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REMARKS ON TOPOLOGICALLY ALGEBRAIC FUNCTORS

by Reinhard BÖRGER and Walter THOLEN

1. INTRODUCTION.

In order to describe functors appearing in universal topological algebra, Y.H. Hong [16] introduced the notion of a topologically algebraic functor. Although these functors are defined by a very simple factorization structure of cones, they allow to prove some important properties such as faithfulness, right-adjointness and lifting of existence of limits and colimits.

Independently from Hong's notion the second author introduced the notion of an orthogonal \mathfrak{M} -functor (cf. [19, 20, 21]) and proved general lifting theorems for adjoint functors extending related investigations for topological functors (cf. Wyler [30]). In this paper a short proof is given that both concepts are equivalent: Each orthogonal \mathfrak{M} -functor is topologically algebraic, and for each topologically algebraic functor there is a largest \mathfrak{M} such that one has an orthogonal \mathfrak{M} -structure (Theorem 1).

In [27], Trnkova introduced the notion of a (faithful) functor with «weak inductive generation» which was rephrased as «semi-finally complete functor» by Wischnewsky [28] and as «semi-topological functor» by Hoffmann [14] and the second author [22]. These functors are treated more intensively in [23] and became interesting because of the following two main results proved here:

Each semi-topological functor admits a so-called locally orthogonal factorization structure and, consequently, has a reflective topological completion, which turns to be its MacNeille completion (cf. Herrlich [9]).

It is known that topologically algebraic implies semi-topological (cf. [22]). Herrlich, Nakagawa, Strecker and Titcomb [11] and the first author [3] independently constructed countable counter-examples to the

reverse implication. In this paper we give a concrete counter-example, i. e. we construct a semi-topological functor over \mathfrak{C}_{na} being not topologically algebraic (Theorem 3). Moreover, sufficient conditions for both concepts to be equivalent are given (Theorem 4).

Semi-topological functors are those functors which have a reflective MacNeille completion. By Herrlich and Strecker [10] topologically algebraic functors are characterised as those functors which admit a reflective universal completion. In order to give an effective construction of the universal completion of certain algebraic categories over \mathfrak{C}_{na} we prove a simple criterion for all P -initial cones to be mono-cones, if P is an arbitrary representable functor (Theorem 5).

Besides Theorem 5 the results of this paper are taken from the preprints [4] and [24] which contain further investigations about topologically algebraic functors like external characterizations and lifting of monads and adjoint functors. But these results are omitted in this paper because they are immediate consequences or analogously gotten from corresponding results for semi-topological functors (cf. [25, 26]). Furthermore, the reader is referred to the «Duality Theorem» for topologically algebraic functors recently proved by Wischnewsky [29] and the theorems for lifting tensor products and inner hom-functors given by Greve [6] and Porst-Wischnewsky [18].

2. TOPOLOGICALLY ALGEBRAIC FUNCTORS AND ORTHOGONAL $(\mathfrak{E}, \mathfrak{M})$ -FUNCTORS.

Let $P: \mathfrak{A} \rightarrow \mathfrak{X}$ be a functor. A P -cone (over \mathfrak{D}) is a triple (X, ξ, D) where X is an object of \mathfrak{X} , $D: \mathfrak{D} \rightarrow \mathfrak{A}$ is a functor and $\xi: \Delta X \rightarrow P \circ D$ is a natural transformation whose domain is constant X . Often we write: $\xi: \Delta X \rightarrow P \circ D$ instead of (X, ξ, D) . A P -cone over the one-morphism category $\mathbf{1}$ is called a P -morphism and written as a pair (x, A) , where A is an object of \mathfrak{A} and x a morphism of \mathfrak{X} with codomain PA ; if X is the domain of x , we also write $x: X \rightarrow PA$. The dual notions are P -cocone and P -comorphism. Note that objects of \mathfrak{X} can be regarded as P -(co-)cones over \emptyset .

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$\text{Epi}(P)$ denotes the class of all P -epimorphisms $e: X \rightarrow PA$, i. e. for all $u, v: A \rightarrow B$ with $(Pu)e = (Pv)e$ one has $u = v$. $\text{Iso}(P)$ denotes the class of all P -isomorphisms $i: X \rightarrow PA$, i. e. i is an isomorphism of \mathcal{X} . Note that $\text{Iso}(P) \subset \text{Epi}(P)$ holds if and only if P is faithful. $\text{Init}(P)$ denotes the (meta-)class of all P -initial \mathcal{Q} -cones $\nu: \Delta B \rightarrow D$, i. e. for all \mathcal{Q} -cones $\alpha: \Delta A \rightarrow D$ and all

$$x: PA \rightarrow PB \text{ with } (P \circ \nu)(\Delta x) = P \circ \alpha,$$

there is a unique $t: A \rightarrow B$ with $Pt = x$ and $\nu(\Delta t) = \alpha$.

DEFINITION 1 (Hong [16]). P is called *topologically algebraic* iff any P -cone $\xi: \Delta X \rightarrow P \circ D$ admits a factorization

$$(*) \quad \xi = (P \circ \mu)(\Delta e),$$

with a P -epimorphism $e: X \rightarrow PA$ and a P -initial \mathcal{Q} -cone $\mu: \Delta A \rightarrow D$.

Orthogonal $(\mathcal{E}, \mathfrak{M})$ -functors are defined by assuming a diagonalization property instead of the condition that e has to be P -epimorphic. A P -morphism $e: X \rightarrow PA$ and an \mathcal{Q} -cone $\nu: \Delta B \rightarrow D$ are called *orthogonal*, written $(e, A) \perp (B, \nu, D)$, if for all P -morphisms $x: X \rightarrow PB$ and all \mathcal{Q} -cones $\alpha: \Delta A \rightarrow D$ with

$$(P \circ \alpha)(\bar{\Delta} e) = (P \circ \nu)(\Delta x),$$

one has a unique ¹⁾ $t: A \rightarrow B$ with $(Pt)e = x$ and $\nu(\Delta t) = \alpha$.

$$\begin{array}{ccc}
 \Delta X & \xrightarrow{\Delta e} & \Delta PA \\
 \Delta x \downarrow & \nearrow \Delta Pt & \downarrow P \circ \alpha \\
 \Delta PB & \xrightarrow{P \circ \nu} & P \circ D
 \end{array}$$

For classes \mathcal{E} of P -morphisms and \mathfrak{M} of \mathcal{Q} -cones define a Galois correspondence by

$$\mathcal{E}_\perp := \{ (B, \nu, D) \mid (e, A) \perp (B, \nu, D) \text{ for all } (e, A) \in \mathcal{E} \},$$

$$\mathfrak{M}^\perp := \{ (e, A) \mid (e, A) \perp (B, \nu, D) \text{ for all } (B, \nu, D) \in \mathfrak{M} \}.$$

¹⁾ For functors considered here uniqueness follows necessarily: cf. Lemma 1.

