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A LOWER BOUND FOR REVERSIBLE AUTOMATA

PIERRE-CYRILLE HÉAM[1]

Abstract. A reversible automaton is a finite automaton in which each letter induces a partial one-to-one map from the set of states into itself. We solve the following problem proposed by Pin. Given an alphabet A, does there exist a sequence of languages K_n on A which can be accepted by a reversible automaton, and such that the number of states of the minimal automaton of K_n is in $O(n)$, while the minimal number of states of a reversible automaton accepting K_n is in $O(\rho^n)$ for some $\rho > 1$? We give such an example with $\rho = \left(\frac{9}{8}\right)^{\frac{1}{12}}$.

Résumé. Un automate réversible est un automate fini dans lequel chaque lettre réalise une fonction injective de l'ensemble des états dans lui-même. On résout dans cet article le problème suivant posé par Pin : étant donné un alphabet A, existe-t-il une suite de langages K_n sur A qui peuvent être reconnus par un automate réversible, et tels que le nombre d'états de l'automate minimal de K_n soit en $O(n)$ alors que le nombre minimal d'états d'un automate réversible reconnaissant K_n soit en $O(\rho^n)$ avec $\rho > 1$? On donne un tel exemple avec $\rho = \left(\frac{9}{8}\right)^{\frac{1}{12}}$.

AMS Subject Classification. 68Q45, 68Q70.

INTRODUCTION

In this paper we answer an open question on reversible automata proposed by Pin in [10]. Reversible automata form a natural class of automata with links to artificial intelligence [1], and to biprefix codes [4]. Moreover reversible automata are used to study inverse monoids, inverse automata [12,13] and certain topological problems [7,10,12,13]. We prove that computing a reversible automaton which

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recognizes a language given by its minimal automaton can not be done, in some cases, faster than in exponential time (if the alphabet contains at least two letters).

We denote by $|K|$ the cardinality of a set K.

We assume that the reader is familiar with the basic definitions on words and formal languages. For more information we refer the reader to [2]. Let us denote by $|u|$ the length of a word and by $u^\sim$ its mirror.

Let us recall that a finite automaton is a 5-tuple $\mathcal{A} = (Q, A, E, I, F)$ where Q is a finite set of states, A is the alphabet, $E \subseteq Q \times A \times Q$ is the set of edges (or transitions), $I \subseteq Q$ is the set of initial states and $F \subseteq Q$ is the set of final states. A path in $\mathcal{A}$ is a finite sequence of consecutive edges:

$$p = (q_0, a_0, q_1), (q_1, a_1, q_2), \cdots, (q_{n-1}, a_n, q_n).$$

The label of the path p is the word $a_1 a_2 \cdots a_n$, its origin is q_0 and its end is q_n. A word is accepted by $\mathcal{A}$ if it is the label of a path in $\mathcal{A}$ having its origin in I and its end in F. The set of words accepted by $\mathcal{A}$ is denoted by $L(\mathcal{A})$. For every state q and language K, we denote by $q.K$ the subset of Q of all the states which are the end of a path having its origin in q and its label in K. An automaton is said to be *trim* if for each state q there exists a path from an initial state to q and a path from q to a final state. An automaton is *deterministic* if it has a unique initial state and does not contain any pair of edges of the form (q, a, q_1) and (q, a, q_2) with $q_1 \neq q_2$. An important result of automata theory states that for an automaton $\mathcal{A}$ there exists exactly one (up to isomorphism) deterministic automaton with a minimal number of states which accepts the same language as $\mathcal{A}$. It is called the *minimal automaton* of $L(\mathcal{A})$. A *reversible* automaton is a finite automaton in which each letter induces a partial one-to-one map from the set of states into itself. It also is an automaton which does not contain any pair of edges of the form (q, a, q_1) and (q, a, q_2) with $q_1 \neq q_2$ or (q_1, a, q) and (q_2, a, q) with $q_1 \neq q_2$. It may happen that a reversible automaton is not deterministic because it may have several initial states. Reversible automata were studied by Pin [10] and by Silva [11]. For an automaton $\mathcal{A} = (Q, A, E, I, F)$, let us introduce the following languages, for p and q in Q:

$$L_{p,q}(\mathcal{A}) = \{u \in A^* \mid p.u = q\}\cdot$$

If $I = \{i\}$ we write $L_p(\mathcal{A})$ for $L_{i,p}(\mathcal{A})$, and if $F = \{f\}$ we write $R_q(\mathcal{A})$ for $L_{q,f}(\mathcal{A})$. If there is no ambiguity we just write $L_{p,q}$, L_p and R_q.

