I(\alpha_H^*, \hat{u}_h(\alpha_H^*)|_{\Omega(\alpha_H^*)}) \leq I(\alpha_H, \hat{u}_h(\alpha_H)|_{\Omega(\alpha_H)}) \quad \forall \alpha_H \in U_{ad}^H . \end{cases}$$

Using classical compactness arguments, one can easily prove.

THEOREM 4.1 : *Let the condition* (4.2) *be satisfied. Then the problem* $(\hat{\mathbf{P}})_h$ *has at least one solution for any* $h > 0$.

Proof: The mapping $\alpha_H \mapsto \hat{u}_h(\alpha_H)$, $\alpha_H \in U_{ad}^H$ (h, H fixed) is continuous (see also Lemma 4.3). ■

Next we shall analyze the mutual relation between $(\hat{\mathbf{P}})$ and $(\hat{\mathbf{P}})_h$ when $h \to 0+$. To this end we assume that $h \to 0+$ iff $H \to 0+$. First we prove an auxiliary result.

LEMMA 4.3 : *Let* $\alpha_H \in U_{ad}^H$ *be such that* $\alpha_H \rightrightarrows \alpha$ *in* $[0, 1]$ *and let* $\hat{u}_h(\alpha_H)$ *be solutions of* $(\mathscr{P}(\alpha_H))_h$. *Then*

$$\hat{u}_h(\alpha_H) \to \hat{u} \quad in \quad \hat{V}, \tag{4.4}$$

where $u \equiv \hat{u}|_{\Omega(\alpha)}$ *solves* $(\mathscr{P}(\alpha))$.

Proof: The sequence $\{\|\hat{u}_h(\alpha_H)\|_{1,\hat{\Omega}}\}$ is bounded as follows from (4.1). Thus there exists a subsequence of $\{\hat{u}_h(\alpha_H)\}$ (still denoted by the same symbol) such that

$$\hat{u}_h(\alpha_H) \rightharpoonup \hat{u} \in \hat{V}. \tag{4.5}$$

Let $\hat{\varphi} \in C^\infty(\overline{\hat{\Omega}})$ be such that supp $\hat{\varphi} \subset \Omega(\alpha) \cup \Xi(\alpha)$. As $\alpha_H \rightrightarrows \alpha$ in $[0, 1]$, supp $\hat{\varphi} \subset \Omega(\alpha_H) \cup \Xi(\alpha_H)$ for all H sufficiently small. Let $r_h\hat{\varphi}$ denote the $\hat{V}_h$-interpolation of $\hat{\varphi}$. Then $r_h\hat{\varphi} \in \hat{V}_{0h}(\alpha_H)$ and

$$r_h\hat{\varphi} \to \hat{\varphi} \quad \text{in} \quad \hat{V}. \tag{4.6}$$

Inserting $r_h\hat{\varphi}$ into $(\hat{\mathscr{P}}_0(\alpha_H))_h$ we obtain

$$(\nabla \hat{u}_h(\alpha_H), r_h\hat{\varphi})_{0,\hat{\Omega}} = (f, r_h\hat{\varphi})_{0,\Omega} .$$

Passing to the limit with h, $H \to 0+$ we arrive at

$$(\nabla \hat{u}, \nabla \hat{\varphi})_{0,\hat{\Omega}} = (f, \hat{\varphi})_{0,\hat{\Omega}} , \tag{4.7}$$

making use of (4.5) and (4.6). It remains to show that $\hat{u}|_{\Gamma(\alpha)} = 0$ or

$$\int_0^1 \mu \hat{u}(\alpha(x_2), x_2)\, dx_2 = 0 \quad \forall \mu \in L^2((0,1)) .$$

