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B-catégories and gamuts

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B-CATEGORIES AND GAMUTS

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Introduction.

In [7] R. Street defined "gamuts" as the V-counterpart of cofibrations in V-Cat in order to recover modules as codiscrete cofibrations. In [5] we observed that automata seen as enriched categories ([2]) were an example of gamuts and that the behaviour-realization *local* adjunction could be proved for a general *biclosed* category V , and for arbitrary modules rather than just *endobimodules of the trivial V-catégorie* 1 .

We made however a further step towards generality in [3], where we described tree automata as categories enriched over a *bicategory* B , or rather as gamuts in B-mod. The behaviour module *is not* any longer an endobimodule, and a behaviour-realization theorem holds, very similar to the classical one. Thus, we deem that an exposition of the B-categorical machinery necessary to get to this generalization is useful. This is the content of §1, of a somewhat expository nature, and of §2, where the main results are given.

In §3 we illustrate examples; in particular, in example 3.2 we give a brief account of our categorical treatment of tree automata, following the line of [3] and providing motivation for our general result. Thence, we think that a very short and informal recall of the classical notion of tree automata has an appropriate place in this Introduction.

The classical theory of tree automata (see e. g. [10]) relies on the notion of Σ-algebra, where Σ is a set of operations symbols that, together with an "arity" function from Σ to the non negative integers, constitutes a *similarity type*; a

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Σ-algebra is then a set A (the *carrier* of the algebra) together with a map α assigning to each symbol h of arity (say) n in Σ an actual operation $\alpha_h : A^n \to A$. Nullary operations are called *constants*.

A *deterministic* Σ*-automaton* is essentially a Σ-algebra (the set of states of the automaton is given by the carrier and for each *input symbol* $h \in \Sigma$ the corresponding *transition map* is the operation α_h) plus a subset F (the *final states*) of A . The α-images of the constants are the *initial states*. The inputs of the automaton are given by the set T_Σ of *terms* (or *trees*) of the type Σ , that is the smallest subset of the free monoïd on Σ such that:

a) the constants are terms,

b) if f is an n-ary operation symbol and $t_1, \ldots, t_n \in T_\Sigma$, then $f(t_1, \ldots, t_n) \in T_\Sigma$.

Thus a term corresponds to a computation involving several operations of various arities, the sequence of which is often represented as a graph, namely a tree, where n-ary branchings visualize n-ary operations: the nodes can be either constants or *subtrees*, that is, according to the inductive definition above, again terms.

The notion of *non deterministic automaton* is obtained in a similar way from the notion of *relational algebra*, where transition maps are substituted by *transition relations* $\rho_h : A^n \to A$, relative to the input h .

We call *reachable states* (or definable elements) of a deterministic (resp. non deterministic) automaton the α-images (resp. ρ-images) of the terms. We say an automaton is *reachable* when all the states are reachable. The *behaviour* $\beta\mathbb{A}$ of a non deterministic automaton $\mathbb{A} = (A, \rho, F)$ is the set of trees recognized by $\mathbb{A}$, i. e. $\beta\mathbb{A} = \{t \in T_\Sigma \ / \ \rho(t) \cap F \neq \emptyset\}$.

Of course, since the "dynamics" of an automaton (i. e. forgetting terminal states) is just an algebra, all the notions above can be rephrased in terms of Lawvere's functorial semantics.

A *theory* T is a category whose objects are finite sets $[n] = \{1, \ldots, n\}$, $n = 0, 1, 2, \ldots$, and which admits the category of finite sets as a subcategory. The initial object is $[0] = \emptyset$ and $[1] = \{1\}$ is the terminal one. Notice further that $[m]$ is the m-fold coproduct of $[1]$, so that an arrow $a : [m] \to [n]$ is equivalent to an m-tuple of "injections" $a_i : [1] \to [n]$.

A T-algebra is then a product preserving functor $A : T^{op} \to \mathrm{Set}$ and a morphism of T-algebras is a natural transformation.

