

JUHA HONKALA

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A KLEENE-SCHÜTZENBERGER THEOREM FOR LINDENMAYERIAN RATIONAL POWER SERIES*

JUHA HONKALA^{1, 2}

Abstract. We define L rational and L recognizable power series, and establish a Kleene-Schützenberger theorem for Lindenmayerian power series by showing that a power series is L rational if and only if it is L recognizable.

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1. INTRODUCTION

The study of formal power series constitutes an important area of theoretical computer science (see Eilenberg [2], Kuich and Salomaa [17]). The formal power series considered most often in connection with automata and languages are the rational series. One of the key results in the theory of rational series is Schützenberger’s theorem stating that a formal series is rational if and only if it is recognizable (see Salomaa and Soittola [20], Berstel and Reutenauer [1]). This theorem generalizes Kleene’s theorem giving the equivalence of finite deterministic automata and regular expressions.

In this paper we define Lindenmayerian rational and Lindenmayerian recognizable (shortly, L rational and L recognizable) power series. The definition of L rational series is obtained from the definition of rational series by replacing the Kleene star by morphic star. The basic idea in the definition of L recognizable series is that the coefficients of the terms of the series are computed by a matrix representation similarly as for recognizable series. The definition of L recognizable series is new; L rational series are also considered in Honkala and Kuich [14,15] (see also Honkala [8]).

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¹ Department of Mathematics, University of Turku, 20014 Turku, Finland;
e-mail: juha.honkala@utu.fi

² Turku Centre for Computer Science (TUCS), Lemminkäisenkatu 14, 20520 Turku, Finland.

The purpose of this paper is to prove that a power series r is L rational if and only if r is L recognizable. This gives a Kleene-Schützenberger theorem for L rational power series. In the proof of the theorem the close connection between growth functions of DTOL systems and \mathbf{N} -rational series plays an important role.

For further background and motivation we refer to Honkala [4-13], Honkala and Kuich [14,15] and the references given therein. A very different Kleene theorem for L algebraic power series is obtained in Ésik and Kuich [3].

It is assumed that the reader is familiar with the basics concerning semirings, formal power series and L systems (see Kuich and Salomaa [17], Rozenberg and Salomaa [18,19]). Notions and notations that are not defined are taken from these references.

2. L RATIONAL AND L RECOGNIZABLE SERIES

In this paper A will always be a commutative semiring. If Σ is a finite alphabet, the set of *formal power series* (resp. *polynomials*) with *noncommuting variables* in Σ and coefficients in A is denoted by $A \ll \Sigma^* \gg$ (resp. $A\langle \Sigma^* \rangle$).

Suppose that Σ and Δ are finite alphabets. A semialgebra morphism $h : A\langle \Sigma^* \rangle \rightarrow A\langle \Delta^* \rangle$ is called *semimonomial* if for each $\sigma \in \Sigma$ there exist $a \in A$ and $w \in \Delta^*$ such that $h(\sigma) = aw$. (Here it is possible that $a = 0$.) In what follows, if a morphism h is called semimonomial, it is always assumed that there exist finite alphabets Σ and Δ such that h is a semimonomial morphism from $A\langle \Sigma^* \rangle$ into $A\langle \Delta^* \rangle$ even if the alphabets Σ and Δ are not specified explicitly.

Suppose again that Σ is a finite alphabet and denote by Σ^0 the monoid $\Sigma^* \cup \{0\}$ obtained from the free monoid Σ^* by adjoining a zero. If $h : A\langle \Sigma^* \rangle \rightarrow A\langle \Delta^* \rangle$ is a semimonomial morphism the *underlying monoid morphism* $\bar{h} : \Sigma^0 \rightarrow \Delta^0$ of h is defined by

$$\bar{h}(\sigma) = \begin{cases} w & \text{if } h(\sigma) = aw \text{ and } a \neq 0 \\ 0 & \text{if } h(\sigma) = 0. \end{cases}$$