Let $\mu \in L^2((0,1))$ be given and $\{\mu_H\}$ a sequence of elements from Λ_H such that

$$\mu_H \to \mu \quad \text{in} \quad L^2((0,1)) . \tag{4.8}$$

From the definition of $(\mathscr{P}(\alpha_H))_h$ it follows that

$$\int_0^1 \mu_H \hat{u}_h(\alpha_H(x_2), x_2)\, dx_2 = 0 . \tag{4.9}$$

It is easy to show, see [4], that (4.5), (4.8) and the fact that $\alpha_H \rightrightarrows \alpha$ in [0, 1] yield

$$\int_0^1 \mu_H \hat{u}_h(\alpha_H(x_2), x_2)\, dx_2 \to \int_0^1 \mu \hat{u}(\alpha(x_2), x_2)\, dx_2 = 0$$

because of (4.9). Consequently $\hat{u} \in \hat{V}_0(\alpha)$ and it solves (4.7) for any $\hat{\varphi} \in \hat{V}_0(\alpha)$. As such $\hat{u}$ is unique, the whole sequence $\{\hat{u}_h(\alpha_H)\}$ tends weakly to $\hat{u}$. On the other hand

$$(\nabla \hat{u}_h(\alpha_H), \nabla \hat{u}_h(\alpha_H))_{0,\hat{\Omega}} = (f, \hat{u}_h(\alpha_H))_{0,\hat{\Omega}} \to (f, \hat{u}(\alpha))_{0,\hat{\Omega}} = (\nabla \hat{u}(\alpha), \nabla \hat{u}(\alpha))_{0,\hat{\Omega}}$$

from which (4.4) follows.

In order to prove the relation between $(\hat{\mathbf{P}})$ and $(\hat{\mathbf{P}})_h$, we shall suppose that the cost functional I is *continuous* in the following sense :

$$(\mathbf{B}) \quad \left\{ \begin{array}{l} \alpha_n \rightrightarrows \alpha \quad \text{in} \quad [0,1],\ \alpha_n, \alpha \in U_{ad} \\ \hat{y}_n \to \hat{y} \quad \text{in} \quad \hat{V}, \hat{y}_n, \hat{y} \in \hat{V} \end{array} \right\} \Rightarrow \lim_{n \to \infty} I(\alpha_n, \hat{y}_n|_{\Omega_n}) = I(\alpha, \hat{y}|_{\Omega\Omega(\alpha)})$$

where $\Omega_n = \Omega(\alpha_n)$.

The main result of this section is

THEOREM 4.2 : *Let* (4.2) *and* (**B**) *be satisfied. Let* $\alpha_H^* \in U_{ad}^H$ *be a solution of* $(\hat{\mathbf{P}})_h$ *and* $\hat{u}_h(\alpha_H^*) \in \hat{V}_h$ *the solution of* $(\mathscr{P}(\alpha_H^*))_h$. *Then there exist subsequences* $\{\alpha_{H_j}^*\} \subset \{\alpha_H^*\}$, $\{\hat{u}_{h_j}(\alpha_{H_j}^*)\} \subset \{\hat{u}_h(\alpha_H^*)\}$ *and elements* $\alpha^* \in U_{ad}$ *and* $\hat{u} \in \hat{V}$ *such that*

$$\alpha_{H_j}^* \rightrightarrows \alpha^* \quad in \quad [0, 1]\,, \tag{4.10}$$

$$\hat{u}_{h_j}(\alpha_{H_j}^*) \to \hat{u}(\alpha^*)\,, \quad j \to \infty\,. \tag{4.11}$$

Moreover, α^* *is a solution of* (**P**) *and* $u^* \equiv \hat{u}(\alpha^*)|_{\Omega(\alpha^*)}$ *solves* $(\mathscr{P}(\alpha^*))$.

