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CENTRAL DECOMPOSITIONS FOR COMPACT CONVEX SETS

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1. Introduction

In this paper we continue the investigation, begun in [8], into facial decompositions for compact convex sets K. In particular we study conditions on K under which the Bishop decomposition determines $A(K)$, or at least determines the centre of $A(K)$; the special case when K is the state space of a unital C^*-algebra is investigated in this connection. In the final section we prove a result for function algebras which is related to facial decompositions, and we also give a simple geometrical proof of the Hoffman-Wermer theorem.

We are indebted to several mathematicians for discussions concerning the contents of this paper, and in particular to E. G. Effros and E. Størmer.

2. Terminology and preliminaries

Let K be a compact convex subset of a locally convex Hausdorff space and let $A(K)$ denote the Banach space of all continuous real-valued affine functions on K, endowed with the supremum norm. The set of extreme points of K will be denoted by ∂K, and its closure by $\overline{\partial K}$. The sets of constancy in ∂K for the central functions in $A(K)$ form a decomposition $\{E_\alpha\}$ of ∂K, such that $E_\alpha = \partial F_\alpha$ for some closed split face F_α of K (cf. [1]). It was shown [8] that the disjoint faces F_α always cover $\overline{\partial K}$ and that they *determine* $A(K)$ in the sense that

$$A(K)|\overline{\partial K} = \{f \in C_{\boldsymbol{R}}(\overline{\partial K}) : f|(F_\alpha \cap \overline{\partial K}) \in A(F_\alpha)|(F_\alpha \cap \overline{\partial K}), \forall\alpha\}.$$

The family $\{F_\alpha\}$ is called the *Šilov decomposition* for K.

A subset E of ∂K is a set of *antisymmetry* if f is constant on E whenever $f \in A(K)$ and $f|\overline{\mathrm{co}}E$ belongs to the centre of $A(\overline{\mathrm{co}}E)$. (Here, $\overline{\mathrm{co}}E$ denotes the closed convex hull of E.) $A(K)$ is said to be *antisymmetric* if ∂K is a set of antisymmetry. It was shown [8] that each x in ∂K belongs to a maximal set of antisymmetry E_β and that each E_β has the form ∂F_β, where $\{F_\beta\}$ is a family of pairwise-disjoint closed split faces of K. The family $\{F_\beta\}$ is called the *Bishop decomposition* for K.

The *essential set* for $A(K)$ is the smallest closed subset E of $\overline{\partial K}$ such that $A(K)|\overline{\partial K}$ contains the ideal $\{f \in C_{\boldsymbol{R}}(\overline{\partial K}) : f(x) = 0, \forall x \in E\}$. The set $F_E = \overline{\mathrm{co}}E$ is the smallest closed (split) face F of K such that whenever B and C are closed subsets of $\overline{\partial K}$, with $B \supseteq F \cap \overline{\partial K}$ and $C \cap F = \emptyset$, then $\mathrm{co}B$ and $\overline{\mathrm{co}}C$ are split faces of K (cf. [8]). The annihilator in $A(K)$ of F_E is the *essential ideal* of $A(K)$.

K is said to satisfy *Størmer's axiom* if the closed convex hull of an arbitrary family of closed split faces of K is again a split face of K. For x in ∂K let F_x denote the smallest closed split face of K containing x, and let U denote the family of all continuous affine bijections $\alpha : K \to K$ such that $\alpha(F_x) = F_x$ for all x in ∂K. Then K is said to admit *sufficiently many inner automorphisms* if $F_x = \overline{\mathrm{co}}\,\{\alpha x : \alpha \in U\}$ for all x in ∂K.

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In [8, Example 10] an example was given of a simplex K for which the Bishop decomposition fails to determine $A(K)$. However, using an adaptation of a proof of Glicksberg [10], we proved that if ∂K is closed then the Bishop decomposition does determine $A(K)$; we now extend that result.

THEOREM (1): *If the Bishop decomposition for K covers $\overline{\partial K}$ then the decomposition determines $A(K)$.*

PROOF: Let μ be an extreme point of the unit ball of the space of all Radon measures on $\overline{\partial K}$ which annihilate $A(K)|\overline{\partial K}$. As in [8, Theorem 8] the result will follow if we can show that the support D of μ is contained in some F_β.