For a language L which is recognized by a reversible automaton we introduce two invariants: $m(L)$, the number of states of its minimal automaton and $c(L)$ the minimal number of states of a reversible automaton accepting L. We denote by $r(n)$ and $R(n)$ the following functions:

$$r(n) = \min\{c(L) \mid m(L) = n\} \text{ and } R(n) = \max\{c(L) \mid m(L) \leq n\}\cdot$$

It is proved in [10] that $r(n) = O\left(\frac{\ln n}{\ln \ln n}\right)^2$. Let us remark that $R(n)$ is an increasing function of n. We prove here that $R(n) = \Omega(\rho^n)$ for some $\rho > 1$.

1. Main theorem

Theorem 1.1. *The relation $R(n) = \Omega(\rho^n)$ holds with $\rho = \left(\frac{3\sqrt{2}}{4}\right)^{\frac{1}{6}} \simeq 1.001$ if the alphabet contains at least two letters.*

For the proof, let us consider the language $L = \{aa, ab, ba\}$ on the alphabet $A = \{a, b\}$. Before we proceed, let us note the following technical result.

Lemma 1.2. *Let $\mathcal{A} = (Q, A, E, i, F)$ be a reversible trim automaton accepting a non-empty subset of $L = \{aa, ab, ba\}$. For each state $s \in Q$, let L_s be the set of words which can be read from state i to s. Then one of the following holds:*

(a) $|F| = 3$ *and* $|L_s| = 1$ *for each state* $s \in F$;
(b) $|F| = 2$, $F = \{s_1, s_2\}$, $|L_{s_1}| = 2$ *and* $|L_{s_2}| = 1$;
(c) $|F| = 2$ *and* $|L_s| = 1$ *for each state* $s \in F$;
(d) $|F| = 1$, $F = \{s\}$ *and* $|L_s| = 2$;
(e) $|F| = 1$, $F = \{s\}$ *and* $|L_s| = 1$.

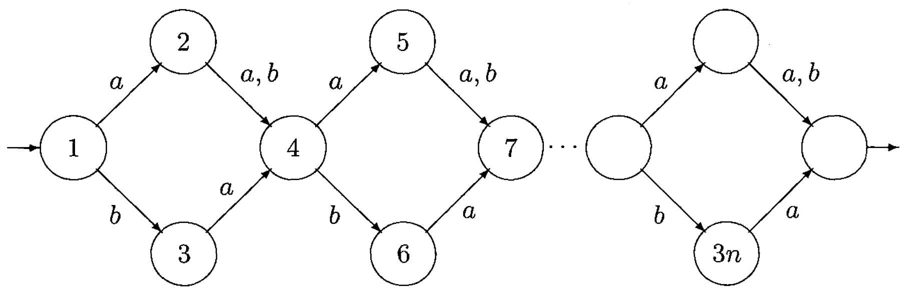
Proof. First we observe that in a reversible automaton on 2 letters, each state is the origin (resp. the end) of at most 2 transitions. Since $|L| = 3$ and $\mathcal{A}$ is trim, F has at most 3 elements, and if $|F| = 3$, then $|L_s| = 1$ for each final state s, that is *(a)* holds.

If $|F| = 2$, say $F = \{s_1, s_2\}$, then $|L_{s_1}| + |L_{s_2}| \leq 3$. Since $\mathcal{A}$ is trim $|L_s| > 0$ for every state, so *(b)* or *(c)* holds.

