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A NOTE ON r-EMBEDDINGS

by Luciano STRAMACCIA

Résumé. Dans cet article on étudie la classe des espaces **r**-compacts [10,12] et la notion connexe de sous-espace **r**-plongé, où **r** est un épiréflecteur topologique. Cette notion permet d'obtenir des résultats dans le contexte du problème de la conservation des produits topologiques par **r**. De plus, elle donne une caractérisation des réflecteurs totaux, définis en [2,3].

Introduction.

Let $\mathbf{r} : \mathbf{Top} \to \mathbf{R}$ be a topological epireflector. In [10,12] we have introduced the category $\mathbf{r} - \mathbf{Comp}$ of **r**-compact spaces, and the related **r**-closure operator.

In this note we go on studing such classes of spaces. Since it turns out that **r**-compactness is not an absolute property, then we are concerned, in particular, with a concept that overcomes such a difficulty, namely that of **r**-embedded subset. A subset A of a space X is **r**-embedded in X whenever $\mathbf{r}(S)$ is a subspace of $\mathbf{r}(X)$.

After presenting the general properties of **r**-embeddings, we concentrate on two arguments.

First, we describe the close relation existing between the classes of **r**-embeddings in **Top** and that of embeddings in **R** with respect to which **r** is total, in the sense of [2]. Actually, **r** is total with respect to the class of open embeddings if and only if every **r**-open subset of a given space is **r**-embedded there.

Furthermore, we give a characterization of the maximal class $T_{\mathbf{r}}$ of embeddings in **R**, such that **r** is total with respect to $T_{\mathbf{r}}$. We obtain a somewhat surprising result (Th.2.3) in this direction, by making use

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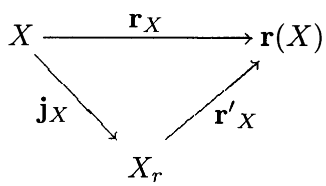
of the notion of pullback complement [3].
Secondly, the concept of **r**-embedding turns out to be useful in connection with the question whether **r** preserves products. Given two **r**-compact spaces X and Y, and two **r**-embedded subspaces $A \subset X$, $B \subset Y$, then $\mathbf{r}(A \times B) = \mathbf{r}(A) \times \mathbf{r}(B)$ holds. Such kind of result applies to a great variety of cases and, in particular, when $\mathbf{r} = \tau$ is the Tychonoff modification functor.

1. r-Compact Spaces and r-Embeddings.

In this paper we shall be concerned with an epireflective subcategory **R** of the category **Top** of all topological spaces and maps, with reflector $\mathbf{r} : \mathbf{Top} \to \mathbf{R}$.
For every space $X \in \mathbf{Top}$, let $\mathbf{r}_X : X \to \mathbf{r}(X)$ be its onto reflection map, and let us decompose it as follows



where X_r is the space having the same underlying set as X, with the initial topology induced by $\mathbf{r}_X$. Then $\mathbf{j}_X$ acts as the identity, and $\mathbf{r}_X = \mathbf{r}'_X$ as set maps.
Such a procedure is functorial and gives a decomposition $\mathbf{r} = \mathbf{r}' \cdot \mathbf{j}$ of the reflector **r** by means of the two functors $\mathbf{j} : \mathbf{Top} \to \mathbf{Top}_r$, and $\mathbf{r}' : \mathbf{Top}_r \to \mathbf{R}$. Here $\mathbf{Top}_r = \{X_r | X \in \mathbf{Top}\}$ is the bireflective hull of **R** in **Top** with bireflector $\mathbf{j} = \{j_X\}$, while $\mathbf{r}'$ is the restriction of **r** to $\mathbf{Top}_r$, in fact $\mathbf{r}'_X = \mathbf{r}_{X_r}$, for every space X. The topology on X_r is generated by what we called in [12] the $\mathbf{r}$ – *closure* operator, defined as follows

$$cl_r(M) = \mathbf{r}_X^{-1}(cl(\mathbf{r}_X(M))),$$

where $M \subset X$ and cl denotes ordinary closure. M is r – *closed* in X whenever $M = cl_r(M)$. A subset U of X will be called r – *open* if

$X - U$ is r-closed there.

