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IVAN KOLAR

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ON SPECIAL TYPES OF NONHOLONOMIC JETS

by Ivan KOLAR

Résumé. Le concept d'un type spécial de r-jets non-holonomes et de (k, r)-vitesses non-holonomes est discuté d'un point de vue général. Une attention particulière est porte à la composition de r-jets non-holonomes du même type. Les cas où le produit est préservé sont caractérisés en terme d'algèbres de Weil.

Let $\mathcal{M}f$ be the category of all manifolds and all smooth maps, $\mathcal{M}f_l$ be the category of all manifolds and local diffeomorphisms and $\mathcal{M}f_m$ be the category of m-dimensional manifolds and local diffeomorphisms. The constructions $J^r(M, N)$ of holonomic r-jets and $\widetilde{J}^r(M, N)$ of non-holonomic r-jets between two manifolds M, N are bundle functor on the product category $\mathcal{M}f_l \times \mathcal{M}f$, provided for every smooth map $f : N \to \overline{N}$, every local diffeomorphism $g : M \to \overline{M}$ and every $Z \in \widetilde{J}^r_x(M, N)_y$ we define

$$\widetilde{J}^r(g, f)(Z) = (j^r_y f) \circ Z \circ (j^r_x g)^{-1}, \tag{1}$$

where $\circ$ denotes the composition of nonholonomic r-jets, [3], [6]. Further, $J^r(g, f)$ is the restriction and corestriction of $\widetilde{J}^r(g, f)$ to the holonomic subbundles, [7]. Clearly, both functors J^r and $\widetilde{J}^r$ preserve products in the second factor, i.e.

$$\widetilde{J}^r_x(M, N_1 \times N_2) = \widetilde{J}^r_x(M, N_1) \times \widetilde{J}^r_x(M, N_2), \qquad x \in M, \tag{2}$$

and the same holds in the holonomic case. Many special types of nonholonomic r-jets were studied in differential geometry, [5], [9], the semiholonomic r-jets, introduced already by C. Ehresmann, [3], are the best known example.

In [8], we described the bundle functors on $\mathcal{M}f_m \times \mathcal{M}f$ preserving products in the second factor by means of Weil algebras. Using this

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point of view, we introduced the general concept of r-jet functor E as a bundle functor on $\mathcal{M}f_m \times \mathcal{M}f$ satisfying $J^r \subset E \subset \widetilde{J}^r$ and preserving products in the second factor, [5]. Then we deduced a complete characterization of E in terms of Weil algebras. However, the composition of nonholonomic r-jets cannot be included into this framework. That is why we discuss the composition problem in the present paper.

In Section 1 we summarize the basic properties of nonholonomic r-jets and nonholonomic (k,r)-velocities from the functorial point of view. In Section 2 we treat a general nonholonomic r-jet m-functor F_m on $\mathcal{M}f_m \times \mathcal{M}f$ and a general nonholonomic r-velocity m-functor V_m on $\mathcal{M}f$ without the assumption of preserving products. Such functors are said to be "weak". A weak total r-jet functor is defined on the category $\mathcal{M}f_I \times \mathcal{M}f$ and has the composition property. In Proposition 1 we present a simple condition for a sequence (F_m), $m \in \mathbb{N}$, of weak r-jet m-functors to form a weak total r-jet functor F. Section 3 is devoted to another characterization of a weak total r-jet functor in terms of a sequence (V_m), $m \in \mathbb{N}$, of weak r-velocity m-functors. In Section 4 we add the condition of preserving products. The general theory of Weil bundles, [6], [7], yields directly a characterization of an r-jet m-functor F_m by means of a Weil algebra $\mathbb{D}^F_m$. (We remark that this approach is close to the synthetic differential geometry, [4].) In Proposition 4 we deduce a complete description of a total r-jet functor F in terms of a sequence $(\mathbb{D}^F_m)$, $m \in \mathbb{N}$, of Weil algebras and their homomorphisms.