Proof: The set U_{ad} is compact in $C([0, 1])$ norm. As $U_{ad}^H \subset U_{ad}$ for any $H > 0$, the existence of a subsequence such that (4.10) holds, follows. At the same time the sequence $\{\hat{u}_{h_j}(\alpha_{Hj}^*)\}$ tends to a function $\hat{u}(\alpha^*) \in \hat{V}$ such that $u^* = \hat{u}(\alpha^*)|_{\Omega(\alpha^*)}$ solves $(\mathscr{P}(\alpha^*))$ by virtue of Lemma 4.3. From the definition of $(\hat{\mathbf{P}})_{h_j}$ now follows :

$$I(\alpha_{H_j}^*, \hat{u}_{h_j}(\alpha_{H_j}^*)|_{\Omega(\alpha_{Hj}^*)}) \leq I(\alpha_{H_j}, \hat{u}_{h_j}(\alpha_{H_j})|_{\Omega(\alpha_{Hj})}) \tag{4.12}$$

holds for any $\alpha_{H_j} \in U_{ad}^{H_j}$. Let $\bar{\alpha} \in U_{ad}$ be given. Then there exists a sequence $\{\bar{\alpha}_H\}$, $\bar{\alpha}_H \in U_{ad}^H$ such that

$$\bar{\alpha}_H \rightrightarrows \bar{\alpha} \quad \text{in} \quad [0, 1] \tag{4.13}$$

(see [4]) and at the same time

$$\hat{u}_h(\bar{\alpha}_H) \to \hat{u}(\bar{\alpha}) \quad \text{in} \quad \hat{V}\,. \tag{4.14}$$

Then $\hat{u}(\bar{\alpha})|_{\Omega(\bar{\alpha})}$ solves $(\mathscr{P}(\bar{\alpha}))$ as follows from Lemma 4.3 again. Now passing to the limit with $h_j, H_j \to 0+$ in (4.12) and taking into account (4.10), (4.11), (4.13) and (4.14) as well as (**B**) we arrive at

$$I(\alpha^*, \hat{u}(\alpha^*)|_{\Omega(\alpha^*)}) \leq I(\bar{\alpha}, \hat{u}(\bar{\alpha})|_{\Omega(\bar{\alpha})})\,.$$

As $\bar{\alpha} \in U_{ad}$ is arbitrary we conclude that $\alpha^* \in U_{ad}$ solves (**P**). ∎

5. MATRIX FORMULATION

We will present a matrix formulation of problem $(\hat{\mathbf{P}})_h$ for fixed $h > 0$ and $H > 0$. The state problem $(\mathscr{P}(\alpha_H))_h$ reads as follows in matrix form :

$$\mathbf{A}\mathbf{u}(\boldsymbol{\alpha}) = \mathbf{F} + \mathbf{G}(\boldsymbol{\alpha})^T \boldsymbol{\lambda}(\boldsymbol{\alpha}) \tag{5.1}$$

$$\mathbf{G}(\boldsymbol{\alpha})\,\mathbf{u}(\boldsymbol{\alpha}) = \mathbf{0}\,. \tag{5.2}$$

Here, $\mathbf{u}(\boldsymbol{\alpha})$ is a vector of nodal values of the approximate displacement field $\hat{u}_h \in \hat{V}_h$ and $\boldsymbol{\lambda}(\boldsymbol{\alpha}) \in \mathbf{R}^{m(H)}$ is a vector containing the constant values of the field $\lambda \in \Lambda_H$. Both of these vectors are considered as functions of $\boldsymbol{\alpha} \in \mathbf{R}^{m(H)+1}$, the vector of nodal values of $\alpha_H \in U^H_{ad}$, since for given such $\boldsymbol{\alpha}$ the system (5.1) and (5.2) is uniquely solvable for $\mathbf{u}(\boldsymbol{\alpha})$ and $\boldsymbol{\lambda}(\boldsymbol{\alpha})$. Furthermore, the elements of the symmetric positive definite stiffness matrix $\mathbf{A}$, the force matrix $\mathbf{F}$ and the kinematic transformation matrix $\mathbf{G}(\boldsymbol{\alpha})$ are given by

$$(\mathbf{A})_{ij} = (\nabla\hat{\varphi}_i, \nabla\hat{\varphi}_j)_{0,\hat{\Omega}}$$

$$(\mathbf{F})_j = (f, \hat{\varphi}_j)_{0,\hat{\Omega}}$$

$$(\mathbf{G}(\boldsymbol{\alpha}))_{ki} = \int_{\alpha_H^k} \hat{\varphi}_i(\alpha_H(x_2), x_2)\, dx_2$$

where $\hat{\varphi}_i$ is the Courant base function of node i.