EXAMPLES 1.1. *(a) When* $\mathbf{r} : \mathbf{Top} \to \mathbf{Top}_o$ *is the* T_o*-identification functor, then* $U \subset X$ *is* $\mathbf{r}$*-open if and only if* U *is open and* $x \in U$ *implies* $cl\{x\} \subset U$.
In general, for a quotient reflector $\mathbf{r}$*, the* $\mathbf{r}$*-open subsets of* $X \in \mathbf{Top}$ *coincide with those subsets* S *that are open and saturated with respect to the reflection map* $\mathbf{r}_X$*, that is* $S = \mathbf{r}_X^{-1}(\mathbf{r}_X(S))$.
(b) Let $\tau : \mathbf{Top} \to \mathbf{Tych}$ *be the Tychonoff modification functor [5,6]. The* τ*-open subsets of a space* X *are those which are union of cozero sets of* X.
(c) Let $\mathbf{r} : \mathbf{Top} \to \mathbf{0-dim}$ *be the reflector functor on the category of zero-dimensional spaces; in such a case the* $\mathbf{r}$*-open subsets of* $X \in \mathbf{Top}$ *are those subsets that are union of clopen subsets of* X.
(d) Let $\mathbf{R} \subset \mathbf{Top}$ *be such that the Salbany closure operator [9](called* $\mathbf{R}$*-closure) induced by it, coincides with the ordinary closure on each* $X \in \mathbf{R}$*. This happens, e.g., for* $\mathbf{R} = \mathbf{Haus}$*, the category of Hausdorff spaces,* $\mathbf{R} = \mathbf{Reg}$*, the category of regular spaces,* $\mathbf{R} = \mathbf{Tych}$*,* $\mathbf{R} = \mathbf{O-dim}$*, etc. In such a case the* $\mathbf{r}$*-open subsets of* X *are the same as the* $\mathbf{R}$*-open subsets. For the general situation see [11].*

A topological space X is said to be $\mathbf{r}-$ *compact* whenever its reflection $\mathbf{r}(X)$ in $\mathbf{R}$ is a compact space (no separation axiom is assumed); this amounts to say that every cover of X by $\mathbf{r}$-open subsets has a finite subcover.
The category $\mathbf{r} - \mathbf{Comp} \subset \mathbf{Top}$ of $\mathbf{r}$-compact spaces has been studied in [10,12], where we are concerned with the question whether $\mathbf{r}$ preserves topological products.
Recently, $\mathbf{r}$-compact spaces have been studied further in [8].

$\mathbf{r}$-compactness is not an absolute property, as shown by the following example concerning the Tychonoff reflector τ.

EXAMPLE 1.2. *Let* X *be the space having the closed unit interval* I *as underlying set and a topology which differs from the usual one in that the neighbourhoods of the point* 0 *do not contain points of the*

set $A = \{\frac{1}{n}: n \in \mathbf{N}\}$.
X is a non-compact, τ-compact space, in fact one has $\tau(X) = X_\tau = I$. If we consider the subset $S = A \cup \{0\}$ of X, then S is relatively τ-compact with respect to X, but it is not τ-compact considered as a topological space, with the topology inherited from X.

In order to avoid this kind of situation, it is useful to introduce the notion of $\mathbf{r}$ – *embedding*, which also turns out to be of interest in connection with other concepts, as we shall see below.

DEFINITION 1.3. *A subset S of a space X is said to be $\mathbf{r}$ – embedded in X provided that every $\mathbf{r}$-open subset of S is the intersection of an $\mathbf{r}$-open subset of X with S.*

THEOREM 1.4. *For a subset S of a space X, the following are equivalent:*

(1) *S is $\mathbf{r}$-embedded in X.*
(2) *S_r is a subspace of X_r.*
(3) *$\mathbf{r}(S)$ is a subspace of $\mathbf{r}(X)$.*
(4) *$cl_{r,S}(M) = cl_{r,X}(M) \cap S$, for every $M \subset S$.*