Obviously one has

$$\text{Iso}(P)_1 = \text{Init}(P).$$

Now we assume \mathfrak{E} and \mathfrak{M} to be closed under composition with \mathfrak{A} -isomorphisms, i. e. if $i: A \rightarrow B$ is an isomorphism, let $((Pi)e, B) \in \mathfrak{E}$ for all $(e, A) \in \mathfrak{E}$ and let $(A, \mu(\Delta i)) \in \mathfrak{M}$ for all $(B, \mu) \in \mathfrak{M}$.

DEFINITION 2 (Tholen [19, 20, 21]). P is called an *orthogonal* $(\mathfrak{E}, \mathfrak{M})$ -functor if

$$\text{Iso}(P) \subset \mathfrak{E} \subset \mathfrak{M}^\perp$$

and if any P -cone $\xi: \Delta X \rightarrow P \circ D$ admits a factorization (*) with

$$(e, A) \in \mathfrak{E} \quad \text{and} \quad (A, \mu, D) \in \mathfrak{M}.$$

Because of $\text{Iso}(P)_1 = \text{Init}(P)$ one obviously gets $\mathfrak{M} \subset \text{Init}(P)$. Moreover one easily proves the equations

$$\mathfrak{E} = \mathfrak{M}^\perp \quad \text{and} \quad \mathfrak{M} = \mathfrak{E}_1.$$

Therefore we also speak of an orthogonal \mathfrak{E} -functor or an orthogonal \mathfrak{M} -functor, if the other parameter is not explicitly given.

Topologically algebraic functors and orthogonal $(\mathfrak{E}, \mathfrak{M})$ -functors are faithful and right adjoint and lift the existence of all types of limits and colimits from \mathfrak{X} to \mathfrak{A} (cf. [20, 22]). These properties hold more generally for semi-topological functors (see 3 below).

The connection between Definitions 1 and 2 is clarified by Theorem 1 below. For this we need the following Lemma 1 whose proof is based on a trick outlined more generally in [5] (cf. also [23], Corollary 6.4).

LEMMA 1. If P is an orthogonal $(\mathfrak{E}, \mathfrak{M})$ -functor, then $\mathfrak{E} \subset \text{Epi}(P)$.

PROOF. Let $e: X \rightarrow PA$ be in \mathfrak{E} and assume $(Pu)e = (Pv)e$, with $u, v: A \rightarrow B$ in \mathfrak{A} . Let $I = \text{Mor } \mathfrak{A}$ and define a discrete P -cone

$$(X, x_i, B_i)_I \quad \text{by} \quad x_i = (Pu)e \quad \text{and} \quad B_i = B \quad \text{for all } i \in I.$$

It factorizes into $(C, m_i, B_i)_I$. Then $J := \{t: A \rightarrow C \mid m_i t \in \{u, v\}\}$ is not empty. With any surjection $\sigma: I \rightarrow J$ define $h_i: A \rightarrow B_i$, $i \in I$, by

$$h_i = \begin{cases} u & \text{for } m_i \sigma(i) = v \\ v & \text{for } m_i \sigma(i) = u. \end{cases}$$

Then there exists a morphism t with $m_i t = h_i$, $i \in I$, hence $t \in J$, say $t = \sigma(i_0)$. So we get

$$m_{i_0} t = v \Leftrightarrow h_{i_0} = u \Leftrightarrow m_{i_0} t = u.$$

THEOREM 1. *The following conditions are equivalent:*

- (i) P is topologically algebraic.
- (ii) P is an orthogonal $\text{Init}(P)$ -functor.
- (iii) P is an orthogonal \mathfrak{M} -functor for some \mathfrak{M} .

(iv) P has a left adjoint and \mathfrak{A} is an orthogonal \mathfrak{M} -category (i. e., $\text{Id}_{\mathfrak{A}}$ is an orthogonal \mathfrak{M} -functor) for some subclass $\mathfrak{M} \subset \text{Init}(P)$.

PROOF. (ii) \Rightarrow (iii) \Leftrightarrow (iv) is trivial and (iii) \Rightarrow (i) follows from Lemma 1. So it remains to prove (i) \Rightarrow (ii). Let $\xi: \Delta X \rightarrow P \circ D$ be a P -cone and consider all P -epimorphisms $e_i: X \rightarrow P A_i$ such that there is a P -initial cone $\mu_i: \Delta A_i \rightarrow D$ with

$$(P \circ \mu_i)(\Delta e_i) = \xi, \quad i \in I$$

(μ_i is uniquely determined by e_i). The discrete P -cone $(e_i: X \rightarrow P A_i)_I$ has again an epimorphic and initial factorization

$$(P m_i) e = e_i \quad \text{with } e: X \rightarrow P A, \quad i \in I.$$

Because of $I \neq \emptyset$ and $(e, A) \in \text{Epi}(P)$ we can define $\mu := \mu_i(\Delta m_i)$ independently from the choice of $i \in I$. Hence we get a factorization (*) which turns out to be the desired one as can be shown in 3 steps.

Step 1. $\mu: \Delta A \rightarrow D$ is P -initial.

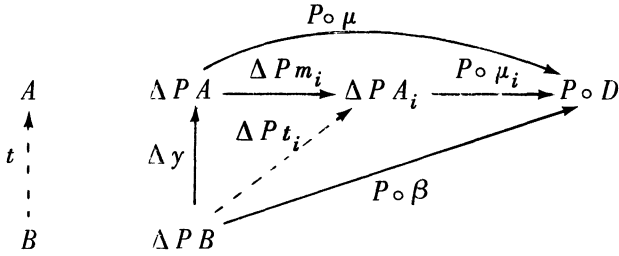
Proof 1. For all $\beta: \Delta B \rightarrow D$ and

$$\gamma: P B \rightarrow P A \quad \text{with } (P \circ \mu)(\Delta \gamma) = P \circ \beta$$

and any $i \in I$ one gets

$$t_i: B \rightarrow A_i \quad \text{with } (P m_i) \gamma = P t_i.$$

Therefore we have a $t: B \rightarrow A$ with $P t = \gamma$. ($\mu(\Delta t) = \beta$ and uniqueness of t follow from the faithfulness of P).