All manifolds and maps are assumed to be infinitely differentiable. Unless otherwise specified, we use the terminology and notation from the book [7].

1. Nonholonomic jets and velocities.

The r-th nonholonomic prolongation $\widetilde{J}^rY$ of a fibered manifold $Y \to M$ is defined by the iteration

$$\widetilde{J}^rY = J^1(\widetilde{J}^{r-1}Y \to M),$$

$\widetilde{J}^1Y = J^1Y$. The canonical inclusion $J^rY \hookrightarrow \widetilde{J}^rY$ is determined by $j^r_x s \longmapsto j^1_x(u \mapsto j^{r-1}_u s)$ for every local section s of Y, $u \in M$. The space $\widetilde{J}^r(M,N)$ of nonholonomic r-jets of M into N is the r-th nonholonomic prolongation of the product fibered manifold $M \times N \to M$. So we have $J^r(M,N) \subset \widetilde{J}^r(M,N)$. Let Q be a third manifold. Ehresmann introduced the composition of nonholonomic r-jets by the following induction, [3]. For $r = 1$, we have the composition of 1-jets. Write $\beta : \widetilde{J}^{r-1}(M, N) \to N$ for the canonical projection. Let $X = j^1_x s(u) \in \widetilde{J}^r_x(M, N)_y$, $u \in M$, and $Z = j^1_y \sigma \in \widetilde{J}^r_y(N, Q)_z$, $y = \beta\big(s(x)\big)$. Then

$$Z \circ X := j^1_x\big(\sigma(\beta(s(u))) \circ s(u)\big) \in \widetilde{J}^r_x(M, Q)_z \tag{3}$$

with the composition of nonholonomic $(r-1)$-jets on the right hand side. If X and Z are holonomic r-jets, then (3) coincides with the classical jet composition. Using induction, one deduces directly from (3) that the composition of nonholonomic r-jets is associative. We write

$$\widetilde{L}^r_{m,n} = \widetilde{J}^r_0(\mathbb{R}^m, \mathbb{R}^n)_0 , \quad \widetilde{L}^r = \bigcup_{m,n \in \mathbb{N}} \widetilde{L}^r_{m,n} .$$

Hence $\widetilde{L}^r$ is the skeleton of the category of nonholonomic r-jets. The composition maps will be denoted by

$$\varkappa_{m,n,q} : \widetilde{L}^r_{m,n} \times \widetilde{L}^r_{n,q} \to \widetilde{L}^r_{m,q} . \tag{4}$$

The functor $\widetilde{T}^r_k$ of nonholonomic (k, r)-velocities is defined by

$$\widetilde{T}^r_k M = \widetilde{J}^r_0(\mathbb{R}^k, M) , \quad \widetilde{T}^r_k f(Z) = (j^r_x f) \circ Z \tag{5}$$

for every manifold M, every map $f : M \to N$ and every $Z \in (\widetilde{T}^r_k M)_x$. By (2), $\widetilde{T}^r_k$ is a product preserving bundle functor on $\mathcal{M}f$. Every $X \in \widetilde{L}^r_{l,k}$ defines a natural transformation (denoted by the same symbol)

$$X : \widetilde{T}^r_k \to \widetilde{T}^r_l , \quad X_M(Z) = Z \circ X , \quad Z \in (\widetilde{T}^r_k M)_x . \tag{6}$$

Indeed, $X_M\big(\widetilde{T}^r_k f(Z)\big) = \big((j^r_x f) \circ Z\big) \circ X = (j^r_x f) \circ (Z \circ X)$ by the associativity of the composition of nonholonomic r-jets.

Replacing $\widetilde{J}^r_0(\mathbb{R}^k, M)$ by $J^r_0(\mathbb{R}^k, M)$, we obtain the classical functor T^r_k of holonomic (k, r)-velocities, [2], [7]. Even T^r_k preserves products. In the holonomic case, all natural transformations $T^r_k \to T^r_l$ are of the form (6) with $X \in L^r_{l,k} = J^r_0(\mathbb{R}^l, \mathbb{R}^k)_0$. However, in the nonholonomic case there are further natural transformations beside (6). The simpliest example is the case of the functor $\widetilde{T}^2_1$, which is naturally isomorphic to the iterated tangent functor TT. Using the general procedure for finding the homomorphisms of Weil algebras, one evaluates easily all natural transformations $TT \to TT$, [6]. Then one sees directly that not all of them are of the form (6).