Suppose that no single F_β contains D. Let G be the smallest closed split face of K containing all sets F_β which intersect D. Then ∂G is not a set of antisymmetry, and so there exists a central function f in $A(G)$ which is not constant. Since the restriction of f to F_β is central in $A(F_\beta)$, f must be constant on each F_β. For each g in $A(G)$ there exists an h in $A(G)$ such that $h(x) = f(x)g(x)$ for all x in ∂G. If y belongs to D then

y lies is some F_β contained in G, and since f is constant on F_β we have $h(y) = f(y)g(y)$.

Assuming that $0 < f < 1$, define Radon measures μ_1, μ_2 on $\overline{\partial K}$ by

$$\mu_1(u) = \int fu\, \mathrm{d}\mu, \qquad \mu_2(u) = \int (1-f)u\, \mathrm{d}\mu$$

for all u in $C_{\boldsymbol{R}}(\overline{\partial K})$. Then $\mu = \mu_1 + \mu_2$ and $\|\mu_1\| + \|\mu_2\| = 1$. For each g in $A(K)$ there exists, by the above argument together with the fact that G is a closed split face of K, an h in $A(K)$ such that $h(y) = f(y)g(y)$ for all y in D. Therefore

$$\int g\, \mathrm{d}\mu_1 = \int fg\, \mathrm{d}\mu = \int h\, \mathrm{d}\mu = 0,$$

so that μ_1, and similarly μ_2, annihilates $A(K)|\overline{\partial K}$. Since μ is extreme it follows that $\mu_1 = \mu = \mu_2$, and that f is constant on D.

If $f(y) = \lambda$ for all y in D, let $H = \overline{\mathrm{co}}\, \{x \in \partial G : f(x) = \lambda\}$. Then H is a split face of G, and hence a split face of K. Moreover H contains all the faces F_β which intersect D, so that $H = G$. But then f is constant on G, and this contradiction completes the proof.

A consequence of Theorem 1 is that if every point of $\overline{\partial K}$ belongs to some closed split antisymmetric face of K then the Bishop decomposition determines $A(K)$. The following example shows that the converse of Theorem 1 is false even for metrizable simplexes.

EXAMPLE: Let K be a compact metrizable simplex such that $\overline{\partial K} = K$ (cf. [14]). Define $\tilde{K}$ such that

$$A(\tilde{K}) = \{\underline{f} = \{f_n\}, n \geqq 0 : f_n \in A(K),\, f_n \to f_0 \in L\},$$

where L is a fixed two-dimensional subspace of $A(K)$ containing the constants and $\|\underline{f}\| = \sup\, \{\|f_n\| : n \geqq 0\}$. Then $\tilde{K}$ is the closed convex hull of countably many copies K_n of K, together with a closed line segment I. The Bishop decomposition for $\tilde{K}$ consists of the faces K_n and the extreme points of I, and $\overline{\partial \tilde{K}}$ is the union of the K_n together with I. Hence the Bishop decomposition fails to cover $\overline{\partial \tilde{K}}$. However, if $\tilde{f} \in C_{\boldsymbol{R}}(\overline{\partial \tilde{K}})$ is such that $\tilde{f}|K_n \in A(K_n)$ for each n it is straightforward to check that $\tilde{f}$ is affine on I, and that $\tilde{f}$ is the restriction of some $\underline{f}$ in $A(\tilde{K})$; therefore the Bishop decomposition determines $\tilde{K}$. Finally, since K is a simplex $A(K)$ has the Riesz interpolation property, and if $\underline{f}_1, \underline{f}_2, \underline{g}_1, \underline{g}_2 \in A(\tilde{K})$ with

$$\underline{f}_1 + \underline{\varepsilon},\ \underline{f}_2 + \underline{\varepsilon} \leqq \underline{g}_1, \underline{g}_2$$

for some $\varepsilon > 0$, it is easy to construct coordinately an $h \in A(\tilde{K})$ with $\underline{f_1}, \underline{f_2} \leqq \underline{h} \leqq \underline{g_1}, \underline{g_2}$. It follows that $\tilde{K}$ is a simplex.

It has been pointed out to us that if K is the state space of the C^*-algebra in [16, p. 136] then, in a similar manner, it can be shown that the Bishop decomposition determines $A(K)$ but does not cover $\overline{\partial K}$. We do not know any necessary and sufficient conditions for the Bishop decomposition to determine $A(K)$.

Every closed linear subspace L of $C_{\mathbf{R}}(\overline{\partial K})$ which contains $A(K)|\overline{\partial K}$ is isometrically order-isomorphic to a space $A(H)$, for some compact convex set H. The next result shows that there always exists a smallest such space L for which the Bishop decomposition for H determines $A(H)$.