Step 2. If $(Pm)\hat{e} = e$ with $\hat{e}: X \rightarrow PC$ P -epimorphic and $m: C \rightarrow A$ P -initial, then m must be an isomorphism.

Proof 2. $m: C \rightarrow A$ in $\text{Init}(P)$ and $\mu: \Delta A \rightarrow D$ in $\text{Init}(P)$ imply $\mu(\Delta m)$ in $\text{Init}(P)$, hence $\hat{e} = e_i$ with an $i \in I$. Then the equations

$$(Pm_i m)\hat{e} = \hat{e} \quad \text{and} \quad (Pmm_i)v = e$$

prove the assertion.

Step 3. $(v, A) \in \text{Init}(P)^\downarrow$.

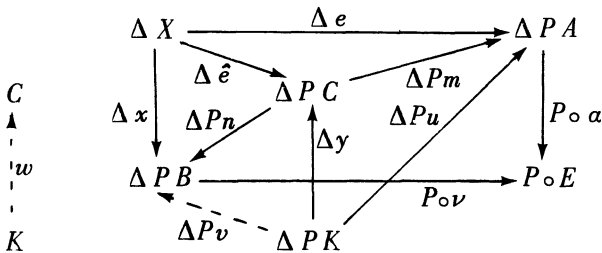
Proof 3. Let $v: \Delta B \rightarrow E$ be P -initial and let $a: \Delta A \rightarrow E$ and

$$x: X \rightarrow PB \quad \text{with} \quad (Pc v)(\Delta x) = (P \circ a)(\Delta e)$$

be given. Then $(e: X \rightarrow PA, x: X \rightarrow PB)$ is a P -cone indexed by a two point set. Hence we get a P -epimorphism $\hat{e}: X \rightarrow PC$ and a P -initial \mathcal{U} -cone $(m: C \rightarrow A, n: C \rightarrow B)$ with

$$(Pm)\hat{e} = e \quad \text{and} \quad (Pn)\hat{e} = x.$$

Because of Step 2 it suffices to show that m is P -initial as a single morphism; then $t := nm^{-1}$ is a suitable diagonal with $(Pt)e = x$. Consider $u: K \rightarrow A$ and $y: PK \rightarrow PC$ with $(Pm)y = Pu$. Since v is P -initial, we get $v: K \rightarrow B$ with $Pv = (Pn)y$, and since (m, n) is P -initial, we get $w: K \rightarrow C$ with $Pw = y$.



REMARKS. 1° Theorem 1 was proved independently and at the same time by Herrlich and Strecker [10]. Their proof is very different from the more «direct» proof given here.

2° Obviously $\text{Init}(P)$ is the greatest class \mathfrak{M} such that a topologically algebraic functor P is an orthogonal \mathfrak{M} -functor. Correspondingly there is a smallest class \mathfrak{E} such that P is an orthogonal \mathfrak{E} -functor, namely $\mathfrak{E} = \text{Init}(P)^\perp$. Herrlich and Strecker [10] call P -morphisms of $\text{Init}(P)^\perp$ semi-universal.

3. SEMI-TOPOLOGICAL FUNCTORS AND LOCALLY ORTHOGONAL $(\mathfrak{E}, \mathfrak{M})$ -FUNCTORS.

Semi-topological functors arise from topologically algebraic functors very naturally. Namely, in the same way as Herrlich has done for $(\mathfrak{E}, \mathfrak{M})$ -topological functors (cf. [8], Lemma 6.1), one can prove the following «semi-final» property for topologically algebraic functors.

LEMMA 2. Let $P: \mathfrak{A} \rightarrow \mathfrak{X}$ be a topologically algebraic functor. Then each P -cocone $\xi: P \circ D \rightarrow \Delta X$ has a « P -semi-final lifting», i. e. there exists a P -morphism $e: X \rightarrow PA$ and an \mathfrak{A} -cocone

$$a: D \rightarrow \Delta A \quad \text{with} \quad (\Delta e)\xi = P \circ a$$

such that for all $\gamma: X \rightarrow PB$ and all $\beta: D \rightarrow \Delta B$ with $(\Delta \gamma)\xi = P \circ \beta$ there is a unique $t: A \rightarrow B$ with

$$(Pt)e = \gamma \quad \text{and} \quad (\Delta t)a = \beta.$$

DEFINITION 3 (Trnkova [27], Wischnewsky [28], Hoffmann [14], Tholen [22]). P is called *semi-topological* if any P -cocone admits a P -semi-final lifting.

By the «Duality Theorem» and the «Diagonal Lemma» proved in [23] it is possible to describe semi-topological functors also by a factorization property which is nearly as beautiful as the orthogonal one for topological algebraic functors (see Theorem 2 below).

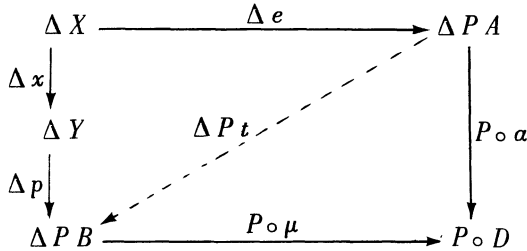
DEFINITION 4 (Tholen [23]). Let \mathfrak{E} and \mathfrak{M} be closed under composition with \mathfrak{A} -isomorphisms. P is called a *locally orthogonal $(\mathfrak{E}, \mathfrak{M})$ -functor* if

$\text{Iso}(P) \subset \mathfrak{E}$ and any P -cone $\xi : \Delta Y \rightarrow P \circ D$ admits a factorization

$$\xi = (P \circ \mu)(\Delta p),$$

with $p : Y \rightarrow PB$ in \mathfrak{E} and $\mu : \Delta B \rightarrow D$ in \mathfrak{M} such that, for all $e : X \rightarrow PA$ in \mathfrak{E} , $x : X \rightarrow Y$ in \mathfrak{X} and $\alpha : \Delta A \rightarrow D$ with $(P \circ \alpha)(\Delta e) = \xi(\Delta x)$, one has a unique $t : A \rightarrow B$ with

$$(Pt)e = px \quad \text{and} \quad \mu(\Delta t) = \alpha.$$



If \mathfrak{M} is not specified we briefly speak of a locally orthogonal \mathfrak{E} -functor. But note that \mathfrak{M} is not determined by \mathfrak{E} or vice versa. Clearly, every orthogonal $(\mathfrak{E}, \mathfrak{M})$ -functor is a locally orthogonal $(\mathfrak{E}, \mathfrak{M})$ -functor. The precise connection between Definitions 2 and 4 is given by the following Lemma 3 which can be proved in the same way as the little more special Lemma 7.3 of [23].