Finally, let us assume that $F = \{s\}$ is a singleton. We need to verify that $|L_s| \neq 3$, that is, $\mathcal{A}$ does not accept all of L. If there is no a-labeled transition out of i, then $L_s = L(\mathcal{A}) = \{ba\}$ and *(e)* holds. If there is no b-labeled transition out of i, then $L_s \subseteq \{ab, aa\}$ and *(d)* or *(e)* holds. We now consider the case where both an a-labeled transition and a b-labeled transition start in state i. If $i.a = i.b$, then there is no b-labeled transition from $i.a$ to the final state s since bb cannot be accepted, and hence $L_s = \{aa, ba\}$ and *(d)* holds. On the other hand, suppose that $i.a \neq i.b$. Since $\mathcal{A}$ is trim, there is an a-labeled transition from state $i.b$ to s. But $\mathcal{A}$ is reversible, so there is no a-labeled transition from state $i.a$ to s, and the word aa is not accepted. In particular, $L_s = \{ab, ba\}$ and *(d)* holds. □

For every integer n, L^n is a finite language which can be recognized by a reversible automaton (it is easy to check that every finite language can be recognized by a reversible automaton). We prove that $c(L^{2n}) \geq \rho^n$, while $m(L^{2n}) = O(n)$.

First observe that, for every positive integer n, $|L^n| = 3^n$, $L^n = (L^n)^{\sim}$ and every word of L^n has length $2n$. Furthermore it is easy to check that the minimal automaton recognizing L^n is the following one:



Thus the minimal automaton of L^n has $m(L^n) = 3n+1$ states. Moreover, if $\mathcal{A}$ is a trim automaton which recognizes a sublanguage of L^n, then for all pairs of states p and q of $\mathcal{A}$, all the words of $L_{p,q}$ have the same length.

Let $\mathcal{A} = (Q, A, E, \{i\}, \{f\})$ be a reversible automaton with a unique initial state and a unique final state recognizing a sublanguage of L^{2n}. Let $M(\mathcal{A}) = i.A^{2n}$ be the set of states of $\mathcal{A}$ we can reach from i by reading a word of length $2n$ (half of the length of the words of L^{2n}).

Proposition 1.3. *With the above notation, the following relation holds*

$$|L(\mathcal{A})| \leq \frac{1}{2}\left(\sum_{q\in M(\mathcal{A})} |L_q|^2 + |R_q^{\sim}|^2\right) \cdot \tag{1}$$

Proof. By definition we have:

$$L(\mathcal{A}) = \bigcup_{q\in M(\mathcal{A})} L_q R_q.$$

But $\mathcal{A}$ is deterministic and hence, if $p \neq q$ then $L_p \cap L_q = \emptyset$. Consequently, if $p, q \in M(\mathcal{A})$ then $L_pR_p \cap L_qR_q = \emptyset$. Indeed the prefix of length $2n$ of a word of $L_pR_p \cap L_qR_q$ would have to be in $L_p \cap L_q$. Thus we get:

$$|L(\mathcal{A})| = \sum_{q\in M(\mathcal{A})} |L_q||R_q|.$$

Since R_q is finite we have $|R_q| = |R_q^{\sim}|$. But for every pair of real numbers (x, y) we have $xy \leq \frac{1}{2}(x^2 + y^2)$. From this we deduce (1). $\square$

For the next proposition and the next three lemmas we consider a reversible automaton $\mathcal{A} = (Q, A, E, \{i\}, F)$ recognizing a sublanguage of L^n (where n is a strictly positive integer), and having a unique initial state i. For every state q of Q let us denote by n_q the cardinal of L_q.

Proposition 1.4. *The following equality holds:*

$$\sum_{q\in F} n_q^2 \leq 8^n. \tag{2}$$

To prove Proposition 1.4 we proced by induction on n and we need some technical lemmas. For the proof we can assume without loss of generality that $\mathcal{A}$ is trim.