PROOF: The equivalence of (1) and (2) follows directly from the definitions.
(2) $\rightarrow$ (3). Let s be the embedding of S in X. We have the following situation: $\mathbf{r}(s_r) \cdot \mathbf{r}_{S_r} = \mathbf{r}_{X_r} \cdot s_r$, where s_r is the embedding of S_r in X_r and $\mathbf{r}_{X_r} \cdot s_r$, $\mathbf{r}(s_r)$ are initial maps. Our first task is to show that $\mathbf{r}(s_r)$ is also initial. To this end, let $V \subset \mathbf{r}(S)$ be an open subset; then there exists an open subset $T \subset \mathbf{r}(X)$ such that

$$\mathbf{r}_{S_r}^{-1}(V) = (\mathbf{r}_{S_r} \cdot s_r)^{-1}(T) = (\mathbf{r}(s_r) \cdot \mathbf{r}_{S_r})^{-1}(T) = \mathbf{r}_{S_r}^{-1}(\mathbf{r}(s_r)^{-1}(T)).$$

Since $\mathbf{r}_{S_r}$ is onto, we obtain that $V = \mathbf{r}(s_r)^{-1}(T)$. For $\mathbf{r}(s_r)$ to be an embedding it remains to show that it is injective. In order to do this let us recall [7] that $\mathbf{R}$ is either bireflective in **Top** or it is contained in the category $\mathbf{Top}_o$ of T_o-spaces; in the former case $\mathbf{r}(s)$ must be injective, hence an embedding. In the latter case $\mathbf{r}(S)$ is a T_o-space. If x and y are two distinct points in $\mathbf{r}(S)$ then there is an open subset U of $\mathbf{r}(S)$ such that $x \in U$ and $y \notin U$. By the first part of the proof there

exists an open set $V \subset \mathbf{r}(X)$ such that $U = \mathbf{r}(s)^{-1}(V)$. It follows that $\mathbf{r}(s)(x)$ and $\mathbf{r}(s)(y)$ must be different.
(3) $\to$ (4). Let us recall that $cl_{r,S}(M)$ is given by the intersection of all $\mathbf{r}$-closed subsets F of S that contain M. Every such F is of the form $F = \mathbf{r}_S^{-1}(F')$, being F' a closed subset of $\mathbf{r}(S)$. Note also that, by assumption, F' is of the form $F' = \mathbf{r}(s)^{-1}(F'')$ for some closed subset F'' of $\mathbf{r}(X)$. Then one has $\mathbf{r}_S^{-1}(\mathbf{r}(s)^{-1}(F'')) = s^{-1}(\mathbf{r}_X^{-1}(F'')) = \mathbf{r}_X^{-1}(F'') \cap S$. It follows

$$\begin{aligned} cl_{r,S}(M) &= \cap\{\mathbf{r}_S^{-1}(F') : M \subset F\} = \cap\{\mathbf{r}_S^{-1}(\mathbf{r}(s)^{-1}(F'')) : M \subset F\} = \\ &= \cap\{s^{-1}(\mathbf{r}_X^{-1}(F'')) : M \subset F\} = \cap\{\mathbf{r}_X^{-1}(F'') \cap S : M \subset F\} = \\ &= \cap\{\mathbf{r}_X^{-1}(F'') : M \subset F\} \cap S = cl_{r,S}(M) \cap S. \end{aligned}$$

(4) $\to$ (1). This is immediate.

COROLLARY 1.5. *Let S be an $\mathbf{r}$-embedded subspace of X. Then:*

1. $\mathbf{r}(S) = \mathbf{r}_X(S)$ *and* $\mathbf{r}_S = \mathbf{r}_{X|S}$.
2. *S is $\mathbf{r}$-compact iff it is $\mathbf{r}$-compact relatively to X.*

COROLLARY 1.6. *Every retract of X is $\mathbf{r}$-embedded in X.*

EXAMPLES 1.7. *(a) Let $\mathbf{r}$ be a quotient reflector with $\mathbf{r}_X : X \to \mathbf{r}(X) = X/\sim$, for every $X \in$ **Top**. A subset $M \subset X$ is then $\mathbf{r}$-embedded exactly when $M/\sim$ is a subset of $X/\sim$. It follows that a sufficient condition in order that M be $\mathbf{r}$-embedded in X is that M is either open or closed in X and saturated with respect to the quotient map $\mathbf{r}_X$.*
If $\mathbf{r}$ is the T_o-identification functor, then every closed subset of X is $\mathbf{r}$-embedded in X, as one can easily verify.
(b) Recall that a subset S of a space X is $z-$ embedded [1] whenever every cozero set in S is the intersection with S of a cozero set in X. Hence it is clear that every z-embedded subset is also τ-embedded. Moreover, since every cozero set is z-embedded, it follows that every τ-open subset of X is τ-embedded.