Next, assume that $h_i : A\langle \Sigma_i^* \rangle \rightarrow A\langle \Delta_i^* \rangle$, $1 \leq i \leq m$, are semimonomial morphisms where Σ_i and Δ_i are finite alphabets. Then, if $1 \leq i_1, \dots, i_k \leq m$ and w is a word or $w = 0$ we say that $h_{i_k} \dots h_{i_1}(w)$ exists if $w \in \Sigma_{i_1}^0$ and $h_{i_t} \dots h_{i_1}(w) \in \Sigma_{i_{t+1}}^0$ for all $1 \leq t \leq k - 1$.

In the sequel any finite set H of semimonomial (resp. monoid) morphisms is also regarded as a finite alphabet with $\text{card}(H)$ letters. Then, if $L \subseteq H^*$ is a language and $r \in A \ll \Sigma^* \gg$ we say that $L(r)$ exists if $u(w)$ exists whenever $u \in L$ and $w \in \text{supp}(r)$, and the family

$$\{u(w)\}_{u \in L, w \in \text{supp}(r)}$$

is locally finite. If $L(r)$ exists we denote

$$L(r) = \sum_{u \in L, w \in \text{supp}(r)} (r, w)u(w).$$

(Here $\varepsilon(w) = w$ if ε is the empty word.)

In what follows Σ_∞ will be a fixed countably infinite alphabet. We denote $\Sigma_\infty^0 = \Sigma_\infty^* \cup \{0\}$ and

$$A\{\{\Sigma_\infty^*\}\} = \bigcup_{\Sigma \subset \Sigma_\infty, \Sigma \text{ finite}} A \ll \Sigma^* \gg .$$

We are now in a position to define L rational series.

Definition 1. The family of Lindenmayerian rational (shortly, L rational) series over A is the least family $\mathcal{R} \subseteq A\{\{\Sigma_\infty^*\}\}$ satisfying the following conditions:

- (i) $A(\Sigma^*) \subseteq \mathcal{R}$ if $\Sigma \subset \Sigma_\infty$ is finite;
- (ii) if $r_1, r_2 \in \mathcal{R}$ then $r_1 + r_2 \in \mathcal{R}$ and $r_1 r_2 \in \mathcal{R}$;
- (iii) if $r \in \mathcal{R}$ and h is a semimonomial morphism, then $h(r) \in \mathcal{R}$ if $h(r)$ exists;
- (iv) if $r \in \mathcal{R}$ and $H = \{h_1, \dots, h_m\}$ is a set of semimonomial morphisms, then $H^*(r) \in \mathcal{R}$ if $H^*(r)$ exists.

The definition of L rational series is a modification of the definition of rational series obtained by replacing the Kleene star by morphic star. In this context it turns out to be necessary to postulate separately closure under morphic image; for rational series this is a consequence of the other conditions.

In Honkala and Kuich [14,15] arbitrary semialgebra morphisms are used instead of semimonomial morphisms in the definition of L rational series. The choice depends upon whether we are interested in generalizing DTOL or TOL languages.

Next we define L recognizable series. The basic idea is that an L recognizable series is a sum of terms obtained by morphic iteration where the coefficient of a given term can be computed by a matrix representation as in the definition of recognizable series (see Salomaa and Soittola [20], Berstel and Reutenauer [1]).

A *Lindenmayerian representation* (shortly, *L representation*) is a triple (T, μ, α) such that the following conditions hold:

- (i) There exist finite alphabets Σ and Δ , a finite set H of morphisms of the monoid Δ^0 , a morphism $h : \Delta^0 \rightarrow \Sigma^0$ and $w \in \Delta^0$ such that T is the mapping $T : H^* \rightarrow \Sigma^0$ defined by

$$T(u) = hu(w)$$

for all $u \in H^*$.