With this said, it is obvious to define a *deterministic automaton* as a pair given by a T-algebra and a set of final states F which is a subset of $A[1]$, where $A[1]$ is again the carrier of the algebra.
Non deterministic automata correspond then analogously to *relationnal T-algebra*, i. e. *lax functors* $A:T^{op}\to Rel(S)$ ($Rel(S)$ is the bicategory of sets and relations) sending coproducts in T to quasi-products in $Rel(S)$. We will describe in 3.2 a bicategory $B(T)$ constructed out of the theory T and such that T-algebras corresponds to $B(T)$-categories, while automata correspond to certain diagrams, to take into account final states and behaviour.
In fact, the behaviour itself is a functor and, as a consequence of our general result, it is possible to construct a "realization" functor which assigns to each behaviour a canonical automaton with a "quasi-universal" property which exhibits the realization as a very weak adjoint to the behaviour. This is the "best" analogue to the classical Goguen's behaviour-realization adjunction in this non deterministic context.

Finally, let us observe further that gamuts and cofibrations, crucial in the work of Rosebrugh and Wood ([6]), are even in this case an instance of gamuts in a bicategory of the form B-mod (see the remark at the end of §1).

1. Gamuts and B-categories.

Let B be a locally complete and cocomplete *biclosed* bicategory, i. e. with right liftings and right extensions.

Namely, given 1-cells with common domain $f:a\to c$ and $g:a\to d$, there is a 1-cell from c to d satisfying the universal property of a right Kan extension. The extension will be denoted by $[f,g]$.

As for the lifting of 1-cells with common codomain, it is defined to be an extension in B^{coop} (the bicategory obtained from B by reversing both the 1-cells and the 2-cells). The lifting will be denoted by $\{f,g\}$.

We recall here briefly the basic notions of B-category theory, referring the reader to (e. g.) [9] and [1] for further details and applications. Our general reference for bicategories is [1].

A B-*category* X is a set X together with a function $e: X \to \mathrm{Obj}B$ and a function $X(-,-): X \times X \to \mathrm{Morph}B$ such that:

(i) $X(x_1, x_2): e(x_1) \to e(x_2)$,

(ii) $\delta: 1_{e(x)} \to X(x,x)$,

(iii) $\mu: X(x_2, x_3).X(x_1, x_2) \to X(x_1, x_3)$,

satisfying the obvious axioms of left and right identities and associativities.
If x is an object of a B-category X, $e(x)$ is called the "underlying object" of x in B or also the "extent" of x. The 2-cells δ and μ are the identity and composition 2-cells. Notice that if we consider a monoïdal (biclosed) category V as a one object bicategory whose 1-cells are the "objects" of V and 2-cells are the "arrows" of V, then the definition above specializes to the classical notion of V-category.

Example ([11]). If L is a locale, the bicategory of relations $\mathrm{Rel}(L)$ is defined as follows:
- objects of $\mathrm{Rel}(L)$: opens u in L,
- arrows from u to v : elements $w \leq u \quad v$,
- 2-cells: order in L,
- composition of arrows: intersection.

In [11] Walters showed that the category Sheaves(L) is equivalent to a suitable subcategory of Cauchy-complete (see below for a definition) Rel(L)-categories.

A B-*functor* F from X to a B-category Y is a function $F: X \to Y$ such that:

(i) $e(F(x)) = e(x)$,

(ii) $F_{xx'}: X(x,x') \to Y(Fx, Fx')$.

The 2-cell $F_{xx'}$ is called the *strength* of the functor; notice again that the definition above generalizes the notion of V-functor.

We call B-cat the bicategory of B-categories and B-functors.