A fundamental result reads that every product preserving bundle functor F on $\mathcal{M}f$ is a Weil functor T^A, whose Weil algebra is $A = F\mathbb{R}$, [6], [7]. The Weil algebra of T_k^r is $\mathbb{D}_k^r = J_0^r(\mathbb{R}^k, \mathbb{R})$. Using translations on $\mathbb{R}^k$, one identifies $\widetilde{T}_k^r$ with $T_k^1 \circ \cdots \circ T_k^1$. Hence the Weil algebra $\widetilde{\mathbb{D}}_k^r = \widetilde{J}_0^r(\mathbb{R}^k, \mathbb{R})$ is identified with the tensor product

$$\widetilde{\mathbb{D}}_k^r = \mathbb{D}_k^1 \underbrace{\otimes \cdots \otimes}_{r\text{-times}} \mathbb{D}_k^1 .$$

The canonical projection $\mathbb{D}_k^1 \to \mathbb{R}$ combined with the identities on the other factors defines r projections

$$\pi_i : \widetilde{\mathbb{D}}_k^r \to \mathbb{D}_k^1 \underbrace{\otimes \cdots \otimes}_{(r-1)\text{-times}} \mathbb{D}_k^1 = \widetilde{\mathbb{D}}_k^{r-1} , \quad i = 1, \dots, r .$$

The Weil algebra $\overline{\mathbb{D}}_k^r = \overline{J}_0^r(\mathbb{R}^k, \mathbb{R})$ corresponding to the semiholonomic case is the subalgebra of all elements $Z \in \widetilde{\mathbb{D}}_k^r$ satisfying $\pi_i(Z) = \pi_j(Z)$ for all $i, j = 1, \dots, r$.

2. Weak velocity and jet functors.

For a bundle functor E on $\mathcal{M}f$, the condition $T_m^r \subset E \subset \widetilde{T}_m^r$ means

$$J_0^r(\mathbb{R}^m, N)_x \subset (EN)_x \subset \widetilde{J}_0^r(\mathbb{R}^m, N)_x , \qquad x \in N ,$$
$$Ef(Z) = (j_x^r f) \circ Z , \qquad Z \in (EN)_x$$

for every manifold N and every map $f : N \to \overline{N}$.

Let G_m^r be the r-th jet group in dimension m.

Definition 1. A weak r-velocity m-functor V_m is a bundle functor on $\mathcal{M}f$ such that $T_m^r \subset V_m \subset \widetilde{T}_m^r$ and

(7) $\qquad Z \in (V_m N)_x , \quad X \in G_m^r \quad \text{implies} \quad Z \circ X \in (V_m N)_x .$

Hence we have an action of G_m^r on each $(V_m N)_x$. In particular, for every m-dimensional manifold M we can construct the associated fiber bundle $P^r M[V_m N]$, where $P^r M$ is the r-th order frame bundle of M.

For a bundle functor E on $\mathcal{M}f_m \times \mathcal{M}f$, the condition $J^r \subset E \subset \widetilde{J}^r$ means

$$J_x^r(M, N)_y \subset E_x(M, N)_y \subset \widetilde{J}_x^r(M, N)_y , \qquad x \in M, \ y \in N ,$$
$$E(g, f)(Z) = (j_y^r f) \circ Z \circ (j_x^r g)^{-1} \qquad Z \in E_x(M, N)_y ,$$

for every map $f : N \to \overline{N}$ and every local diffeomorphism $g : M \to \overline{M}$.

Definition 2. A weak r-jet m-functor F_m is a bundle functor on $\mathcal{M}f_m \times \mathcal{M}f$ satisfying $J^r \subset F_m \subset \widetilde{J}^r$.