LEMMA 3. *The following conditions are equivalent:*

- (i) P is an orthogonal \mathfrak{E} -functor.
- (ii) P is a locally orthogonal \mathfrak{E} -functor and \mathfrak{E} is closed under composition, i. e. for all $(p, A) \in \mathfrak{E}$ and $e : A \rightarrow B$ in \mathfrak{A} with $(Pe, B) \in \mathfrak{E}$, one has $((Pe)p, B) \in \mathfrak{E}$.

Very similarly to Theorem 1 one has the following characterization of semi-topological functors which is proved in [23]. There we denoted the class of all P -morphisms occurring in some P -semi-final lifting by: $\text{Quot}(P)$.

THEOREM 2. *The following conditions are equivalent:*

- (i) P is semi-topological.
- (ii) P is a locally orthogonal $\text{Quot}(P)$ -functor.

(iii) P is a locally orthogonal \mathfrak{E} -functor for some \mathfrak{E} .

(iv) P has a left adjoint and \mathfrak{A} is a locally orthogonal \mathfrak{E} -category (i. e. $Id_{\mathfrak{A}}$ is a locally orthogonal \mathfrak{E} -functor) for some $\mathfrak{E} \subset \text{Mor } \mathfrak{A}$ such that the counit of P belongs pointwise to \mathfrak{E} .

REMARKS. 1° $\text{Quot}(P)$ is the smallest class \mathfrak{E} such that a semi-topological functor P is a locally orthogonal \mathfrak{E} -functor.

2° The P -morphisms of $\text{Quot}(P)$ are called P -quotients in [23] and semi-final in [10]. For P faithful one has $\text{Quot}(P) \subset \text{Init}(P)^\perp$ and the following «cone-free» characterization of each $e: X \rightarrow PA$ in $\text{Quot}(P)$: For any $y: X \rightarrow PB$ such that, for all

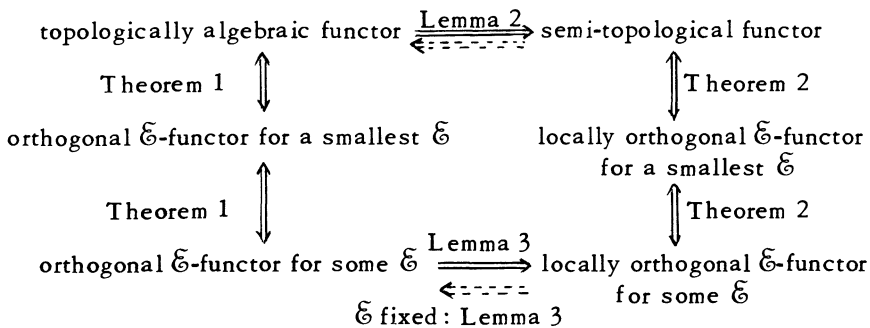
$$x: PC \rightarrow X \quad \text{and} \quad a: C \rightarrow A \quad \text{with} \quad Pa = ex,$$

there is a $b: C \rightarrow B$ with $yx = Pb$, then there exists a unique

$$t: A \rightarrow B \quad \text{with} \quad (Pt)e = y.$$

3° By M.B. Wischnewsky and the second author it was shown that every semi-topological functor $P: \mathfrak{A} \rightarrow \mathfrak{X}$ admits a factorization $P = T \circ E$ with T topological (= initially complete in the sense of Brümmer [2] and Herrlich [9]) and E an embedding of a full reflective subcategory (cf. [23]). Both T and E are trivially topologically algebraic. Since compositions of semi-topological functors are again semi-topological (cf. [23]), in $\mathcal{C}at$ the class of all semi-topological functors is the subcategory generated by all topologically algebraic functors.

The following diagram summarizes the results of Sections 2 and 3 and shows which problems are to be investigated in the following.



4. A CONCRETE COUNTER-EXAMPLE.

All examples of semi-topological functors given in [23] are already topologically algebraic. In the next section we shall prove the equivalence of both concepts under very mild additional conditions. Therefore, it is rather difficult to find semi-topological functors being not topologically algebraic. It is impossible to construct finite examples since Herrlich, Nakagawa, Strecker and Titcomb [11] have shown that a semi-topological functor $P: \mathcal{A} \rightarrow \mathcal{X}$ with \mathcal{A} finite is already topologically algebraic. (Another proof for this can be given using the pushout characterization 6.3 of [23].) So they constructed a counter-example with categories having finitely many objects and countable hom-sets. Another example using countable preordered sets (i.e. categories whose classes of objects are countable and whose hom-sets contain at most one morphism) was independently constructed by the first author of the present paper (cf. [3]). Answering a question of J. Adamek we now prove:

THEOREM 3. *There exists a semi-topological functor $P: \mathcal{A} \rightarrow \mathcal{E}_{nd}$ being not topologically algebraic.*

PROOF. Objects of \mathcal{A} are triples (X, A, ϕ) where X is a set, $A \subset X$ a subset and $\phi: \mathcal{P}A \rightarrow X$ a map defined on the power set of A such that

$$\phi(\{a\}) = a \text{ for all } a \in A.$$

A morphism $f: (X, A, \phi) \rightarrow (Y, B, \psi)$ is given by a map $f: X \rightarrow Y$ such that $f[A] \subset B$ and

$$f \circ \phi(M) = \psi(f[M]) \text{ for all } M \subset A.$$

$P: \mathcal{A} \rightarrow \mathcal{E}_{nd}$ is obvious: $P(X, A, \phi) = X$.

I. P is semi-topological: It suffices to construct P -semi-final liftings of discrete P -cocones. Let (Y_i, B_i, ψ_i) be objects of \mathcal{A} , and let $f_i: Y_i \rightarrow Z$, $i \in I$ be maps. Define

$$U = \bigcup_{i \in I} f_i[B_i], \quad V = \mathcal{P}U \cup \{ \{z\} \mid z \in Z \}$$

and consider the smallest equivalence relation \sim on V with the following properties:

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- (1) $f_i[N] \sim \{f_i \circ \psi_i(N)\}$ for all $i \in I, N \subset B_i$.
 (2) If $R \subset U \times U$ with $\{x\} \sim \{y\}$ for all $(x, y) \in R$, then

$$\{x \mid \exists y : (x, y) \in R\} \sim \{y \mid \exists x : (x, y) \in R\}.$$

Now let $s: Z \rightarrow V$ be the singleton map and $p: V \rightarrow X =: V/\sim$ be the projection map. Define $q = p \circ s, A = q[U]$ and

$$\phi: \mathcal{P}A \rightarrow X \text{ by } \phi(M) = p(q^{-1}[M] \cap U).$$

We shall prove that (X, A, ϕ) and q yield a semi-final lifting.