Lemma 1.5. *If $n = 1$, Proposition 1.4 is true.*

Proof. Applying Lemma 1.2 to $\mathcal{A}$, we get:

In case *(a)*, $\sum_{q\in F} n_q^2 = |L_{i,s_1}|^2 + |L_{i,s_2}|^2 + |L_{i,s_3}|^2 = 3$.
In case *(b)*, $\sum_{q\in F} n_q^2 = |L_{i,s_1}|^2 + |L_{i,s_2}|^2 = 4 + 1 = 5$.
In case *(c)*, $\sum_{q\in F} n_q^2 = |L_{i,s_1}|^2 + |L_{i,s_2}|^2 = 1 + 1 = 2$.
In case *(d)*, $\sum_{q\in F} n_q^2 = |L_{i,s_1}|^2 = 4$.
In case *(e)*, $\sum_{q\in F} n_q^2 = |L_{i,s_1}|^2 = 1$.

This completes the proof. □

We denote by S the set of states which we can reach by reading a word of length $2n-2$ (every word of L^n has length $2n$).

Lemma 1.6. *For every q in F we have*

$$n_q = \sum_{s\in S} n_s |L_{s,q}|. \tag{3}$$

Proof. Since every word of $L(\mathcal{A})$ has length $2n$ we have

$$L_q = \bigcup_{s\in S} L_s L_{s,q}. \tag{4}$$

But $\mathcal{A}$ is deterministic, so as in the proof of Proposition 1.3 the union in (4) is disjoint. Since $n_q = |L_q|$ for every state q in Q, we can now deduce formula (1.6) from formula (4). □

We now prove that:

Lemma 1.7. *For all $s \in S$ we have*

$$\sum_{q\in F} \sum_{t\in S} |L_{s,q}||L_{t,q}| \leq 8. \tag{5}$$

Proof. Let $s \in S$ and let F_s be the set of states in F, which we can reach from s. If $q \in F$, we denote by I_q the set of states of S from which q can be reached. The sum in formula (5) is in fact equal to

$$\sum_{q\in F_s} \sum_{t\in I_q} |L_{s,q}||L_{t,q}|.$$

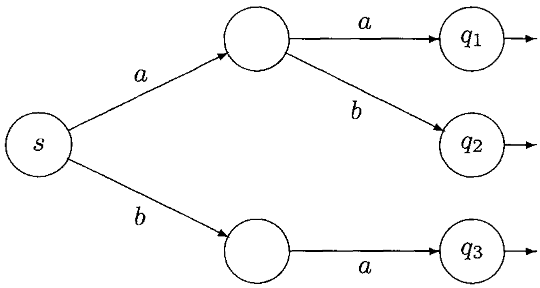
Since $\mathcal{A}$ is trim, the words which can be read from a state in S to a state in F, are all in L. Moreover, as $\mathcal{A}$ is reversible, if $q \in F$, then the languages $L_{t,q}$ $(t \in I_q)$ are pairwise disjoint and hence

$$\sum_{t\in I_q} |L_{t,q}| \leq 3.$$

Consider the automaton obtained from $\mathcal{A}$ by first making s the initial state and then trimming. This automaton satisfies the hypothesis of Lemma 1.2 and one of the following five cases arises.

(a) $|F_s| = 3$ and $|L_{s,q}| = 1$ for each $q \in F_s$.

We have, say, the following configuration



Note that there is no state $t \in I_{q_1}$ such that $t.b = s.a$: if there was, we would have $t.bb = q_2$, so $bb \in L_{t,q_2} \subseteq L$, a contradiction. Since $\mathcal{A}$ is trim, it follows that $ba \notin \bigcup_{t\in I_{q_1}} L_{t,q_1}$ and hence

$$\sum_{t\in I_{q_1}} |L_{t,q_1}| \leq 2.$$

We already observed that

$$\sum_{t\in I_{q_2}} |L_{t,q_2}| \leq 3 \text{ and } \sum_{t\in I_{q_3}} |L_{t,q_3}| \leq 3$$

so

$$\sum_{q\in F_s} \sum_{t\in I_q} |L_{s,q}||L_{t,q}| = \sum_{q\in F_s} \sum_{t\in I_q} |L_{t,q}| \leq 2+3+3 = 8.$$

(b) $F_s = \{q_1, q_2\}$ with $|L_{s,q_1}| = 2$ and $|L_{s,q_2}| = 1$. Since $\mathcal{A}$ is reversible we have either $I_{q_1} = \{s\}$ or $I_{q_1} = \{s, u\}$ with $|L_{u,q_1}| = 1$.