Given V_m, we define a bundle functor jV_m on $\mathcal{M}f_m \times \mathcal{M}f$ by

$$jV_m(M, N) = P^r M[V_m N], \quad jV_m(g, f) = P^r g[V_m f],$$

where $P^r g : P^r M \to P^r \overline{M}$ is the principal bundle morphism induced by g. According to (6) and (7), $V_m f$ is a G^r_m-equivariant map, so that $P^r g[V_m f]$ is a well defined morphism of associated bundles.

Given F_m, we define a bundle functor vF_m on $\mathcal{M}f$ by

$$vF_m(N) = (F_m)_0(\mathbb{R}^m, N), \quad (vF_m)f(Z) = F_m(\mathrm{id}_{\mathbb{R}^m}, f)(Z),$$

$Z \in vF_m(N)$. For $X = j^r_0\gamma \in G^r_m$, we have

$$Z \circ X = F_m(\gamma^{-1}, \mathrm{id}_N)(Z),$$

so that vF_m is a weak r-velocity m-functor. Thus, we have established a bijection between the weak r-velocity m-functors and weak r-jet m-functors.

Since $\mathcal{M}f_I \times \mathcal{M}f$ is the union of $\mathcal{M}f_m \times \mathcal{M}f$ for all $m \in \mathbb{N}$, every bundle functor F on $\mathcal{M}f_I \times \mathcal{M}f$ is a sequence $F = (F_m)$ of bundle functors on $\mathcal{M}f_m \times \mathcal{M}f$.

Definition 3. A weak total r-jet functor is a sequence $F = (F_m)$ of weak r-jet m-functors with the composition property: if $Z_1 \in F_x(M, N)_y$ and $Z_2 \in F_y(N, Q)_z$, then $Z_2 \circ Z_1 \in F_x(M, Q)_z$.

If we write

$$L^F_{m,n} = F_0(\mathbb{R}^m, \mathbb{R}^n)_0, \qquad L^F = \bigcup_{m,n\in\mathbb{N}} L^F_{m,n},$$

then L^F is a category satisfying $L^r \subset L^F \subset \widetilde{L}^r$. The elements of

$$J^F(M, N) := F_m(M, N), \qquad m = \dim M,$$

will be called F-jets of M into N. Every element $Z \in J^F_x(M, N)_y$ is of the form

$$Z = v \circ X \circ u^{-1}, \quad u \in P^r_x M, \ v \in P^r_y N, \ X \in L^F_{m,n},$$

$n = \dim N$. Given another $\overline{Z} \in J_y^F(N, Q)_z$, $\overline{Z} = w \circ \overline{X} \circ v^{-1}$, $w \in P_z^r Q$, $\overline{X} \in L_{n,q}^F$, we have

$$\overline{Z} \circ Z = w \circ (\overline{X} \circ X) \circ u^{-1} .$$

This defines the category of F-jets and L^F is its skeleton.

Conversely, if we have a subcategory $L^r \subset \mathscr{L} \subset \widetilde{L}^r$, we construct a weak total r-jet functor $\mathcal{F}$ as follows. The jet composition yields an action of $G_m^r \times G_n^r$ on $\mathscr{L}_{m,n}$ and we define $\mathcal{F}(M, N)$ to be the associated fiber bundle

$$\mathcal{F}(M, N) = \big(P^r M \times P^r N\big)[\mathscr{L}_{m,n}] ,$$

$m = \dim M$, $n = \dim N$. Given a map $f : N \to \overline{N}$, a local diffeomorphism $g : M \to \overline{M}$ and $Z \in F_x(M, N)_y$, we can write $Z = v \circ X \circ u^{-1}$, $j_y^r f = \overline{v} \circ X_2 \circ v^{-1}$, $j_x^r g = \overline{u} \circ X_1 \circ u^{-1}$, $\overline{v} \in P_{f(y)}^r \overline{N}$, $\overline{u} \in P_{g(x)}^r \overline{M}$, $X_2 \in \mathscr{L}_{n,\overline{n}}$, $X_1 \in \mathscr{L}_{m,m}$. Then we set

$$\mathcal{F}(g, f)(Z) = \overline{v} \circ \big(X_2 \circ X \circ X_1^{-1}\big) \circ \overline{u}^{-1} .$$