THEOREM (2): *If $\hat{K}$ denotes the state space of the ordered Banach space $L = \{f \in C_{\mathbf{R}}(\overline{\partial K}) : f|(F_\beta \cap \overline{\partial K}) \in A(F_\beta)|(F_\beta \cap \overline{\partial K}), \forall \beta\}$, then the Bishop decomposition for $\hat{K}$ determines $A(\hat{K})$.*

PROOF: Since the Choquet boundary $\partial \hat{K}$ for L contains ∂K, the Šilov boundaries $\overline{\partial \hat{K}}$ for L and $\overline{\partial K}$ for $A(K)|\overline{\partial K}$ coincide. Let $x \in \partial K$ and let C be the maximal $A(\hat{K})$-antisymmetric subset of $\partial \hat{K}$ which contains x. If $x \in F_\beta$ and if G is the smallest closed split face of $\hat{K}$ which contains ∂F_β, then we will show that $A(G)$ is antisymmetric.

In fact, let f be central in $A(G)$, so that f extends to a function in $A(\hat{K})$. Then, if $g \in A(F_\beta) = A(K)|F_\beta$, there exists an h in $A(\hat{K})$ such that $h(y) = f(y)g(y)$ for all y in ∂F_β. The definition of L, together with the fact that F_β belongs to the Bishop decomposition for K, now implies that f is constant on ∂F_β. Since f is central in $A(G)$ and G is the smallest closed split face of $\hat{K}$ containing ∂F_β, we conclude that f is constant on G, therefore C contains ∂F_β.

If $f \in C_{\mathbf{R}}(\overline{\partial K})$ and if $f|C \in A(\overline{\text{co}}\, C)|C$ (where $\overline{\text{co}}\, C$ denotes the closed convex hull of C in $\hat{K}$, and hence belongs to the Bishop decomposition for $\hat{K}$) then $f|C$ has an extension $f \in L$. Hence $f|\partial F_\beta$ has an extension belonging to $A(F_\beta)$. Since $x \in \partial K$ was chosen arbitrarily it follows that the Bishop decomposition for $\hat{K}$ determines $A(\hat{K})$.

For the simplex K in [8, Example 10] the set $\hat{K}$ is a Bauer simplex, identifiable with the base of the positive cone in ℓ_1. In this example the centres of $A(K)$ and $A(\hat{K})$ are distinct, and the essential ideals of $A(K)$ and $A(\hat{K})$ are also distinct. If K (not necessarily a simplex) satisfies certain conditions, then the following result shows that these distinctions do not occur.

THEOREM (3): *Let K satisfy Størmer's axiom and admit sufficiently many inner automorphisms. Then*

$$(centre\ A(K))|\overline{\partial K} = (centre\ A(\hat{K}))|\overline{\partial K}$$

$$= \{f \in C_{\mathbf{R}}(\overline{\partial K}) : f|(F_\beta \cap \overline{\partial K}) \text{ is constant}, \forall \beta\}.$$

The essential ideals of $A(K)$ *and* $A(\hat{K})$ *coincide and, in particular, K is a Bauer simplex if and only if* $\hat{K}$ *is a Bauer simplex.*

PROOF: For each $x \in \partial K$ denote by J_x the primitive ideal

$$(F_x)_\perp = \{f \in A(K) : f(y) = 0, \forall y \in F_x\},$$

so that $J_x \in \text{Prim}\, A(K)$. By [1, Theorem II 6.30] the map $x \rightsquigarrow J_x$ is open from the relative topology of ∂K to the hull-kernel topology of Prim $A(K)$. Now if $f \in C_{\mathbf{R}}(\overline{\partial K})$ is constant on each $F_\beta \cap \overline{\partial K}$, then f is constant on each ∂F_x since ∂F_x is a set of antisymmetry for $A(K)$. Therefore the equation

$$\tilde{f}(J_x) = f(x), \qquad x \in \partial K,$$

defines a continuous function $\tilde{f}$ on Prim $A(K)$, and hence Alfsen and Andersen's version of the Dauns-Hofmann theorem [1, Theorem II 7.20] shows that $f|\partial K$ belongs to (the restriction of) the centre of $A(K)$. If h belongs to $A(\hat{K})$ then the definition of $A(\hat{K})$ shows that $f.h.$ belongs to $A(\hat{K})|\overline{\partial K}$, and therefore f belongs also to (the restriction of) the centre of $A(\hat{K})$.