(b1) If $I_{q_1} = \{s\}$ we have

$$\sum_{t \in I_{q_1}} |L_{t,q_1}||L_{s,q_1}| = 4$$

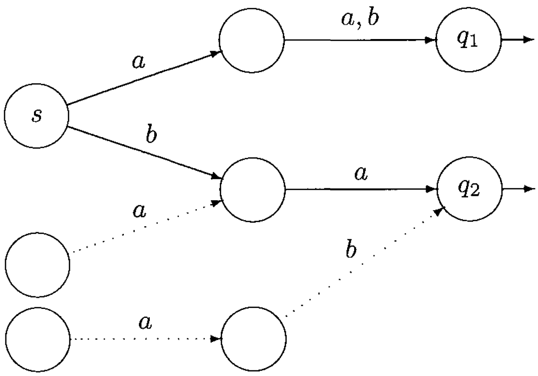
and

$$\sum_{t \in I_{q_2}} |L_{t,q_2}||L_{s,q_2}| = \sum_{t \in I_{q_2}} |L_{t,q_2}| \leq 3$$

so

$$\sum_{q \in F_s} \sum_{t \in I_q} |L_{s,q}||L_{t,q}| \leq 4 + 3 = 7.$$

This case can be illustrated in the following figure:



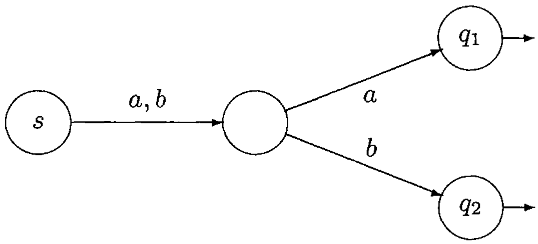
(b2) Assume that $I_{q_1} = \{s, u\}$. By reversibility, we have $L_{u,q_1} = L \setminus L_{s,q_1}$. Since $|L_{s,q_1}| = 2$, it follows that $|L_{u,q_1}| = 1$ and

$$\sum_{t \in I_{q_1}} |L_{s,q_1}||L_{t,q_1}| = 2\,(|L_{s,q_1}| + |L_{u,q_1}|) = 6.$$

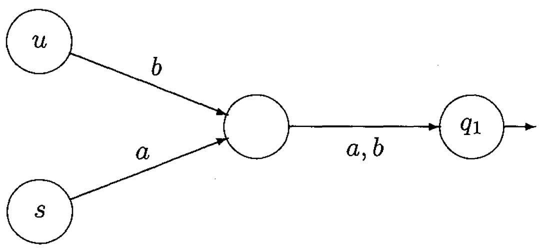
If $I_{q_2} = \{s\}$, we are finished:

$$\sum_{q \in F_s} \sum_{t \in I_q} |L_{s,q}||L_{t,q}| = 6 + 1 = 7.$$

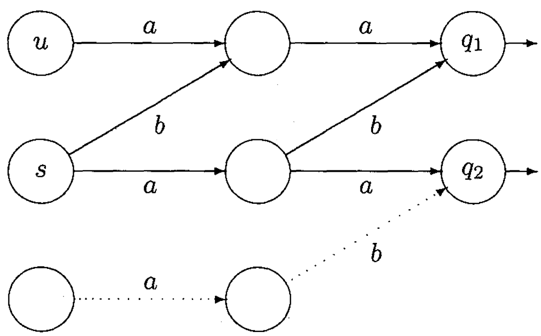
In order to go further, let us analyze this situation in more detail. If $L_{s,q_1} = \{aa, ba\}$, then by reversibility we have $s.b = s.a$. Moreover in that case $L_{s,q_2} = \{ab\}$, so that $(s.a).b = q_2$. This implies in turn that $s.bb = q_2$, $bb \in L_{s,q_2}$, a contradiction.