- (ii) There exists a positive integer k such that $\mu : H^* \rightarrow \mathbf{N}^{k \times k}$ is a morphism.
- (iii) $\alpha : \mathbf{N}^{k \times k} \rightarrow A$ is a morphism from the additive monoid of $\mathbf{N}^{k \times k}$ into the multiplicative monoid of A .

Note that if $\alpha : \mathbf{N}^{k \times k} \rightarrow A$ is as in (iii) there exist $a_{ij} \in A$, $1 \leq i, j \leq k$, such that

$$\alpha(M) = \prod_{1 \leq i, j \leq k} a_{ij}^{M_{ij}}$$

for $M \in \mathbf{N}^{k \times k}$. (As usual in group theory we have here $a^0 = 1$ for any $a \in A$.)

Now we can define L recognizable series and state the Kleene-Schützenberger theorem for L rational series.

Definition 2. A series $r \in A\{\{\Sigma_\infty^*\}\}$ is called Lindenmayerian recognizable (shortly, L recognizable) if there exists an L representation (T, μ, α) such that the family $\{\alpha\mu(u)T(u)\}_{u \in H^*}$ is locally finite and

$$r = \sum_{u \in H^*} \alpha\mu(u)T(u).$$

Theorem 1. A series $r \in A\{\{\Sigma_\infty^*\}\}$ is L rational if and only if r is L recognizable.

The proof of Theorem 1 requires several lemmas and will be given in the next section.

3. PROOFS

We show first that L rational series are L recognizable.

Lemma 1. Suppose $r \in A\{\{\Sigma_\infty^*\}\}$ is L rational. Then there exist a finite set H of semimonomial morphisms, a rational language $L \subseteq H^*$ and $w \in \Sigma_\infty^0$ such that $L(w)$ exists and

$$r = L(w).$$

Proof. The proof is by L rational induction. Suppose first that r is a polynomial. Choose a letter $\sigma \in \Sigma_\infty$. It is clear that there exists a finite set H of semimonomial morphisms such that $r = H(\sigma)$.

Suppose then that $r_1 = L_1(w_1)$ and $r_2 = L_2(w_2)$ where $L_1 \subseteq H_1^*$ and $L_2 \subseteq H_2^*$ are rational languages and $w_1, w_2 \in \Sigma_\infty^0$. Without restriction we assume that H_1 and H_2 are disjoint. First, let $\sigma \in \Sigma_\infty$ be a letter and choose two new semimonomial morphisms h_1, h_2 such that $h_1(\sigma) = w_1$ and $h_2(\sigma) = w_2$. Then

$$r_1 + r_2 = (L_1 h_1 \cup L_2 h_2)(\sigma).$$

Consider then the product $r_1 r_2$. Suppose first that there exist disjoint alphabets Σ_1 and Σ_2 such that $u(w_1) \in A \ll \Sigma_1^* \gg$ and $v(w_2) \in A \ll \Sigma_2^* \gg$ for all $u \in H_1^*$ and $v \in H_2^*$. Furthermore, suppose that each element $h_1 \in H_1$ satisfies $h_1(\sigma_2) = \sigma_2$ if $\sigma_2 \in \Sigma_2$ and each element $h_2 \in H_2$ satisfies $h_2(\sigma_1) = \sigma_1$ if $\sigma_1 \in \Sigma_1$. (If necessary we are free to change the pertinent images.) Then, if $u \in L_1$, $v \in L_2$, we have

$$uv(w_1 w_2) = u(w_1 v(w_2)) = u(w_1)v(w_2).$$

Hence $L_1 L_2(w_1 w_2)$ is defined and

$$r_1 r_2 = L_1 L_2(w_1 w_2).$$

Now the claim for the product in the general case follows by renaming. More precisely, we first replace r_1 and r_2 by letter-equivalent series s_1 and s_2 over

disjoint alphabets and then apply a letter-to-letter morphism to s_1s_2 to obtain r_1r_2 .