If X and Y are B-categories a *bimodule* $\phi: X \to\!\!\!\!+\; Y$ assigns:
- to each pair of objects x in X and y in Y over u, v respectively, a 1-cell $\phi(x,y): u \to v$ in B,
- to objects x, x' in X and y in Y, a 2-cell (right action) $\rho: \phi(x',y)X(x,x') \Rightarrow \phi(x,y)$,
- to objects x in X and y, y' in Y, a 2-cell (left action) $\lambda: Y(y,y')\phi(x,y) \Rightarrow \phi(x,y')$,

satisfying the axions expressing the compatibilities of the left and right actions.
Again this definition generalizes the notions of V-module and distributor. Further, as in the V-categorical case, a B-functor $F:X\to Y$ yields a pair of bimodules $F^*:X\text{-+->}Y$ and $F_*:Y\text{-+->}X$ given by:

- $F^*(x,y) = Y(Fx,y)$,
- $F_*(y,x) = Y(y,Fx)$.

Given bimodules $\phi,\psi:A\text{-+->}C$, a 2-cell $\theta:\phi\Rightarrow\psi$ is a family of 2-cells $\theta_{ac}:\phi(a,c)\Rightarrow\psi(a,c)$ in B compatible with the actions.
The composite $\psi\phi:A\text{-+->}X$ of bimodules $\phi:A\text{-+->}C$ and $\psi:C\text{-+->}X$ is defined as follows:

- $\psi\phi(a,x)$ is the colimit in $B(ea,ex)$ of the diagram

$$\psi(c,x)\phi(a,c) \xleftarrow[\rho.\phi(a,c)]{\qquad\qquad} (c',x)C(c,c')(a,c) \xrightarrow[\psi(c',x)]{\qquad\qquad} \psi(c',x)\phi(a,c')$$

where c, c' vary over all objects of C. The actions are the obvious ones induced by the actions of ϕ and ψ. Notice that the existence of the colimit is guaranteed by the assumption of the local cocompleteness of B. We write this colimit with the coend notation:

$$\int^{c}\psi(c,x).\phi(a,c).$$

Thus we can define the bicategory B-mod of B-categories and bimodules.
It is also possible to speak of *adjoint* bimodules.
An important example is provided by the bimodules F^* and F_* induced by a B-functor F: we have the adjonction $F_* \dashv F^*$.
A B-category is *Cauchy-complete* if every adjoint pair of bimodules $\phi,\psi:X\rightleftarrows Y$ arises from a functor from X to Y.
To test Cauchy-completeness is sufficient to consider bimodules to/from *trivial* B-categories, that is one object B-categories (say $u\hat{}$) over an object u, with $u\hat{}(*,*) = 1_u$.
Notice that in our assumptions, B-mod is again a biclosed bicategory. Indeed in the diagramm:

$$\begin{tikzcd}
A \arrow[rr, "\phi" description] \arrow[dr, "\psi"'] & \Leftarrow & C \arrow[dl, "{[\phi,\psi]}"] \\
& D &
\end{tikzcd}$$

the components $[\phi,\psi](c,d)$ of the bimodule $[\phi,\psi]$ are given by the equalizer of the following diagram:

$$\prod_a [\phi(a,c),\psi(a,d)] \rightrightarrows \prod_{a,a'} [\phi(a,c).A(a',a),\psi(a',d)]$$

and are 1-cells $e(c)\to e(d)$.
We will use for this equalizer the end notation:

$$[\phi,\psi](c,d) = \int_a [\phi(a,c),\psi(a,d)].$$

Notice also that the various forms of Yoneda lemma, involving integral notation, hold.

In [7] Ross Street defined gamuts in V-mod , which are actually defineable in any bicategory, being just diagrams defined by lax functors from the bicategory 3 .
So, a *gamut* in B-mod (or a B-gamut) from A to D is a diagram $(C,\phi,\psi,\theta,\lambda)$:

$$\begin{tikzcd}
& C \arrow[dr, "\psi"] \arrow[d, phantom, "\Downarrow\lambda"] & \\
A \arrow[ur, "\phi"] \arrow[rr, "\theta"'] & {} & D
\end{tikzcd}$$