REMARK 1.8. *In the non-surjective case of the Hewitt realcompactification functor ν :* **Top** $\to$ **r − Comp**, *the concept of ν-embedding coincides with that defined in [1].*

The situation described in 1.7.(b) above is not typical of τ, but it is fairly general, since it is shared by all those reflectors that are *total* (in the sense of [2]), with respect to the class of open embeddings. Besides τ, other examples are the reflectors onto the categories **Haus**, **Reg**, **0 − dim**.

Let us recall that the epireflector $\mathbf{r}$ is said to be *total* [2,3] with respect to a class $\mathbf{S}$ of topological embeddings, whenever, given an embedding $s : S \to \mathbf{r}(X)$, with $s \in \mathbf{S}$, then the restriction of $\mathbf{r}_X$ to $\mathbf{r}_X^{-1}(S)$, that is the map

$$\mathbf{r}_{X|\mathbf{r}_X^{-1}(S)} : \mathbf{r}_X^{-1}(S) \to S$$

is uniquely $\mathbf{R}$-extendable, hence a reflection map.

PROPOSITION 1.9. *The epireflector* $\mathbf{r}$ *is total with respect to the class of open embeddings iff, for every* $X \in \mathbf{Top}$, *every* $\mathbf{r}$*-open subset of* X *is* $\mathbf{r}$*-embedded in* X.

PROOF: Let $\mathbf{r}$ be total with respect to the class of open embeddings and let A be an $\mathbf{r}$-open subset of X. Then $\mathbf{r}_X(A)$ is open in $\mathbf{r}(X)$ and $A = \mathbf{r}_X^{-1}(\mathbf{r}_X(A))$. It follows that $\mathbf{r}_{X|A}: A = \mathbf{r}_X^{-1}(\mathbf{r}_X(A)) \to \mathbf{r}_X(A)$ is the reflection map for A, hence $\mathbf{r}(A) = \mathbf{r}_X(A)$ and $\mathbf{r}_A = \mathbf{r}_{X|A}$, that is A is $\mathbf{r}$-embedded in X.
Conversely, assume that, for any space X, every $\mathbf{r}$-open subset of X is $\mathbf{r}$-embedded. Let $s : S \to \mathbf{r}(X)$ be an open embedding; then $\mathbf{r}_X^{-1}(S)$ is $\mathbf{r}$-open in X. It follows that $\mathbf{r}_{X|\mathbf{r}_X^{-1}(S)}: \mathbf{r}_X^{-1}(S) = \mathbf{r}_X^{-1}(\mathbf{r}_X(\mathbf{r}_X^{-1}(S))) \to S$ is a reflection map. This shows that $\mathbf{r}$ is total with respect to the class of open embeddings.

2. Further Properties of r-Embeddings.

Let the epireflector $\mathbf{r} : \mathbf{Top} \to \mathbf{R}$ be given and let us denote by $T_{\mathbf{r}}$ the maximal class of topological embeddings such that $\mathbf{r}$ is total with respect to it.
A natural question that arises is then to characterize the class $T_{\mathbf{r}}$.
The next results deal with this problem.

PROPOSITION 2.1. *Let $X \in \mathbf{Top}$. An embedding $s : S \to \mathbf{r}(X)$ is an element of T_r if and only if there exists a subset A of X such that:*

(1) $S = \mathbf{r}_X(A)$.
(2) *A is saturated with respect to $\mathbf{r}_X$.*
(3) *A is $\mathbf{r}$-embedded in X.*

PROOF: The proof is straitforward and depends essentially on the definition of $\mathbf{r}$-embedding.

The following theorem indicates how one must choose the subset A of X, involved in the previous proposition. It make use of the notion of pullback complement [3] which we recall for sake of completeness.