Step 1. $K \sim q^{-1}[q[K]] \cap U$ for all $K \subset U$.

Proof 1. Consider

$$R := \{(x, y) \in K \times U \mid \{x\} \sim \{y\}\}$$

and apply property (2) of \sim .

Step 2. $q \circ f_i: (Y_i, B_i, \psi_i) \rightarrow (X, A, \phi)$ in \mathcal{A} for all $i \in I$.

Proof 2. $(X, A, \phi) \in \text{Cb } \mathcal{A}$ since

$$\phi(\{q(x)\}) = p(q^{-1}[\{q(x)\}] \cap U) = p(\{x\}) = q(x)$$

for all $x \in U$ because of Step 1. Trivially,

$$q \circ f_i[B_i] \subset A \text{ for all } i \in I.$$

Finally, for $N \subset B_i$ we have

$$\begin{aligned} q \circ f_i \circ \psi_i(N) &= p(\{f_i \circ \psi_i(N)\}) \\ &= p(f_i[N]) && \text{(property (1))} \\ &= p(q^{-1}[q \circ f_i[N]]) && \text{(Step 1)} \\ &= \phi(q \circ f_i[N]). \end{aligned}$$

Step 3. For $(Y, B, \psi) \in \text{Ob } \mathcal{A}$ and $g: Z \rightarrow Y$ with

$$g \circ f_i: (Y_i, B_i, \psi_i) \rightarrow (Y, B, \psi) \text{ in } \mathcal{A} \text{ for all } i \in I,$$

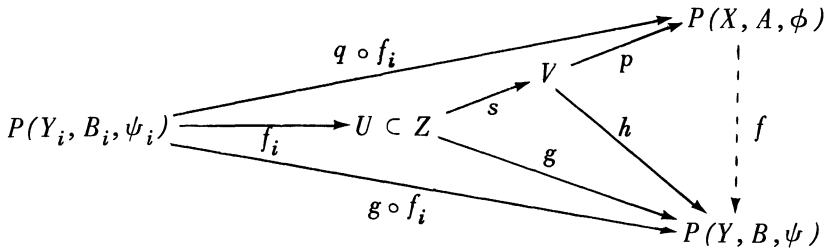
there exists a unique $f: (X, A, \phi) \rightarrow (Y, B, \psi)$ in \mathcal{A} with $f \circ q = g$.

Proof 3. Define a map $h: V \rightarrow Y$ by

$$h(\{z\}) = g(z) \text{ for all } z \in Z \text{ and } h(K) = \psi(g[K]) \text{ for all } K \subset U$$

(h is well defined). In order to get f with $f \circ p = h$ we have to show that the equivalence relation induced by h fulfills properties (1) and (2).

This is easily done using the fact that all $g \circ f_i$ are morphisms of \mathcal{A} .



$g \circ f_i[B_i] \subset B$ for all $i \in I$ implies $f[A] \subset B$. Furthermore, for $M \subset A$, one has $M = p[q^{-1}[M] \cap U]$ and therefore

$$\begin{aligned} f \circ \phi(M) &= h(q^{-1}[M] \cap U) = \psi(g[q^{-1}[M] \cap U]) = \\ &= \psi(f \circ p[q^{-1}[M] \cap U]) = \psi(f[M]). \end{aligned}$$

Let $f': (X, A, \phi) \rightarrow (Y, B, \psi)$ be any \mathcal{U} -morphism with $f' \circ q = g$. Then for all $K \subset U$ one has

$$\begin{aligned} f' \circ p(K) &= f' \circ \phi(q[K]) && \text{(Step 1)} \\ &= \psi(f' \circ q[K]) = \psi(g[K]) = f \circ p(K). \end{aligned}$$

Together with $f' \circ p(\{z\}) = f \circ p(\{z\})$ for all $z \in Z$ one has $f' = f$.

II. P is not topologically algebraic: For all ordinals $\alpha \geq 2$ define: $Y_\alpha := \{\xi \mid \xi < \alpha\}$ and $\psi_\alpha: \mathcal{P} Y_\alpha \rightarrow Y_\alpha$ by

$$\psi_\alpha(N) := \begin{cases} \xi & \text{for } N = \{\xi\} \\ 0 & \text{for } N = Y_\alpha \\ \min(Y_\alpha \setminus N) & \text{otherwise.} \end{cases}$$

Clearly, all $(Y_\alpha, Y_\alpha, \psi_\alpha) \in \text{Ob } \mathcal{U}$. Assume there exists a P -initial \mathcal{U} -cone

$$(h_\alpha: (X, A, \phi) \rightarrow (Y_\alpha, Y_\alpha, \psi_\alpha))_{\alpha \geq 2}$$

such that $\{1\} \hookrightarrow Y_\alpha$ factorizes over h_α , $\alpha \geq 2$.

Step 1. $A = X$.

Proof 1. Consider

$$(\{0, 1\}, \{1\}, \psi) \in \text{Ob } \mathcal{U} \quad \text{with } \psi(\emptyset) = 0 \text{ and } \psi(\{1\}) = 1.$$

Let $x \in X$ and define

$$g: \{0, 1\} \rightarrow X \quad \text{by } g(0) = \phi(\emptyset) \text{ and } g(1) = x.$$

Then all $h_\alpha \circ g$ are morphisms of \mathcal{U} , hence g itself, and therefore $x = g(1)$

is in A .

Step 2. h_α is surjective, $\alpha \geq 2$.

Proof 2. If not, form $\eta = \min(Y_\alpha \setminus h_\alpha[X])$. Obviously, $h_\alpha[X]$ is not a singleton, since

$$1 \in h_\alpha[X] \quad \text{and} \quad 0 = \psi_\alpha(\emptyset) = h_\alpha \circ \phi(\emptyset) \in h_\alpha[X].$$

Therefore we get

$$\eta = \psi_\alpha(h_\alpha[X]) = h_\alpha \circ \phi(X) \in h_\alpha[X].$$

This is impossible.

From the assertion of Step 2 we get $\text{card } X \geq \alpha$ for all ordinals α , so that our assumption must be wrong.

REMARKS. 1° In Part II of the above proof we do not have to assume that $(\{1\} \rightarrow Y_\alpha)_{\alpha \geq 2}$ factorizes over a P -epimorphism followed by a P -initial \mathcal{A} -cone. We could take any P -morphism and got the contradiction, too. But note that, if one generalizes Definition 1 in this way, one gets a properly greater class of functors.