Thus, we have proved

Proposition 1. *A sequence $F = (F_m)$ of weak r-jet m-functors form a weak total r-jet functor, if and only if the values of the restriction of $\varkappa_{m,n,q}$ to $L_{m,n}^F \times L_{n,q}^F$ lie in $L_{m,q}^F$.* □

In this case we write

$$\varkappa_{m,n,q}^F : L_{m,n}^F \times L_{n,q}^F \to L_{m,q}^F . \tag{8}$$

Remark. In differential geometry, the velocity and jet functors are constructed by iterating the jet prolongation procedure in various ways (in the case of all nonholonomic r-jets we iterate the construction of 1-jets in each step) and by adding natural conditions similar to the well known case of semiholonomic jets. None of these constructions yields a functor that is weak in the proper sense, i.e. not product preserving. However, the weakness assumption is necessary for a systematic approach to the subject.

3. Another characterization of weak total r-jet functors.

Consider a sequence $V = (V_m)$ of weak r-velocity m-functors.

Definition 4. The sequence $V = (V_m)$ is said to be admissible, if for every manifold N, every $x \in N$ and every $k \in \mathbb{N}$,

$$Z \in (V_m N)_x \quad \text{and} \quad X \in L^r_{k,m} \quad \text{implies} \quad Z \circ X \in (V_k N)_x \,.$$

Clearly, if $F = (F_m)$ is a weak total r-jet functor, then (vF_m) is an admissible sequence.

For an admissible sequence V, we define $L^V_{k,m}$ as the set of all $X \in \widetilde{L}^r_{k,m}$ such that $Z \circ X \in (V_k N)_x$ for every manifold N, every $x \in N$ and every $Z \in (V_m N)_x$.

Lemma. *If V is an admissible sequence, then*

$$X_1 \in L^V_{m,n} \quad \text{and} \quad X_2 \in L^V_{n,q} \quad \text{implies} \quad X_2 \circ X_1 \in L^V_{m,q} \,.$$

Proof. We have $W \circ X_1 \in (V_m N)_x$ for all $W \in (V_n N)_x$ and $Z \circ X_2 \in (V_n N)_x$ for all $Z \in (V_q N)_x$. Hence $Z \circ (X_2 \circ X_1) \in (V_m N)_x$. □

If we write $V_{m,n} = (V_m \mathbb{R}^n)_0$, then $L^V_{m,n} \subset V_{m,n}$. Indeed, for $X \in L^V_{m,n}$ we take $Z = j^r_0 \operatorname{id}_{\mathbb{R}^n} \in V_n(\mathbb{R}^n)_0$. Then $Z \circ X = X \in V_m(\mathbb{R}^n)_0$.

Assume that each $L^V_{m,n}$ is a submanifold of $V_{m,n}$. Clearly, the r-th jet group G^r_n in dimension n satisfies $G^r_n \subset L^V_{n,n}$. Hence the jet composition defines an action of G^r_n on $L^V_{m,n}$ and we construct the associated fiber bundle

$$\mathcal{V}_m N = P^r N[L^V_{m,n}] \,.$$

For a map $f : N \to Q$, we define $\mathcal{V}_m f : \mathcal{V}_m N \to \mathcal{V}_m Q$ as follows. We write $j^r_x f = w \circ Z \circ u^{-1}$, $u \in P^r_x N$, $Z \in L^r_{n,q}$, $w \in P^r_{f(x)} Q$. For an equivalence class $\{u, X\} \in \mathcal{V}_m N$, we set

$$\mathcal{V}_m f(\{u, X\}) = \{w, Z \circ X\} \,.$$

Then each $\mathcal{V}_m$ is a weak r-velocity m-functor. In the same way as in Section 2, we deduce

Proposition 2. *The sequence $(j\mathcal{V}_m)$ is a weak total r-jet functor.*

Since $L^V_{m,n} \subset V_{m,n}$ we have $\mathcal{V}_m N \subset V_m N$ for every manifold N.