Now if $L_{s,q_1} = \{aa, ab\}$ then by determinism both the a-labeled and the b-labeled transitions into q_1 come from $s.a$. Moreover $L_{u,q_1} = \{ba\}$, so $u.b = s.a$ and we now have $u.bb = q_1$, $bb \in L_{u,q_1}$, again a contradiction.



Thus $aa \notin L_{s,q_1}$ and we have $L_{s,q_1} = \{ab, ba\}$, $L_{s,q_2} = L_{u,q_1} = \{aa\}$ and the following configuration



Suppose that $v \in I_{q_2}$, $v \neq s$. Then $aa \notin L_{v,q_2}$ by reversibility. Also $ba \notin L_{v,q_2}$: otherwise $v.b = s.a$ and $v.bb = q_1$. This implies $bb \in L_{v,q_1} \subseteq L$, a contradiction. Thus if $v \neq s$ lies in I_{q_2}, then $L_{v,q_2} = \{ab\}$. This shows that

$$\sum_{t \in I_{q_2}} |L_{s,q_2}||L_{t,q_2}| = \sum_{t \in I_{q_2}} |L_{t,q_2}| \leq 2$$

and hence

$$\sum_{q\in F_s}\sum_{t\in I_q}|L_{s,q}||L_{t,q}|\leq 8.$$

(c) $|F_s|=2$ and $|L_{s,q}|=1$ for each $q\in F_s$. Since $\sum_{t\in I_q}|L_{t,q}|\leq 3$ for each $q\in F$, we have

$$\sum_{q\in F_s}\sum_{t\in I_q}|L_{s,q}||L_{t,q}|=\sum_{q\in F_s}\sum_{t\in I_q}|L_{t,q}|\leq 2\times 3=6.$$

(d–e) $F_s=\{q\}$ and $|L_{s,q}|=2$ (resp. 1). Again using that $\sum_{t\in I_q}|L_{t,q}|\leq 3$, we find that

$$\sum_{q\in F_s}\sum_{t\in I_q}|L_{s,q}||L_{t,q}|\leq 6 \text{ (resp. 3)}.$$

□

Now we can prove Proposition 1.4.

Proof. We proceed by induction on n. The proposition is true for $n=1$ by Lemma 1.5.

Assume now that it is true for n. Let $\mathcal{A}=(Q,A,E,\{i\},F)$ be a reversible trim automaton with a unique initial state i recognizing a subset of L^{n+1}. According to Lemma 1.6

$$n_q^2=\sum_{(s,t)\in S^2}n_sn_t|L_{s,q}||L_{t,q}|. \tag{6}$$

Thus we have:

$$\begin{aligned}\sum_{q\in F}n_q^2&=\sum_{q\in F}\sum_{(s,t)\in S^2}n_sn_t|L_{s,q}||L_{t,q}|=\sum_{(s,t)\in S^2}\left(\sum_{q\in F}|L_{s,q}||L_{t,q}|\right)n_sn_t\\&\leq\frac{1}{2}\sum_{(s,t)\in S^2}\left(\sum_{q\in F}|L_{s,q}||L_{t,q}|\right)n_s^2+\frac{1}{2}\sum_{(s,t)\in S^2}\left(\sum_{q\in F}|L_{s,q}||L_{t,q}|\right)n_t^2\\&\leq\sum_{(s,t)\in S^2}\left(\sum_{q\in F}|L_{s,q}||L_{t,q}|\right)n_s^2.\end{aligned}$$

From this, by Lemma 1.7, we have

$$\sum_{q\in F}n_q^2\leq 8\sum_{s\in S}n_s^2.$$

Now we can apply the induction hypothesis to the automaton deduced from $\mathcal{A}$ by taking S for the set of final states rather than F, and by making it trim. We obtain:

$$\sum_{s\in S} n_s^2 \leq 8^n.$$

This establishes the proposition. □

Now we prove the main theorem.

Proof. Let n be a positive integer and let $\mathcal{A} = (Q, A, E, I, F)$ be a reversible automaton recognizing L^{2n}. For each $i \in I$ and $f \in F$ let us denote by $\mathcal{A}_{if}$ the automaton $(Q, A, E, \{i\}, \{f\})$.