2° The functor P , being semi-topological, has, in particular, a left adjoint which induces a monad on \mathfrak{E}_{ns} . Its Eilenberg-Moore algebras are just pointed sets.

From Theorem 3 and Remark 3 of Section 3 we get:

COROLLARY. 1° *There exists a topological functor $T: \mathfrak{B} \rightarrow \mathfrak{E}_{\text{ns}}$ and a full reflective subcategory \mathcal{A} of \mathfrak{B} such that T/\mathcal{A} is not topologically algebraic.*

2° *Compositions of topologically algebraic functors need not be topologically algebraic, even if their composition ends at \mathfrak{E}_{ns} .*

REMARK. In [24], Corollary 31.4, the second author has studied the problem of cancellability of topologically algebraic functors:

Let $P = T \circ U$ be topologically algebraic such that

$$U(\text{Init}(P)) \subset \text{Init}(T).$$

Then U is semi-topological; moreover U is topologically algebraic, pro-

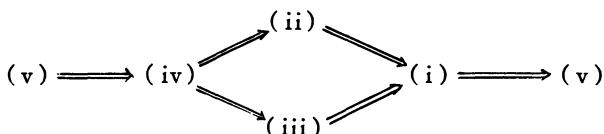
5. AN EQUIVALENCE THEOREM.

Having the counter-example of Section 4 it is natural to look for conditions under which both, topologically algebraic and semi-topological, is the same. The conditions we give in the following are also sufficient to get a largest \mathfrak{E} such that one has a (locally) orthogonal \mathfrak{E} -functor, namely $\mathfrak{E} = \text{Epi}(P)$.

THEOREM 4. *Let \mathfrak{X} be (co)complete and let \mathfrak{A} be cowell-powered and well-powered with respect to extremal monomorphisms. Then the following conditions are equivalent:*

- (i) *P is semi-topological.*
- (ii) *P is topologically algebraic.*
- (iii) *P is a locally orthogonal $\text{Epi}(P)$ -functor.*
- (iv) *P is an orthogonal $\text{Epi}(P)$ -functor.*
- (v) *\mathfrak{A} is (co)complete, P is faithful and right-adjoint.*

PROOF. One looks at the diagram



and can restrict oneself to the first implication because the rest is well-known. We have to distinguish two cases: \mathfrak{A} is cocomplete and \mathfrak{A} is complete. The first case was considered in [23], Corollary 6.5: \mathfrak{A} is a (locally) orthogonal $\text{Epi}(\mathfrak{A})$ -category, hence, by Theorem 2 and Lemma 3, P is an orthogonal $\text{Epi}(P)$ -functor. Now let \mathfrak{A} be complete. Let \mathfrak{M} be the class of P -initial extremal monomorphisms of \mathfrak{A} . Because P is faithful, \mathfrak{M} will contain all regular monomorphisms, and \mathfrak{M} is closed under (arbitrary) pullbacks. So \mathfrak{A} has $(\text{Epi}(\mathfrak{A}), \mathfrak{M})$ -factorizations of morphisms fulfilling the diagonalization property. Now it remains to construct factorizations of cones from factorizations of morphisms. For this one uses the following lemma (an analogous result was proved in [20], Lemma 3.4), which completes the proof.

LEMMA 4. *Let \mathfrak{A} have products and $(\mathfrak{E}, \mathfrak{M})$ -factorizations of morphisms*

fulfilling the diagonalization property with

$$\mathfrak{E} \subset \text{Epi}(\mathfrak{A}) \text{ and } \mathfrak{M} \subset \text{Init}(P).$$

Let \mathfrak{A} be \mathfrak{E} -cowell-powered, and let P be faithful and right-adjoint. Then P is an orthogonal $\hat{\mathfrak{E}}$ -functor with $\hat{\mathfrak{E}}$ containing all P -morphisms

$$e: X \rightarrow P A \text{ with } (\epsilon A)(F e): F X \rightarrow A \text{ in } \mathfrak{E}$$

(F being the left adjoint and ϵ the counit).

REMARKS. 1° In Theorem 4 the assumption of well-poweredness with respect to extremal monomorphisms is only needed in case \mathfrak{X} is complete. Generally, all the completeness and smallness assumptions of Theorem 4 are only needed to prove that \mathfrak{A} is an orthogonal $\text{Epi}(\mathfrak{A})$ -category. This property also yields equivalence of conditions (i) - (iv) (cf. Herrlich, Nakagawa, Strecker, Titchcomb [11]).

2° One observes that we have proved in Theorem 4, (v) \Rightarrow (iv) that P is an orthogonal $(\text{Epi}(P), \mathfrak{M})$ -functor with $\mathfrak{M} \subset \text{Mono}(\mathfrak{A})$, where we denote by $\text{Mono}(\mathfrak{A})$ the class of all *mono-cones* $\mu: \Delta A \rightarrow D$, i. e., for

$$u, v: B \rightarrow A \text{ in } \mathfrak{A} \text{ with } \mu(\Delta u) = \mu(\Delta v)$$

one has $u = v$. From this fact alone it follows that \mathfrak{A} has coequalizers. (cf. [23], Proposition 5.3). This little observation proves once more Adamek's useful result on colimits of algebras (cf. [1]):

COROLLARY. Let \mathfrak{X} be a cocomplete category with $(\mathfrak{E}, \mathfrak{M})$ -factorizations of morphisms fulfilling the diagonalization property with $\mathfrak{E} \subset \text{Epi}(\mathfrak{X})$ and $\mathfrak{M} \subset \text{Mono}(\mathfrak{X})$ and let \mathfrak{X} be \mathfrak{E} -cowell-powered. Then for every functor $T: \mathfrak{X} \rightarrow \mathfrak{X}$ which preserves \mathfrak{E} the category of T -algebras has coequalizers.

PROOF. \mathfrak{X} is an orthogonal $(\mathfrak{E}, \mathfrak{M})$ -category with $\mathfrak{M} \subset \text{Mono}(\mathfrak{X})$. Because of $T\mathfrak{E} \subset \mathfrak{E}$ the same property holds for the category of T -algebras.