A morphism of gamuts $(C,\phi,\psi,\theta,\lambda)\to(C',\phi',\psi',\theta',\lambda')$ is then given by (S,α,β,γ) where S is a bifunctor $C\to C'$ and α, β, γ are 2-cells $\alpha:\phi\Rightarrow S^*.\phi'$, $\beta:\psi.S^*\Rightarrow\psi'$, $\gamma:\theta\Rightarrow\theta'$ such that:

$$
\begin{tikzcd}
 & C \arrow[dr, "\psi"] \arrow[d, phantom, "\Downarrow\lambda"] & \\
A \arrow[ur, "\phi"] \arrow[rr, "\theta"] \arrow[rr, bend right=50, "\theta'"'] \arrow[rr, phantom, bend right=25, "\Downarrow\gamma"] & {} & D
\end{tikzcd}
=
\begin{tikzcd}
 & C \arrow[dr, "\psi"] & \\
A \arrow[ur, "\phi"] \arrow[r, "\phi'"] \arrow[rr, bend right=50, "\theta'"'] \arrow[rr, phantom, bend right=25, "\Downarrow\lambda'"] & C' \arrow[u, "S^*"] \arrow[r, "\psi'"] \arrow[u, phantom, shift right=4, "\alpha\Downarrow"] \arrow[u, phantom, shift left=4, "\Downarrow\beta"] & D
\end{tikzcd}
$$

where S^* denotes the right adjoint in the pair of modules induced by the functor S .

As an aside, we mention the fact that the result of Street ([7]) concerning the (bi)equivalence between the (bi)category of V-gamuts from A to D and the (bi)category of cofibrations from A to D can be extended to the B-categorical context. However this is beyond the scope of the present paper.

Gamuts have been also considered by Rosebrugh and Wood in the context of their "proarrow equipments" ([6]) Top→LTop , where Top is the bicategory of topoï and geometric morphisms and LTop is the bicategory of topoï and left exact functors.

What is interesting for us is that this is one example of gamuts in bicategories *of the form* B-mod . In fact, by Barr's theorem (see [4], p. 252) BLTop (boolean topoï) is a *dense* subbicategory of LTop , and we have LTop ≃ BLTop-mod_{fin} , where "$_{fin}$" means that we are considering just *finite* BLTop-categories.

2. The canonical decomposition.

For each small category A , there exists a B-category PA which represents modules, in the sense that, for modules $\theta: A \dashrightarrow C$, we have:

$$
(1) \qquad \frac{\theta: A \dashrightarrow C}{T: C \to PA}
$$

Notice that $T(c)$ is $\theta(-,c)$, which is a module from A to $e^{\wedge}(c)$.

The B-category PA is defined as follows (see also [9]):

- objects are modules $\phi: A \mathrel{-\!\!+\!\!\!\to} u^{\wedge}$, u ranging in B, and for $\psi: A \mathrel{-\!\!+\!\!\!\to} v^{\wedge}$ we set, using the biclosed structure of B-mod,

$$PA(\phi,\psi) = [\phi,\psi] = \int_m [\phi_m,\psi_m].$$

Since we will consider also gamuts with a large "top" category (in fact PA), we have to suppose B embedded in a B' corresponding to a higher universe Set', in such a way that the inclusion of B in B' preserves local limits and Set'-small local colimits.

Let us recall now some classical propositions of V-category theory (see [5]) which extend easily to the B-categorical case.

Proposition 2.1. *Let* $F: A \to C$ *be a fully faithful functor between two small* B*-categories and let* ϕ *be a module from* A *to* D. *Then we have* $F_*[F_*,\phi] \simeq \phi$.

Proposition 2.2. *Given a module* $\theta: A \mathrel{-\!\!+\!\!\!\to} C$, *the gamut below has an invertible 2-cell:*

$$\begin{tikzcd}
& PA \arrow[dr, "T^*", "|" marking] \arrow[d, phantom, "\Downarrow\wr"] & \\
A \arrow[ur, "Y_*", "|" marking] \arrow[rr, "\theta", "|" marking] & & C
\end{tikzcd}$$

(here T *is the functor corresponding to* θ *in (1) and* Y *is the Yoneda embedding).*

Notice that the proofs rely essentialy on the validity of the B-categorical forms of Yoneda lemma.