The matrix form of $I(\alpha_H, \hat{u}_h(\alpha_H)|_{\Omega(\alpha_H)})$ is $\mathcal{J}(\boldsymbol{\alpha}, \mathbf{u}(\boldsymbol{\alpha}))$ and problem $(\hat{\mathbf{P}})_h$ now reads

$$(\overline{\mathbf{P}}) \quad \begin{cases} \text{find } \boldsymbol{\alpha}^* \in \mathcal{U} \text{ such that} \\ \mathcal{J}(\boldsymbol{\alpha}^*, \mathbf{u}(\boldsymbol{\alpha}^*)) \le \mathcal{J}(\boldsymbol{\alpha}, \mathbf{u}(\boldsymbol{\alpha})) \quad \forall \alpha \in \mathcal{U} \end{cases}$$

where $\mathbf{u}(\boldsymbol{\alpha})$ is part of the solution of (5.1) and (5.2) and $\mathcal{U}$ is a subset of $\mathbf{R}^{m(H)+1}$ isometrically isomorphic with U^H_{ad}.

6. SOLUTION METHODS

Problem $(\overline{\mathbf{P}})$ contains two state equations, (5.1) and (5.2). In our treatment so far these have been treated as defining the function $\boldsymbol{\alpha} \mapsto \mathbf{u}(\boldsymbol{\alpha})$. Another possibility is to regard one or both of these equations as explicit constraints. Here we will use the possibility of regarding (5.2) as a constraint which is then added to the objective function as a penalty term. It will then be natural to regard the multiplier vector $\boldsymbol{\lambda}(\boldsymbol{\alpha})$ as a control variable and $\mathbf{u}(\boldsymbol{\alpha})$ as a function of both the « old » control variable $\boldsymbol{\alpha}$ and this new one. Consequently, we write in the sequel $\boldsymbol{\lambda}$ and $\mathbf{u}(\boldsymbol{\alpha}, \boldsymbol{\lambda})$ for these vectors. A penalty approach to the solution of $(\overline{\mathbf{P}})$ is to solve the following problem

$$(\overline{\mathbf{P}})_\varepsilon \quad \begin{cases} \text{find } (\boldsymbol{\alpha}^*, \boldsymbol{\lambda}^*) \in \mathcal{U} \times \mathbf{R}^{m(H)} \text{ such that} \\ \mathcal{E}_\varepsilon(\boldsymbol{\alpha}^*, \boldsymbol{\lambda}^*) \le \mathcal{E}_\varepsilon(\boldsymbol{\alpha}, \boldsymbol{\lambda}) \quad \forall (\boldsymbol{\alpha}, \boldsymbol{\lambda}) \in \mathcal{U} \times \mathbf{R}^{m(H)} \end{cases}$$

where

$$\mathscr{E}_{\varepsilon}(\alpha, \lambda) = \mathscr{J}(\alpha, \mathbf{u}(\alpha, \lambda)) + \frac{1}{2\,\varepsilon}\mathbf{u}(\alpha, \lambda)^T \mathbf{G}(\alpha)^T \mathbf{G}(\alpha)\,\mathbf{u}(\alpha, \lambda)$$

and $\mathbf{u}(\alpha, \lambda)$ is a solution of

$$\mathbf{Au}\,(\alpha\,.\,\lambda) = \mathbf{F} + \mathbf{G}(\alpha)^T \lambda\,. \tag{6.1}$$

The solution of $(\overline{\mathbf{P}})$ is approached by the solution of $(\overline{\mathbf{P}})_{\varepsilon}$ as $\varepsilon \to \infty$.