In the remaining cases the inductive step is clear. □

If $u = h_1h_2 \dots h_m \in H^*$ where $h_i \in H$ for $1 \leq i \leq m$, we denote $u^T = h_mh_{m-1} \dots h_1$. By definition, $\varepsilon^T = \varepsilon$. This notation is extended to languages by $L^T = \{u^T \mid u \in L\}$.

Lemma 2. *Suppose H is a finite set of semimonomial morphisms, $L \subseteq H^*$ is a rational language and $w \in \Sigma_\infty^0$. If $L(w)$ exists, there are finite alphabets Δ and Σ , semimonomial morphisms $g_1, \dots, g_m : A\langle\Delta^*\rangle \rightarrow A\langle\Delta^*\rangle$, $g : A\langle\Delta^*\rangle \rightarrow A\langle\Sigma^*\rangle$ and $w_1 \in \Delta^0$ such that*

$$L(w) = g(g_1 \cup \dots \cup g_m)^*(w_1).$$

Furthermore, $g(x) \in \Sigma^0$ for all $x \in \Delta$.

Proof. Without restriction we assume that there is a finite alphabet Δ_1 such that each $h \in H$ is from $A\langle\Delta_1^*\rangle$ into $A\langle\Delta_1^*\rangle$. Let $\mathcal{A} = (Q, \delta, q_0, F)$ be a finite deterministic automaton such that $L^T = L(\mathcal{A})$. (Here Q, δ, q_0 and F are the state set, transition function, initial state and final state set of \mathcal{A} , respectively.) Consider the elements of Q as new letters and extend each $h \in H$ by

$$h(q) = \delta(q, h)$$

for $q \in Q$. Let G be the set of the extended h s. Let g be a semimonomial morphism such that $g(q) = \varepsilon$ if $q \in F$ and $g(q) = 0$ if $q \in Q - F$. Furthermore, let g act as the identity for the other relevant letters. Then for any $u \in G^*$ we have

$$gu(q_0w) = g(\delta(q_0, u^T)u(w)) = \begin{cases} u(w) & \text{if } u \in L \\ 0 & \text{otherwise.} \end{cases}$$

Hence $gG^*(q_0w)$ is defined and

$$L(w) = gG^*(q_0w).$$

□

Now we have the tools to prove Theorem 1 in one direction.

Lemma 3. *If $r \in A\{\{\Sigma_\infty^*\}\}$ is L rational then r is L recognizable.*

Proof. Suppose $r \in A\{\{\Sigma_\infty^*\}\}$ is L rational. By Lemmas 1 and 2 there exist finite alphabets Δ and Σ , semimonomial morphisms $g_1, \dots, g_m : A\langle\Delta^*\rangle \rightarrow A\langle\Delta^*\rangle$, $g : A\langle\Delta^*\rangle \rightarrow A\langle\Sigma^*\rangle$ and $w_1 \in \Delta^0$ such that

$$r = g(g_1 \cup \dots \cup g_m)^*(w_1).$$

Furthermore, $g(x) \in \Sigma^0$ for all $x \in \Delta$. Denote $G = \{g_1, \dots, g_m\}$. If $1 \leq i \leq m$ and $x \in \Delta$, define the series $s(i, x) \in \mathbf{N} \ll G^* \gg$ by

$$\begin{aligned} (s(i, x), \varepsilon) &= 0 \\ (s(i, x), g_i u) &= (s(i, x), u) + |\bar{u}(w_1)|_x \\ (s(i, x), g_j u) &= (s(i, x), u) \quad \text{if } j \neq i \end{aligned}$$

for $u \in G^*$ where $|\bar{u}(w_1)|_x$ is the number of occurrences of x in $\bar{u}(w_1)$. Intuitively, $(s(i, x), u)$ counts the number of times g_i is applied to the letter x when we compute $u(w_1)$.