The gamut of Proposition 2.2 is called *canonical decomposition of the module* θ.

Obviously, to any gamut from A to C is possible to assign canonically a module, the bottom one. This process amounts to a "forgetful" homomorphism:

$$F\colon B\text{-gam}(A,C)\to B\text{-mod}(A,C)$$

We define also a homomorphism:

$$R\colon B\text{-mod}(A,C)\to B\text{-gam}(A,C)$$

sending a module θ to its canonical decomposition and a 2-cell $\gamma\colon\theta\Rightarrow\theta'$ to the morphism of gamuts $R\gamma = (1_{FA}, \mathrm{id}, \gamma^*, \gamma)$, where $\gamma^*\colon T^*\Rightarrow T'^*$.
Observe that $B\text{-mod}(A,C)$ is a discrete bicategory.

Recalling now (see [2] for the details) that a homomorphism of bicategories $\Omega\colon V\to D$ has a right *local adjoint* $\Delta\colon D\to V$ if for each v in V and d in D there is an adjunction $\Delta_{vd} \dashv \Omega_{vd}$:

$$D(\Omega v, d) \underset{\Omega_{vd}}{\overset{\Delta_{vd}}{\leftrightarrows}} V(v, \Delta d)$$

with appropriate naturality conditions, we have the following:

Theorem 2.3. *The homomorphism* $F\colon B\text{-gam}(A,C)\to B\text{-mod}(A,C)$ *has a right local adjoint* $R\colon B\text{-mod}(A,C)\to B\text{-gam}(A,C)$.

Proof. We have to prove that, for arbitrary θ in $B\text{-mod}(A,C)$ and $X = (D, \phi, \psi, \theta', \lambda)$ in $B\text{-gam}(A,C)$, it holds:

$$B\text{-mod}(FX, \theta) \underset{R_{X\theta}}{\overset{F_{X\theta}}{\leftrightarrows}} B\text{-gam}(X, R\theta)$$

with $R_{X\theta} \dashv F_{X\theta}$.
Let $\gamma\colon\theta'\Rightarrow\theta$ in $B\text{-mod}(FX,\theta)$. Notice that since the latter is just a set, if $(S,\alpha,\beta,\lambda)\colon X\to R\theta$ is a morphism of gamuts, to say that $R_{X\theta}$ is left adjoint to $F_{X\theta}$ amounts to the following:

$$\frac{R_{X\theta}(\bar{\gamma})\Rightarrow(S,\alpha,\beta,\gamma)}{\bar{\gamma}\Rightarrow\gamma}$$

where $\bar{\gamma}\to\gamma$ is in a *discrete* category, so either $\bar{\gamma} = \gamma$ or there is no 2-cell. So there is exactly one 2-cell $R_{X\theta}(\bar{\gamma})\Rightarrow(S,\alpha,\beta,\gamma)$ iff $\bar{\gamma} = \gamma$. This is to say that for each $\gamma\colon\theta'\Rightarrow\theta$, the full subcategory of $B\text{-gam}(X,R)$ given by the $(S,\alpha,\beta,\gamma)\colon X\to R\theta$ with *this* γ , has an initial object. It is easily checked that (just

like in [5], theorem 9) this is obtained observing that to give a morphism of gamuts $(S,\alpha,\beta,\gamma):X\to R\theta$ is to give the module $\sigma:A-{+}->D$ corresponding to S under the bijection (1), plus 2-cells $\alpha:\phi\Rightarrow\sigma$, $\epsilon:\psi\sigma\Rightarrow\theta$ and $\gamma:\theta'\Rightarrow\theta$.
We have the initial object by putting $\sigma = \phi$, $\alpha = id$, $\epsilon = \gamma\lambda$.