Definition 5. An admissible sequence $V = (V_m)$ is said to be distinguished, if $L^V_{m,n} = V_{m,n}$ for all $m, n \in \mathbb{N}$.

Applying Proposition 1, we obtain

Proposition 3. *A sequence (F_m) of weak r-jet m-functors form a weak total r-jet functor, if and only if the sequence (vF_m) is distinguished.*

Example. Consider the sequence $(\overline{T}^r_m)$, $m \in \mathbb{N}$ of semiholonomic m-dimensional r-velocity functors, [9]. Using the formula for the composition of semiholonomic r-jets, [1], one deduces easily that $(\overline{T}^r_m)$ is a distinguished sequence.

4. The product preserving cases.

In the product preserving case, we have a simple description of both velocities and jet functors in terms of Weil algebras.

Definition 6. If a weak r-velocity m-functor V_m on $\mathcal{M}f$ preserves products, then V_m is said to be an r-velocity m-functor.

According to the general theory, [6], [7], V_m is characterized by a Weil algebra $\mathbb{D}^{V_m}$ satisfying

$$\mathbb{D}^r_m \subset \mathbb{D}^{V_m} \subset \widetilde{\mathbb{D}}^r_m .$$

In the jet case, our ideas from [5] and Section 3 can be formulated as follows.

Definition 7. An r-jet m-functor F_m on $\mathcal{M}f_m \times \mathcal{M}f$ or a total r-jet functor F is a weak r-jet m-functor or a weak total r-jet functor that preserves products in the second factor.

This definition yields directly that F_m is an r-jet m-functor, if and only if vF_m is an r-velocity m-functor.

Consider a sequence $F = (F_m)$ of r-jet m-functors. We write

$$\mathbb{D}^{vF_m} =: \mathbb{D}^F_m = \mathbb{R} \times N^F_m ,$$

where N^F_m is the nilpotent part of $\mathbb{D}^F_m$. Since each F_m preserves products, we have $\left(N^F_m\right)^n \subset \widetilde{L}^r_{m,n}$. Hence Proposition 3 implies that F is a total r-jet functor, if and only if (4) restricts and corestricts to

$$\varkappa^F_{m,n,q} : (N^F_m)^n \times (N^F_n)^q \to (N^F_m)^q . \tag{9}$$

An n-tuple X of elements of N_m^F belongs to $\widetilde{L}_{m,n}^r$. By Section 1, X determines a natural transformation $\widetilde{T}_n^r \to \widetilde{T}_m^r$, so an algebra homomorphism (denoted by the same symbol)

$$X : \widetilde{\mathbb{D}}_n^r \to \widetilde{\mathbb{D}}_m^r \,.$$

Thus, (9) can be reformulated as follows.

Proposition 4. *The total r-jet functors are in bijection with the sequences $(\mathbb{D}_m^F)$, $m \in \mathbb{N}$, of Weil algebras such that*

(i) $\mathbb{D}_m^r \subset \mathbb{D}_m^F \subset \widetilde{\mathbb{D}}_m^r$ *for all* $m \in \mathbb{N}$,

(ii) *each algebra homomorphism* $X : \widetilde{\mathbb{D}}_n^r \to \widetilde{\mathbb{D}}_m^r$ *determined by* $X \in (N_m^F)^n \subset \widetilde{L}_{m,n}^r$ *maps* $\mathbb{D}_n^F$ *into* $\mathbb{D}_m^F$.

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Ivan Kolář, Department of Algebra and Geometry, Faculty of Science, Masaryk University, Janáčkovo nám. 2a, 602 00 Brno, Czech Republic

E-mail address: kolar@math.muni.cz