Next, for $1 \leq i \leq m, x \in \Delta$, denote

$$g_i(x) = a(i, x)\bar{g}_i(x), \tag{1}$$

where $a(i, x) \in A$. We claim that

$$u(w_1) = \prod_{1 \leq i \leq m, x \in \Delta} a(i, x)^{(s(i, x), u)} \bar{u}(w_1) \tag{2}$$

for $u \in G^*$. First, (2) is clear if $u = \varepsilon$. Then, if (2) holds for $u \in G^*$, we have for $g_j \in G$

$$\begin{aligned} g_j u(w_1) &= \prod_{1 \leq i \leq m, x \in \Delta} a(i, x)^{(s(i, x), u)} g_j(\bar{u}(w_1)) \\ &= \prod_{1 \leq i \leq m, x \in \Delta} a(i, x)^{(s(i, x), u)} \prod_{x \in \Delta} a(j, x)^{|\bar{u}(w_1)|_x} \bar{g}_j \bar{u}(w_1) \\ &= \prod_{1 \leq i \leq m, x \in \Delta} a(i, x)^{(s(i, x), g_j u)} \bar{g}_j \bar{u}(w_1). \end{aligned}$$

Hence (2) holds for all $u \in G^*$.

Now, if in (1) each $a(i, x)$ is regarded as a new letter, the same computation shows that (2) holds in the free commutative monoid generated by $\Delta \cup \{a(i, x) \mid 1 \leq i \leq m, x \in \Delta\}$. (Here, if $g_i(x) = 0$ for some $1 \leq i \leq m, x \in \Delta$, we take a new letter z and replace each $g_i(x) = 0$ by $g_i(x) = z$. Furthermore, the g_i s act as the identity on the new letters.) Hence there exist a positive integer k , a morphism $\eta : G^* \rightarrow \mathbf{N}^{k \times k}$, row vectors $\beta(i, x) \in \mathbf{N}^{1 \times k}$, $1 \leq i \leq m, x \in \Delta$, and a column vector $\beta \in \mathbf{N}^{k \times 1}$ such that

$$(s(i, x), u) = \beta(i, x)\eta(u)\beta$$

for $1 \leq i \leq m, x \in \Delta, u \in G^*$. Consequently,

$$(s(i, x), u) = \sum_{1 \leq p, q \leq k} \beta(i, x)_p \beta_q \eta(u)_{p, q}$$

and

$$u(w_1) = \prod_{1 \leq p, q \leq k} a_{pq}^{\eta(u)_{p,q}} \bar{u}(w_1)$$

for $u \in G^*$, where

$$a_{pq} = \prod_{1 \leq i \leq m, x \in \Delta} a(i, x)^{\beta(i,x)_p \beta_q} \tag{3}$$

for $1 \leq p, q \leq k$.

It is now easy to see that r is L recognizable by considering the L representation (T, μ, α) defined as follows. Let $H = \bar{G} = \{\bar{g}_1, \dots, \bar{g}_m\}$ and define $T : H^* \rightarrow \Sigma^0$ by

$$T(\bar{u}) = \bar{g}\bar{u}(w_1).$$

Define the morphism $\mu : H^* \rightarrow \mathbf{N}^{k \times k}$ by $\mu(\bar{x}) = \eta(x)$ for $x \in G$. Finally, define $\alpha : \mathbf{N}^{k \times k} \rightarrow A$ by

$$\alpha(M) = \prod_{1 \leq p, q \leq k} a_{pq}^{M_{pq}},$$

for $M \in \mathbf{N}^{k \times k}$, where a_{pq} is given by (3). Then

$$\alpha\mu(\bar{u})T(\bar{u}) = \bar{g}(u(w_1)) = gu(w_1)$$

for $u \in G^*$. Hence the family $\{\alpha\mu(v)T(v)\}_{v \in H^*}$ is locally finite and

$$r = \sum_{v \in H^*} \alpha\mu(v)T(v).$$

This concludes the proof that r is L recognizable. □

To prove that L recognizability implies L rationality one simple lemma is needed.