3. Examples.

3.1. The one object categories A in B are to be thought of as *monads* (v,m) with $m = A(a,a)$ on $v = e(a)$ in B.
Then the fibre $(PA)_u$ is the $B(u,u)$-category of *m-opalgebras with codomain* u. So PA is the (large) B-category of opalgebras with *all* possible codomains.
We can define a morphism of opalgebras with *different* codomains $t:v\to u$ and $s:v\to w$ as $[t,s]$ in B. This is the hom in PA.
Now $Y_*(a,t) = PA(Ya,t) = [m,t]$.
Given a module $\sigma:A-{+}->C$, where $C = (u,n)$, the corresponding functor $S: C\to PA$ is the 1-cell $\sigma:v\to u$.
So $S^*(t,c) = PA(t,\sigma) = [t,\sigma]$ and

$$Y_*S^*(a,c) = \int^{t} PA(m,t)\cdot PA(t,\sigma) \cong PA(m,\sigma) = [m,\sigma]$$

and, by Proposition 2.2, $[m,\sigma] \simeq \sigma$.

3.2. In [3] tree automata are described as categories enriched on a bicategory. We already recall in the Introduction that a (non deterministic) tree automaton is a (relational) T-algebra (plus the initial and terminal states), where T is a Lawvere's theory, i. e. a category whose objects are finite sets and which admits the category of finite sets as a subcategory.
Starting from the theory T, a bicategory $B(T)$ is defined as follows:
- $B(T)$ has the same objects as T,
- the 1-cells from u to v in $B(T)$ are the subsets of $T(u,v)$ and 2-cells are inclusions,
- composition of 1-cells and identities are the obvious one.

B(T) satisfies the assumptions of §1, in particular it is biclosed, with

$$\begin{tikzcd} & v \arrow[dd, "{\in[R,S]}"] \\ u \arrow[ur, "R"] \arrow[dr, "S"'] \arrow[r, phantom, "\Leftarrow"] & {} \\ & w \end{tikzcd}$$

given by $[R,S] = \{k: v \to w \text{ in } T \text{ such that } kR \subseteq S\}$.
In [3] it is shown that the category of "reachable" T-algebras is isomorphic to the category of "reachable" B(T)-categories.
A tree automaton is defined as a gamut:

$$\begin{tikzcd} & X \arrow[dr, "I", "\shortmid" marking] \arrow[d, phantom, "\Downarrow\iota"] & \\ {[1]^\wedge} \arrow[ur, "T", "\shortmid" marking] \arrow[rr, "T.I", "\shortmid" marking] & {} & {[0]^\wedge} \end{tikzcd}$$

where X is a B(T)-category (i. e. an algebra) and $[n]^\wedge$ denotes the trivial category over $[n]$. If $b = (b_1, \ldots, b_n)$ is an X-object over $[n]$, then $I(b) = X(b, x_0)$, i. e. it provides n-tuples of trees (x_0 denotes here the *unique* X-object over 0), while $T(b) = \{g \in T([1],[n]) \ / \ \exists\, a \in F \subseteq X_{[1]} \text{ and } g \in X(a,b)\}$, where F is the set of final states, so that T determines sets of operations which are succesful if performed on those trees (see [3] for all the details). The composite module $T.I$ is now the *behaviour* of the automaton (the set of trees computed by the operations of the automaton which are recognizable, i. e. which belong to the set of final states).
We can apply here our theorem on local adjoints, where the right local adjoint to the "behaviour" (i. e. the forgetful functor) is the *realization* functor, which associates to a behaviour $\theta: [1]^\wedge \text{-+->} [0]^\wedge$ the gamut:

$$
\begin{tikzcd}
 & P[1]^{\wedge} \arrow[dr, "T^*"] & \\
{[1]^{\wedge}} \arrow[ur, "Y_*"] \arrow[rr, "\theta"] & & {[0]^{\wedge}}
\end{tikzcd}
$$

This result is a generalization of both [2] and [5], and provides furthermore a motivation for the "abstract" theorem of [5], where modules were taken between two *different* V-categories, although in the leading example of ordinary automata they both coïncide with the trivial category [1]^. In fact, in the case of tree automata the "behaviour" module is *necessary* defined between two different categories, namely [1]^ and [0]^ .

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