The penalty approach is sensitive to the choice of ε. A means of avoiding (or diminish) this difficulty is to use an augmented Lagrangian method. The functional to be used in such a method is

$$\mathscr{E}_{\varepsilon A}(\alpha, \lambda, \xi) = \mathscr{E}_{\varepsilon}(\alpha, \lambda) + \xi^T \mathbf{G}(\alpha)\,\mathbf{u}(\alpha, \lambda)$$

where ξ is a multiplier vector. Step k of the augmented Lagrangian method consists of solving for fixed ξ^k

$$(\overline{\mathbf{P}})_{\varepsilon}^{k} \quad \begin{cases} \text{find } (\alpha^k, \lambda^k) \in \mathscr{U} \times \mathbf{R}^{m(H)} \text{ such that} \\ \mathscr{E}_{\varepsilon A}(\alpha^k, \lambda^k, \xi^k) \leq \mathscr{E}_{\varepsilon}(\alpha, \lambda, \xi^k) \quad \forall(\alpha, \lambda) \in \mathscr{U} \times \mathbf{R}_{m(H)} \end{cases}$$

and then updating the multipliers by

$$\xi^k = \xi^{k-1} + \frac{1}{\varepsilon}\mathbf{G}(\alpha^k)\,\mathbf{u}(\alpha^k, \lambda^k)\,.$$

The sequence produced by these two steps approaches the solution of $(\overline{\mathbf{P}})$ for a finite ε.

7. SENSITIVITY ANALYSIS

A distinct difficulty in solving $(\overline{\mathbf{P}})_{\varepsilon}^{k}$ numerically is a lack of differentiability of $\mathbf{G}(\alpha)$ which results in a nondifferentiability of $\mathbf{u}(\alpha, \lambda)$. We illustrate this by a small example. Let $\widehat{\mathscr{T}_h}$ consist of 8 finite elements and the partition D_H of two segments of equal length according to figure 2. The design variable α is shown in the figure. Then a simple calculation shows that

$$(\mathbf{G}(\alpha))_{1\,i} = \begin{cases} \alpha/2 & \text{if} \quad 0 < \alpha < 1 \\ (2-\alpha)/2 & \text{if} \quad 1 < \alpha < 2 \end{cases}$$

which shows the non-smoothness at $\alpha = 1$.

The non-smoothness will require use of nondifferentiable optimization methods. In fact, some preliminary numerical experiments using smooth

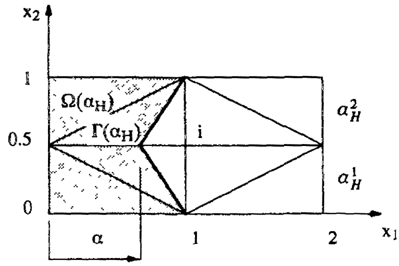

Figure 2. — A simple example showing non-smoothness.

algorithms generally did not give satisfactory results. Nondifferentiable methods require computation of at least one subgradient of $\mathscr{E}_{\varepsilon A}$ (or ($\mathscr{E}_{\varepsilon}$) at each point $(\boldsymbol{\alpha}, \boldsymbol{\lambda})$, see [7, 8]. At smooth points the subgradient coincide with the standard gradient and below we derive this gradient assuming smoothness. At non-smooth points this calculation can be modified following the presentation in [5]. We calculate the derivatives of $\mathscr{E}_{\varepsilon A}(\boldsymbol{\alpha}, \boldsymbol{\lambda}, \boldsymbol{\xi})$ with respect to $\boldsymbol{\alpha}$ and $\boldsymbol{\lambda}$. The derivatives of $\mathscr{E}_{\varepsilon}(\boldsymbol{\alpha}, \boldsymbol{\lambda})$ will be a special case of these derivatives obtained by setting $\boldsymbol{\xi} = \mathbf{0}$. Omitting the arguments the chain rule gives

$$\frac{\partial \mathscr{E}_{\varepsilon A}}{\partial \alpha_i} = \frac{\partial \mathscr{J}}{\partial \alpha_i} + \left(\frac{\partial \mathscr{J}}{\partial \mathbf{u}}\right)^T \frac{\partial \mathbf{u}}{\partial \alpha_i}$$

$$+ \frac{1}{\varepsilon}\left(\frac{\partial \mathbf{u}}{\partial \alpha_i}\right)^T \mathbf{G}^T \mathbf{G} \mathbf{u} + \frac{1}{2\,\varepsilon} \mathbf{u}^T \frac{\partial}{\partial \alpha_i}(\mathbf{G}^T\ \mathbf{G})\,\mathbf{u}$$

$$+ \boldsymbol{\xi}^T \left(\frac{\partial \mathbf{G}}{\partial \alpha_i} \mathbf{u} + \mathbf{G}\,\frac{\partial \mathbf{u}}{\partial \alpha_i}\right). \tag{7.1}$$