DEFINITION 2.2. *Let $f : A \to U$ and $s : U \to Y$ be morphisms in a category $\mathbf{C}$. A pullback complement of the pair (f, s) is a pair (s', f') in such a way that*

$$\begin{array}{ccc} A & \xrightarrow{f} & U \\ {\scriptstyle s'}\downarrow & & \downarrow{\scriptstyle s} \\ P & \xrightarrow[f']{} & Y \end{array}$$

is a pullback square with the property that for every other pullback square

$$\begin{array}{ccc} V & \xrightarrow{g} & U \\ {\scriptstyle t}\downarrow & & \downarrow{\scriptstyle s} \\ T & \xrightarrow[g']{} & Y \end{array}$$

and any morphism $h : V \to A$ with $f \cdot h = g$, there is a unique $h' : T \to P$ such that $f' \cdot h' = g'$ and $s' \cdot h = h' \cdot t$.

THEOREM 2.3. *Let $a : A \to X$ be an embedding that is saturated with respect to $\mathbf{r}_X$. Then a is an $\mathbf{r}$-embedding if and only if the pair $(\mathbf{r}_A, \mathbf{r}(a))$ is a pullback complement of $(a, \mathbf{r}_X)$.*

PROOF: Assume that a is a saturated $\mathbf{r}$-embedding with respect to $\mathbf{r}_X$; then the following diagram is a pullback:

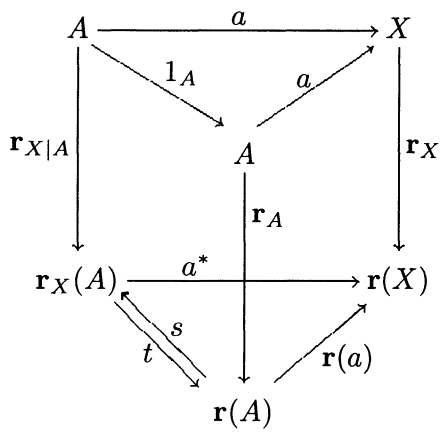
$$\begin{array}{ccc} A & \xrightarrow{a} & X \\ {\scriptstyle \mathbf{r}_A}\downarrow & & \downarrow{\scriptstyle \mathbf{r}_X} \\ \mathbf{r}(A) & \xrightarrow[\mathbf{r}(a)]{} & \mathbf{r}(X) \end{array}$$

Suppose there is another pullback square

$$\begin{array}{ccc} V & \xrightarrow{g} & X \\ {\scriptstyle t}\downarrow & & \downarrow{\scriptstyle \mathbf{r}_X} \\ T & \xrightarrow[g^*]{} & \mathbf{r}(X) \end{array}$$

and a map $h : V \to A$ with $a \cdot h = g$.
By commutativity it follows $g^* \cdot t = \mathbf{r}_X \cdot g = \mathbf{r}_X \cdot a \cdot h = \mathbf{r}(a) \cdot \mathbf{r}_A \cdot h$. Because of Theorem 1.4(3) we know that $\mathbf{r}(a)$ is an embedding. Moreover, t as a pullback of the onto map $\mathbf{r}_X$ is onto as well. Now the fact that onto maps and embeddings form a factorization system implies the existence of the required $h^* : T \to \mathbf{r}(A)$; it arises as a "diagonal fill-in". This concludes the first half of the proof.

Conversely, assume that $a : A \to X$ is a saturated embedding such that the first square is a pullback complement.
Consider the following diagram



where a^* is the inclusion.
By the universal property of the reflection there is a unique $s : \mathbf{r}(A) \to \mathbf{r}_X(A)$ such that $s{\cdot}\mathbf{r}_A = \mathbf{r}_{X|A}$; by the universal property of the pullback complement, there is a unique $t : \mathbf{r}_X(A) \to \mathbf{r}(A)$ such that $\mathbf{r}(a){\cdot}t = a^*$. Then $\mathbf{r}(a){\cdot}t{\cdot}\mathbf{r}_{X|A} = a^* \cdot \mathbf{r}_{X|A} = \mathbf{r}_X \cdot a$; by the universal property of the right outer pullback square, it follows that 1_A must be the unique map which renders the left outer square commutative, that is, $\mathbf{r}_A = t{\cdot}\mathbf{r}_{X|A}$. Finally, the composition $t \cdot s$ must be the identity, hence s is an onto map having a left inverse; s is a homeomorphism, and this concludes the proof.

