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FURTHER CRITERIA FOR TOTALITY
 by B.J. DAY

Résumé. Cet article est une suite de l'article de Kelly [2] sur l'étude de la totalité pour les catégories enrichies.

Introduction. This Note is a sequel to the Kelly survey [2] of totality for enriched categories and some familiarity with the latter is assumed. It is supposed throughout that V is a symmetric monoidal closed category with V_0 admitting all small limits and arbitrary intersections of monics.

Generators and totality.

THEOREM 1. Any cocomplete category A is total if it admits arbitrary cointersections of epics and has a small generating set.

PROOF. It suffices to show that the coend $\int^a fa \otimes a$ can be constructed in A from the generators G of A . First consider the pushout diagram of the canonical map $1 \otimes \epsilon$ and k with $1 \otimes \epsilon$ jointly epic since G generates A :

$$\begin{array}{ccc}
 fa \otimes A(g, a) \otimes g & \xrightarrow{k} & \int^a fg \otimes g \\
 1 \otimes \epsilon \downarrow & \text{p.o.} & \downarrow e_a \\
 fa \otimes a & \xrightarrow{\quad} & q_a
 \end{array}$$

This implies that each e_a is epic; then the pushout of all those epics over a in A is easily seen to be precisely $\int^a fa \otimes a$ in A , as required.

REMARK. In the above result, epics can be replaced by the maps in E for any E - M -factorization system on A ; a general result concerning limits of M -subfunctors can be found in the Lemma of [1], §3.

The adjoint-functor Theorem and totality.

THEOREM 2. A category A is total iff it is complete with all intersections of [strong] monics and there exists a functor r from $[A^{op}, V]$ to A and a natural [strong] monic $\mu: 1 \rightarrow ry$.

PROOF. Necessity is clear. For sufficiency consider the canonical diagram

$$\begin{array}{ccccc}
 fa & \longrightarrow & A(rya, rf) & \xrightarrow{A(\mu, 1)} & A(a, rf) \\
 \alpha_a \downarrow & & \downarrow A(1, r(\alpha)) & & \downarrow A(1, r(\alpha_a)) \\
 & & A(rya, ryb) & \xrightarrow{A(\mu, 1)} & A(a, ryb) \\
 & \nearrow & & \searrow & \\
 yb(a) & \xrightarrow{A(1, \mu)} & & &
 \end{array}$$

The result now follows from the Adjoint-Functor Theorem [1].

As an application, consider the category A of coalgebras for a density comonad on a category B . If such an A is complete then it is total with no assumption on B .

References.

1. B.J. DAY, On adjoint-functor factorization, *Lecture Notes in Math*, 420, Springer (1974), 1-19.
2. G.M. KELLY, A survey of totality for enriched and ordinary categories, *Cahiers Top et Géom, Diff*, XXVII-2 (1986), 109-132 (and references therein).

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