Lemma 4. *Suppose $w \in \Sigma_\infty^*$, $G = \{g_1, \dots, g_m\}$ is a set of semimonomial morphisms and g is a semimonomial morphism such that $r = gG^*(w)$ exists. Then r is L rational.*

Proof. Without restriction we assume that $g_j(\sigma) \in \Sigma_\infty^*$ for all $\sigma \in \Sigma$, $1 \leq j \leq m$. If $G^*(w)$ exists, the claim is an immediate consequence of Definition 1. In the general case, let $x \in \Sigma_\infty$ be a new letter and extend each $g_j \in G$ by $g_j(x) = x^2$. Furthermore, extend g by $g(x) = \varepsilon$. Then $G^*(wx)$ exists and is L rational. Hence also

$$r = gG^*(wx)$$

is L rational. □

Lemma 5. *If $r \in A\{\{\Sigma_\infty^*\}\}$ is L recognizable then r is L rational.*

Proof. Suppose (T, μ, α) is an L representation such that

$$r = \sum_{u \in H^*} \alpha \mu(u) T(u).$$

Here T, μ, α and H are as explained above in the definition of an L representation and

$$\alpha(M) = \prod_{1 \leq i, j \leq k} a_{ij}^{M_{ij}}$$

for $M \in \mathbf{N}^{k \times k}$ where $a_{ij} \in A, 1 \leq i, j \leq k$. Let $H = \{h_1, \dots, h_q\}$.

Because the series $\sum_{u \in H^*} \mu(u)_{ij} u$ are \mathbf{N} -rational for all $1 \leq i, j \leq k$, there exist a DTOL system $(\Sigma_1, f_1, \dots, f_q, w_1)$ and distinct letters $x(i, j) \in \Sigma_1$ for $1 \leq i, j \leq k$ such that

$$\mu(u)_{ij} = |v(w_1)|_{x(i,j)}$$

for all $u \in H^*, 1 \leq i, j \leq k$, where the word v is obtained from u by replacing each h_m by $f_m, 1 \leq m \leq q$. Here we may assume that $\Sigma \cup \Delta$ and Σ_1 are disjoint. For $1 \leq m \leq q$ define $g_m : A\langle(\Delta \cup \Sigma_1)^*\rangle \rightarrow A\langle(\Delta \cup \Sigma_1)^*\rangle$ by

$$g_m(x) = \begin{cases} h_m(x) & \text{if } x \in \Delta \\ f_m(x) & \text{if } x \in \Sigma_1. \end{cases}$$

Let $G = \{g_1, \dots, g_q\}$ and define $g : A\langle(\Delta \cup \Sigma_1)^*\rangle \rightarrow A\langle\Sigma^*\rangle$ by

$$g(x) = \begin{cases} h(x) & \text{if } x \in \Delta \\ a_{ij} \varepsilon & \text{if } x = x(i, j) \\ \varepsilon & \text{if } x \in \Sigma_1 - \{x(i, j) \mid 1 \leq i, j \leq k\}. \end{cases}$$

Then for $v \in G^*$ we have

$$\begin{aligned} gv(w_1 w) &= gv(w_1) gv(w) \\ &= \prod_{1 \leq i, j \leq k} a_{ij}^{|v(w_1)|_{x(i,j)}} gv(w) = \prod_{1 \leq i, j \leq k} a_{ij}^{\mu(u)_{ij}} T(u) = \alpha \mu(u) T(u) \end{aligned}$$

where $u \in H^*$ is the word obtained from $v \in G^*$ by replacing each g_m by $h_m, 1 \leq m \leq q$. Consequently,

$$r = gG^*(w_1 w).$$

Now the L rationality of r follows by Lemma 4. □

Theorem 1 now follows from Lemmas 3 and 5.

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