Let now X and Y be two topological spaces. One says that the epireflector $\mathbf{r}$ preserves their product and writes $\mathbf{r}(X \times Y) = \mathbf{r}(X) \times \mathbf{r}(Y)$, whenever the unique map $t_{X \times Y} : \mathbf{r}(X \times Y) \to \mathbf{r}(X) \times \mathbf{r}(Y)$ such that $t_{X \times Y}.\mathbf{r}_{X \times Y} = \mathbf{r}_X \times \mathbf{r}_Y$, which is induced by the universal property of the reflection, is a homeomorphism. This happens, e.g., whenever X and Y are **r**-compact spaces.
In [12] it was shown that $\mathbf{r}$ preserves the product of X and Y if and only if every **r**-open subset of $X \times Y$ is union of subsets of the form

$A \times B$, where $A \subset X$ and $B \subset Y$ are **r**-open.

THEOREM 2.4. *Let X and Y be topological spaces* **r**-*embedded in* **r**-*compact spaces X^* and Y^*, respectively. Then $\mathbf{r}(X \times Y) = \mathbf{r}(X) \times \mathbf{r}(Y)$ iff $X \times Y$ is* **r**-*embedded in the product space $X^* \times Y^*$.*

PROOF: Let us assume that $X \times Y$ is **r**-embedded in the product $X^* \times Y^*$. Let $P \subset X \times Y$ be an **r**-open subset, then there is a **r**-open subset $Q \subset X^* \times Y^*$ such that $P = (X \times Y) \cap Q$. For every $q \in Q$, there exists a rectangular **r**-open set $A_q \times B_q$ in $X^* \times Y^*$ such that $q \in A_q \times B_q$. It follows that $Q = \cup_{q \in Q} A_q \times B_q$, therefore $P = \cup_{q \in Q}(X \times Y) \cap (A_q \times B_q) = \cup_{q \in Q}((X \cap A_q) \times (Y \cap B_q))$, where $(X \cap A_q)$ and$(Y \cap B_q)$ are **r**-open subsets in X, Y, respectively. Hence P can be written as a union of rectangular **r**-open sets of $X \times Y$.
Conversely, assume that $\mathbf{r}(X \times Y) = \mathbf{r}(X) \times \mathbf{r}(Y)$, then every **r**-open subset of $X \times Y$ is a union of rectangular **r**-open subsets. Let $P \subset X \times Y$ be an **r**-open subset; there are **r**-open subsets $A_i \subset X$ and $B_i \subset Y, i \in I$, such that $P = \cup_{i \in I} A_i \times B_i$. By assumption there are **r**-open subsets $A_i^* \subset X^*$ and $B_i^* \subset Y^*$ with $A_i = X \cap A_i^*$, $B_i = Y \cap B_i^*$. Now $P^* = \cup_{i \in I} A_i^* \times B_i^*$ is an **r**-open subset of $X^* \times Y^*$ with the property that $P = (X \times Y) \cap P^*$.

COROLLARY 2.5. *Assume that* **r** *is total with respect to the class of open embeddings and let X, Y be* **r**-*compact spaces. For any pair of* **r**-*open subsets $A \subset X$ and $B \subset Y$, we have $\mathbf{r}(A \times B) = \mathbf{r}(A) \times \mathbf{r}(B)$.*

REMARKS 2.6. *(a)The category of w – compact spaces, defined and studied in [5,6], is contained in τ –* **Comp**. *Since τ is total with respect to the open embeddings, because of Proposition 1.9, Corollary 2.5 applies to any two (unions of) cozero sets $A \subset X$ and $B \subset Y$, whenever X and Y are w-compact.*
The same is true, e.g., when A, B are Lindeloff subspaces of X, Y, respectively, since then they are z-embedded [1], hence τ-embedded.
(b) The corollary above applies to most of the usual situations; see, in fact, [2] for a list of topological epireflectors that are total with respect to the class of open embeddings.

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