6. CATEGORIES IN WHICH EVERY INITIAL CONE IS A MONO-CONE.

In [23], it is shown that every locally orthogonal factorization structure \mathfrak{E} of a semi-topological functor $P: \mathfrak{A} \rightarrow \mathfrak{X}$ leads to a topological completion of P , i. e. there is a full reflective embedding $E: \mathfrak{A} \rightarrow \mathfrak{B}$ and

a topological functor $T: \mathcal{B} \rightarrow \mathcal{X}$ with $T \circ E = P$. \mathcal{B} can be taken as a full subcategory of the comma-category $\langle \mathcal{A}, P \rangle$ (whose objects are P -morphisms and whose morphisms are obvious commutative diagrams) with $\mathcal{C} \cap \mathcal{B} = \mathcal{E}$. In case $\mathcal{E} = \text{Quot}(P)$ one gets the smallest topological completion, the *Mac Neille completion* of P (cf. Herrlich [9], Porst [17]). Herrlich and Strecker [10] show that for a topologically algebraic functor and $\mathcal{E} = \text{Init}(P)^\perp$, one gets the so-called *universal completion*, which is just the reflection from the (meta-)category of faithful and amnesic functors over \mathcal{X} with initiality preserving functors as morphisms into the full subcategory of topological functors over \mathcal{X} . They prove that for the underlying set functor of a full epireflective subcategory of topological spaces, consisting of T_0 -spaces and containing at least one space with more than one element, the Mac Neille completion and the universal completion are the same because of

$$\text{Quot}(P) = \text{Init}(P)^\perp ;$$

$\text{Quot}(P)$ contains just all P -morphisms $p: X \rightarrow PA$ with p surjective.

We try to get an analogous result for monadic functors and, more generally, regular functors (cf. Herrlich [7]) over \mathcal{E}_{na} . However, already simple examples like semi-groups and groups show that the situation is more complicated for «algebraic» functors. We are only able to give a sufficient condition which allows an effective computation of $\text{Init}(P)^\perp$ and therefore of the universal completion of P . The key for this is the following more general Lemma 5 which seems to be interesting by itself. From [23] we repeat the notion of a *P -semi-initial factorization*

$$(*) \quad \xi = (P \circ \mu)(\Delta p) ,$$

that is the universal property of Definition 4 holds only with e the identity, i. e. for all P -comorphisms $x: PA \rightarrow X$ and \mathcal{A} -cones

$$a: A \rightarrow D \quad \text{with} \quad P \circ a = \xi(\Delta x)$$

one has a unique $t: A \rightarrow B$ with

$$Pt = px \quad \text{and} \quad \mu(\Delta t) = a .$$

Clearly, if $\mu: \Delta B \rightarrow D$ is P -initial, then (*) is P -semi-initial. A P -morphism $e: X \rightarrow PA$ belongs to $\text{Quot}(P)$ iff e allows no proper P -semi-initial factorization, i. e. if $(Pm)q = e$ is a P -semi-initial factorization then m is an isomorphism (cf. [23]).

LEMMA 5. Let $P \approx \mathfrak{Q}(G, -): \mathfrak{Q} \rightarrow \mathfrak{E}_{\text{nd}}$ be a representable functor such that G admits non-trivial epimorphic endomorphisms, i. e. there exists an epimorphism $e: G \rightarrow G$ different from the identity. Let $(f_i: A \rightarrow B_i)_I$ be a (discrete) \mathfrak{Q} -cone and let $(Pm_i) \circ \phi = Pf_i$ be a semi-initial factorization of the P -cone $(Pf_i: PA \rightarrow PB_i)_I$ by an \mathfrak{Q} -cone $(m_i: C \rightarrow B_i)_I$ and a surjective map $\phi: PA \rightarrow PC$. Then $(Pm_i)_I$ is a mono-cone, i. e.

$$m_i u = m_i v \text{ for all } i \in I \text{ with } u, v: G \rightarrow C$$

implies $u = v$.

PROOF. We can assume $P = \mathfrak{Q}(G, -)$. Because of the semi-initiality ϕ can be written as $\phi = Pp$ with

$$p: A \rightarrow C \text{ and } m_i p = f_i \text{ for all } i \in I.$$

There are

$$r, s: G \rightarrow A \text{ with } pr = u, ps = v,$$

since ϕ is surjective. Define a map $\psi: PG \rightarrow PA$ by

$$\psi(g) := \begin{cases} rg & \text{for } g \in PG, g \neq G \\ s & \text{for } g = G. \end{cases}$$

Then

$$(Pf_i) \circ \psi(G) = (Pf_i)(s) = f_i s = m_i v = m_i u = f_i r = P(f_i r)(G),$$

hence (by definition of ψ)

$$(Pf_i) \circ \psi = P(f_i r).$$

By semi-initiality, there exists a morphism

$$w: G \rightarrow C \text{ with } Pw = (Pp) \circ \psi.$$

One gets

$$w = (Pw)(G) = (Pp) \circ \psi(G) = ps = v$$

and

$$u g = p r g = (P p) \circ \psi (g) = (P w)(g) = w g = v g$$

for all $g \neq G$. In case $g = e$ this equation yields $u = v$.

THEOREM 5. *Let $P \approx \mathcal{U}(G, -): \mathcal{U} \rightarrow \mathcal{E}_{na}$ be a representable functor such that G admits non-trivial epimorphic endomorphisms. Then every P -initial \mathcal{U} -cone is a mono-cone.*

PROOF. Application of Lemma 5 with ϕ the identity map yields that the P -image of a P -initial \mathcal{U} -cone is a mono-cone, hence a mono-cone itself. (Note that P need not be faithful.)

Since mono-cones are P -initial for regular functors one gets :

COROLLARY. *Let $P: \mathcal{U} \rightarrow \mathcal{E}_{na}$ be a regular functor and let G be the free object of \mathcal{U} with one generator. If there exists an epimorphism $e: G \rightarrow G$ different from the identity, then $\text{Init}(P) = \text{Mono}(\mathcal{U})$.*

Denote by $\text{Gen}(P)$ the class of all P -generating morphisms $g: X \rightarrow P A$, i.e. the induced morphism $\hat{g}: F X \rightarrow A$ is a regular epimorphism (F being the left adjoint of P).

COROLLARY. *Under the assumptions of the above Corollary, the objects of a universal completion \mathcal{B} of P are P -generating morphisms, and morphisms of \mathcal{B} are given by commutative diagrams*

$$\begin{array}{ccc} X & \xrightarrow{u} & Y \\ g \downarrow & & \downarrow h \\ P A & \xrightarrow{P f} & P B \end{array} .$$

The topological functor $\mathcal{B} \rightarrow \mathcal{E}_{na}$ forgets all of the diagram besides the upper row.