If l is central in $A(K)$ then l is certainly constant on each F_β. If l' is central in $A(\hat{K})$ then $l'|(F_\beta \cap \overline{\partial K})$ is central in $A(F_\beta)$; in fact if $p \in A(F_\beta)$ then $p = q|F_\beta$ for some q in $A(K)$, and so there exists an $s \in A(\hat{K})$ with $s = l' \cdot q$ on $\overline{\partial K}$, that is $s|(F_\beta \cap \overline{\partial K}) \in A(F_\beta)|(F_\beta \cap \overline{\partial K})$ and $s = l' \cdot p$ on $\overline{\partial F_\beta}$. Therefore, since the centre of $A(F_\beta)$ is trivial, l' is constant on $F_\beta \cap \overline{\partial K}$. This completes the proof of the first statement of the theorem.

If f belongs to the essential ideal of $A(\hat{K})$ then f is in the centre of $A(\hat{K})$, and hence belongs to $A(K)$ by the above argument. Therefore the essential ideals of $A(K)$ and $A(\hat{K})$ coincide. Since K and $\hat{K}$ are Bauer simplexes if and only if their essential ideals coincide with $A(K)$ and $A(\hat{K})$ respectively, the final statement is evident.

COROLLARY (4): *If K is the state space of a unital* C^**-algebra* $\mathscr{A}$*, then the Bishop decomposition for K determines the centre of* $\mathscr{A}$.

PROOF: $A(K)$ can be identified with $\mathscr{A}_h$, the set of hermitian elements of $\mathscr{A}$, and K satisfies Størmer's axiom and admits sufficiently many inner automorphisms. The centre of $A(K)$ is naturally identifiable with the hermitian elements in the centre of $\mathscr{A}$, and hence the result follows. (For references see [1].)

When K is the state space of $\mathscr{A}$ the closed split faces of K are the invariant faces, and they comprise the annihilators in K of the closed two-sided ideals of $\mathscr{A}$. Using these facts it is not difficult to see that the Šilov decomposition $\{F_\alpha\}$ for K consists of the annihilators of the closed two-sided ideals $\{I_\alpha\}$ in $\mathscr{A}$ which are generated by the maximal ideals of the centre of $\mathscr{A}$. The Bishop decomposition $\{F_\beta\}$ for K corresponds to the family $\{I_\beta\}$ of closed two-sided ideals in $\mathscr{A}$ which are minimal subject to the property that $\mathscr{A}/I_\beta$ has trivial centre.

In general we have been unable to decide whether the Bishop decomposition determines $\mathscr{A}$. This is, of course, so if ∂K is closed and it is also true if $\mathscr{A}$ is a weakly central algebra.

For an arbitrary compact convex set K we say that $A(K)$ is *weakly central* if $J_1 = J_2$ whenever J_1 and J_2 are maximal near-lattice ideals in $A(K)$ such that $J_1 \cap Z = J_2 \cap Z$, where Z denotes the centre of $A(K)$ (cf. [5]).

THEOREM (5): *If $A(K)$ is weakly central then the Bishop and Šilov decompositions for K coincide. In particular, the Bishop decomposition determines $A(K)$.*

PROOF: Let F_α be a face in the Šilov decomposition for K and suppose that the centre of $A(F_\alpha)$ is non-trivial. Then there exist non-empty disjoint closed split faces G and H of F_α and hence, by Zorn's lemma, there exist disjoint minimal split faces G_1 and H_1 of K contained in F_α. But then $I = (G_1)_\perp$ and $J = (H_1)_\perp$ are maximal near-lattice ideals in $A(K)$ with $I \cap Z = (F_\alpha)_\perp \cap Z = J \cap Z$, since the functions in Z are constant on F_α. Therefore we obtain $I = J$ and $G_1 = H_1$, and this contradiction shows that $A(F_\alpha)$ has trivial centre. Since the Bishop decomposition is at least as fine as the Šilov decomposition the two decompositions must coincide.

COROLLARY (6): *Let $\mathscr{A}$ be a weakly central unital C^*-algebra, for example a W^*-algebra, with state space K and Bishop decomposition $\{I_\beta^\perp\}$. Then a function f in $C_{\mathbf{R}}(\overline{\partial K})$ is the restriction to $\overline{\partial K}$ of an hermitian element of $\mathscr{A}$ if and only if, for each β, f coincides on $I_\beta^\perp \cap \overline{\partial K}$ with an hermitian element of $\mathscr{A}/I_\beta$.*

The result of Corollary 6 can also be deduced from the results of Vesterstrøm [16].