Next we use the equation obtained by taking the derivative of (6.1) :

$$\mathbf{A} \frac{\partial \mathbf{u}}{\partial \alpha_i} = \left(\frac{\partial \mathbf{G}}{\partial \alpha_i}\right)^T \boldsymbol{\lambda}\ .$$

Putting this into (7.1) and identifying the solution $\mathbf{p}$ of the adjoint equation

$$\mathbf{A}\mathbf{p} = \frac{\partial \mathscr{J}}{\partial \mathbf{u}} + \frac{1}{\varepsilon}\mathbf{G}^T\ \mathbf{G}\mathbf{u} + \mathbf{G}^T\ \boldsymbol{\xi}$$

we obtain

$$(7.2) \quad \frac{\partial \mathscr{E}_{\varepsilon A}}{\partial \alpha_i} = \frac{\partial \mathscr{J}}{\partial \alpha_i} + \mathbf{p}^T \left(\frac{\partial \boldsymbol{G}}{\partial \alpha_i} \right)^T \lambda + \frac{1}{2\,\varepsilon} \boldsymbol{u}^T \frac{\partial}{\partial \alpha_i} (\boldsymbol{G}^T \boldsymbol{G}) \, \boldsymbol{u} + \xi^T \frac{\partial \boldsymbol{G}}{\partial \alpha_i} \boldsymbol{u} \, .$$

Similarly to this derivation we obtain

$$\frac{\partial \mathscr{E}_{\varepsilon A}}{\partial \lambda_i} = \mathbf{p}^T \mathbf{G}_i$$

where $\mathbf{G}_i$ is the i^{th} row of $\mathbf{G}$.

REFERENCES

[1] C. ATAMIAN, G. V. DINH, R. GLOWINSKI, HE JIWEN and J. PERIAUX, 1991, On some imbedding methods applied to fluid dynamics and electro-magnetics, *Computer Methods in Applied Mechanics and Engineering,* **91,** pp. 1271-1299.

[2] E. J. HAUG, K. K. CHOI and V. KOMKOV, 1986, *Design sensitivity analysis of structural systems,* Academic Press, Orlando.

[3] O. PIRONNEAU, 1984, *Optimal shape design for eliptic systems,* Springer-Verlag, New York.

[4] J. HASLINGER and P. NEITTAANMÄKI, 1988, *Finite element approximation for optimal shape design : theory and applications,* John Wiley, Chichester.

[5] J. HASLINGER, K.-H. HOFFMANN and M. KOCVARA, 1993, Control/fictitious domain method for solving optimal design problems, *M^2AN* **27**(2), pp. 157-182.

[6] R. GLOWINSKI, T.-W. PAN and J. PERIAUX, 1994, A fictitious domain method for Dirichlet problem and applications, *Computer Methods in Applied Mechanics and Engineering,* **111,** pp. 283-303.

[7] J.-B. HIRIART-URRUTY and C. LEMARÉCHAL, 1993, *Convex analysis and minimization algorithms II,* Springer-Verlag, New York.

[8] H. SCHRAMM and J. ZOWE, 1992, A version of the bundle idea for minimizing a nonsmooth function : conceptual idea, convergence analysis, numerical results, *SIAM J. Optimization,* **2**(1), pp. 121-152.

[9] R. GLOWINSKI, A. J. KEARSLEY, T. W. PAN and J. PERIAUX, 1995, Numerical simulation and optimal shape for viscous flow by a fictitious domain method, to appear.