It is clear that

$$L(\mathcal{A}) = \bigcup_{i\in I, f\in F} L(\mathcal{A}_{if}). \tag{7}$$

According to formula (1) we have

$$|L(\mathcal{A}_{if})| \leq \frac{1}{2}\left(\sum_{q\in M(\mathcal{A}_{if})} |L_q(\mathcal{A}_{if})|^2 + |R_q(\mathcal{A}_{if})^{\sim}|^2\right). \tag{8}$$

Now the reversible automata $(Q, A, E, \{i\}, M(\mathcal{A}_{if}))$ recognizes a sublanguage of L^n. By formula (2), we have

$$\sum_{q\in M(\mathcal{A}_{if})} |L_q(\mathcal{A}_{if})|^2 \leq 8^n.$$

Moreover, let us denote by $E' = \{(p, a_i, q) \mid (q, a_i, p) \in E\}$. The reversible automata $(Q, A, E', \{f\}, M(\mathcal{A}_{if}))$ also recognizes a sublanguage of L^n, and by (2), we have

$$\sum_{q\in M(\mathcal{A}_{if})} |R_q(\mathcal{A}_{if})^{\sim}|^2 \leq 8^n.$$

Thus, $|L(\mathcal{A}_{if})| \leq 8^n$ by formula 8.

By formula (7), it comes $|L(\mathcal{A})| \leq |I||F|8^n$. But $|L(\mathcal{A})| = 9^n$, so

$$|I||F| \geq \left(\frac{9}{8}\right)^n.$$

It follows that I or F contain a least r^n elements with $r = \frac{3\sqrt{2}}{4}$. Since I and F are subsets of Q we have $|Q| \geq r^n$.

We have proved that $c(L^{2n}) \geq r^n$. But $m(L^{2n}) = 6n + 1$. Hence $R(6n + 1) = \Omega(r^n)$ and, since $R(n)$ is an increasing function of n, this proves the theorem with $\rho = r^{\frac{1}{6}}$. □

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References

[1] D. Angluin, Inference of reversible languages. *J. ACM* **29** (1982) 741-765.
[2] J. Berstel, *Transductions and Context-Free Languages.* B.G. Teubner, Stuttgart (1979).
[3] M. Hall, A topology for free groups and related groups. *Ann. of Math.* **52** (1950).
[4] T. Hall, Biprefix codes, inverse semigroups and syntactic monoids of injective automata. *Theoret. Comput. Sci.* **32** (1984) 201-213.
[5] B. Herwig and D. Lascar, Extending partial automorphisms and the profinite topology on free groups. *Trans. Amer. Math. Soc.* **352** (2000) 1985-2021.
[6] S. Margolis and J. Meakin, Inverse monoids, trees, and context-free languages. *Trans. Amer. Math. Soc.* **335** (1993) 259-276.
[7] S. Margolis, M. Sapir and P. Weil, *Closed subgroups in pro-***V** *topologies and the extension problem for inverse automata.* Preprint (1998).
[8] S.W Margolis and J.-E. Pin, Languages and inverse semigroups, edited by J. Paredaens, *Automata, Languages and Programming, 11th Colloquium.* Antwerp, Belgium. Springer-Verlag, *Lecture Notes in Comput. Sci.* **172** (1984) 337-346.
[9] J.-E. Pin, Topologies for the free monoid. *J. Algebra* **137** (1991).
[10] J.-E. Pin, On reversible automata, edited by I. Simon, in *Proc. of Latin American Symposium on Theoretical Informatics (LATIN '92).* Springer, Berlin, *Lecture Notes in Comput. Sci.* **583** (1992) 401-416; A preliminary version appeared in the Proceedings of ICALP'87, *Lecture Notes in Comput. Sci.* **267**.
[11] P.V. Silva, On free inverse monoid languages. *RAIRO: Theoret. Informatics Appl.* **30** (1996) 349-378.
[12] B. Steinberg, *Finite state automata: A geometric approach.* Technical Report, Univ. of Porto (1999).
[13] B. Steinberg, *Inverse automata and profinite topologies on a free group.* Preprint (1999).

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