PROOF. By the definition of a regular functor, P is an orthogonal $(\text{Gen}(P), \text{Mono}(\mathcal{U}))$ -functor, hence

$$\text{Gen}(P) = \text{Mono}(\mathcal{U})^\perp = \text{Init}(P).$$

EXAMPLES. 1° The underlying set functor of the following monadic cat-

egories fulfills the assumption of Theorem 5:

- (abelian) groups ($Z \rightarrow Z, x \mapsto -x$),
- R -modules ($\text{char } R \neq 2$) ($R \rightarrow R, x \mapsto -x$),
- (commutative) rings ($Z[X] \rightarrow Z[X], X \mapsto -X$),
- (commutative) R -algebras ($R[X] \rightarrow R[X], X \mapsto X+1$).

2° The following monadic categories contain initial cones which are not mono-cones:

- (commutative) semi-groups (see below),
- (commutative) monoids (see below),
- pointed sets (cf. Herrlich-Strecker [10]),
- directed graphs (cf. Herrlich-Strecker [10]).

The example which can be used in case of semi-groups is as trivial as intelligent. Consider the map

$$\{o, x, y\} \rightarrow \{o, x\} \text{ with } o \mapsto o \text{ and } x, y \mapsto x$$

and take the following table of composition:

	o	x	y
o	o	o	o
x	o	o	o
y	o	o	o

In case of monoids one has to adjoin a unit element.

3° The category \mathcal{C}_{amp} of compact T_2 -spaces is monadic over \mathcal{E}_{ns} , but does not fulfill the assumption of Theorem 5 (the one point space does not admit a non-trivial endomorphism). Nevertheless, initial cones are mono-cones (and vice versa): Let $(m_i: B \rightarrow A_i)_{i \in I}$ be initial and assume $m_i(x) = m_i(y)$ for all $i \in I$. Consider $w: [0, 1] \rightarrow B$ with

$$w|_{[0, \frac{1}{2}]} = \text{const } x \text{ and } w|_{(\frac{1}{2}, 1]} = \text{const } y.$$

Now, $m_i \circ w$ is continuous for all $i \in I$, hence w , too, and $x = y$.

The Mac Neille completion of \mathcal{C}_{amp} is properly smaller than its universal completion: Consider the dense (hence $\text{Init}(P)^{\perp}$ -)map

$$j: \mathbb{N} \rightarrow \mathbb{N} \cup \{\infty\}$$

from the (discrete) space N of natural numbers into its Alexandroff compactification and prove that j does not belong to $\text{Quot}(P)$. It suffices to show that j admits a proper semi-initial factorization. For this take the factorization of j over the Stone-Čech compactification of N . If there

$$\begin{array}{ccc}
 & \beta N & \\
 \beta_N \nearrow & & \searrow j \\
 N & \xrightarrow{j} & N \cup \{\infty\}
 \end{array}$$

is a map $g: K \rightarrow N$ from a compact Hausdorff space K into N with $j \circ g$ continuous, then $g(K)$ must be finite and discrete, and $\beta_N \circ g$ is continuous, too. Therefore the factorization is semi-initial.

4° Even for monadic categories over \mathfrak{E}_{na} fulfilling the assumption of Theorem 5 the Mac Neille completion can be properly smaller than the universal completion. For instance the categories of (commutative) groups and (commutative) rings contain a P -generating morphism which is not in $\text{Quot}(P)$:

$$\{1\} \hookrightarrow Z_2 = \{0, 1\}.$$

There is, namely, no map $A \rightarrow \{1\}$ such that the composition $A \rightarrow Z_2$ is an homomorphism since 0 does not belong to the image of this map. On the other hand, $\{1\} \hookrightarrow Z_2$ is not universal, hence not the semi-final lifting of the empty P -cocone.

5° The category of Banach spaces over K ($K = \mathbb{R}$ or \mathbb{C}) with norm decreasing maps as morphisms (P being the unit-ball functor

$$B \mapsto \{x \in B \mid \|x\| \leq 1\})$$

is not monadic over \mathfrak{E}_{na} but fulfills the assumption of Theorem 5 ($x \mapsto x$ is a non-trivial epimorphism of K). The category of ordered sets with monotone maps as morphisms is not monadic over \mathfrak{E}_{na} and does not fulfill the assumption of Theorem 5, but initial cones are mono-cones, too (if $(m_i: B \rightarrow A_i)_I$ is initial and $m_i(x) = m_i(y)$ for all $i \in I$, consider the maps $g, h: \{0 \leq 1\} \rightarrow B$ with

$$g(0) = h(1) = y \text{ and } g(1) = h(0) = y).$$

REMARKS ON TOPOLOGICALLY ALGEBRAIC FUNCTORS

REMARKS. 1° The sufficient condition of Lemma 5 that the representing object G admits a non-trivial epimorphic endomorphism can be obviously generalized: One only needs that all endomorphisms g of G different from the identity form an epi-cone, i. e.

$$ug = vg \text{ for all } g \neq G \text{ implies } u = v.$$

Then one gets many new examples for Theorem 5. For instance, let S be the Sierpinski space (i. e. the only two-point space which is neither discrete nor indiscrete). Then the above condition is fulfilled although there is no non-trivial epimorphic endomorphism of S . Hence, with $P = \mathcal{T}_{\text{ap}}(S, -)$ P -initial cones are mono-cones.

2° Herrlich and Strecker [10] characterized topologically algebraic functors as those functors which admit a reflective universal completion. Therefore, from Theorem 3 it follows that there is a functor $P: \mathcal{A} \rightarrow \mathcal{C}_{\text{ns}}$ admitting a reflective Mac Neille completion but not having a *reflective* universal completion. One can sharpen this result by proving that the universal completion of the functor constructed in Theorem 3 fails to exist (in the same universe). However, to prove this one needs a stronger set-theoretical assumption which, if K is a proper class and L is not empty, guarantees the existence of a surjection $\sigma: K \rightarrow L$. (Together with an axiom of foundation the following strong version of an axiom of choice is sufficient: There exists a function τ with $\tau(X) \in X$ for all non-void sets X .) For details the reader is referred to [4]. More generally the question of existence of universal completions is treated in a forthcoming paper by Adamek, Herrlich and Strecker (cf. [31]). In case of the countable counterexamples given in [3] and [11] the universal completion, of course, does exist (but is not reflective), since all the categories in question are small (cf. [9]). Therefore, the above set-based functor P is, in a sense, «less topologically algebraic» than the functors constructed in [3, 11].

3° The functor P of Theorem 3 is (strongly) fibre-small and (hence) has a fibre-small Mac Neille completion. Therefore, besides completeness, cocompleteness, wellpoweredness and existence of a reflective Mac Neille completion, also fibre-smallness of the Mac Neille completion does not

guarantee existence of the universal completion. The essential point is cowellpoweredness as Theorem 4 shows.

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