The Bishop decomposition for K will determine $\mathscr{A}$ if it distinguishes $\mathscr{A}_h$ amongst the Banach subspaces of $C_{\mathbf{R}}(\overline{\partial K})$. If some subspace L of $C_{\mathbf{R}}(\overline{\partial K})$ such that $A(K)|\overline{\partial K} \subseteq L$ is identifiable with $\mathscr{B}_h$ for some unital C^*-algebra $\mathscr{B}$ containing $\mathscr{A}$ as a subalgebra, then $\mathscr{A}$ separates the points of the pure state space $\overline{\partial K}$ of $\mathscr{B}$ so that $\mathscr{A} = \mathscr{B}$ by Glimm's Stone-Weierstrass theorem [11]. In this sense the Bishop decomposition for K always distinguishes $\mathscr{A}$ amongst C^*-algebras.

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Let A be a function algebra on a compact Hausdorff space X, let S be the state space of A and let $K = \text{co}\,(S \cup -iS)$ with the relative w^*-topology. It was shown [8] that the Bishop decomposition for K corresponds to Bishop's decomposition of X into maximal sets of antisymmetry for A [4], and hence the decomposition determines $A(K)$; in fact the Bishop decomposition for K consists of the sets $\overline{\text{co}}\,(E_\beta \cup -iE_\beta)$ together with x_γ and $-ix_\gamma$, where $\{x_\gamma, E_\beta\}$ are the maximal sets of antisymmetry in X for A (the E_β containing more than one point), and so the Bishop decomposition for K covers X and $\overline{\partial K}$.

The map $\theta : A \to A(K)$, where $\theta f(z) = \text{re}\, f(z)$ for $f \in A$, $z \in K$, is a real-linear homeomorphism (cf. [3]). Using this map, we see that $\text{re}\, A = \{\text{re}\, f : f \in A\}$ and $A(K)|S$ are isometrically order isomorphic (cf. [6]).

We now use these facts to give a simple geometrical proof of the Hoffman-Wermer theorem [12].

THEOREM (7): *(Hoffman-Wermer) If A is a function algebra on X such that* re A *is uniformly closed, then $A = C_{\mathbf{C}}(X)$.*

PROOF: We need to show that the Bishop decomposition for K consists of singletons. Suppose that $K_0 = \text{co}\,(S_0 \cup -iS_0)$ belongs to the Bishop decomposition for K, where S_0 is a closed split face of S. Then $A|(S_0 \cap X)$ is a function algebra on $S_0 \cap X$ with state space S_0. Since re A is uniformly closed we have $A(K)|S = A(S)$ and hence

$$A(S_0) = A(S)|S_0 = A(K)|S_0 = A(K_0)|S_0.$$

Therefore lin S_0 is w^*-closed in lin $K_0 = A(K_0)^*$ (cf. [1, II.5]) where 'lin' denotes real-linear hull. Since $A(K_0)$ is antisymmetric $(S_0)_\perp$ is one-dimensional in $A(K_0)$, so that lin S_0 has codimension one in lin K_0. If $-iS_0$ has two distinct extreme points x_1 and x_2 they are both split faces of K_0, and so x_1 and co $(S_0 \cup x_2)$ are disjoint closed split faces of K_0. Hence lin x_1 and lin $(S_0 \cup x_2)$ intersect only at 0, and this contradiction shows that S_0 is a singleton. But then K_0 cannot be antisymmetric, and the theorem is proved.

A result of Fakhoury [9] and Nagel [13] states that if K is a simplex then the centre of $A(K)$ is the largest closed sublattice of $C_{\mathbf{R}}(\overline{\partial K})$ which is contained in $A(K)|\overline{\partial K}$. (See also [15] for the connections with facial topologies of ∂K.) If A is a non-trivial function algebra and K is the associated set defined above we will show that a largest closed sublattice of $C_{\mathbf{R}}(\overline{\partial K})$ contained in $A(K)|\overline{\partial K}$ always exists, but is never equal to the centre.

THEOREM (8): *If A is a function algebra on X and if $A_1 = A \cap C_{\mathbf{R}}(X)$ then $\theta(A_1 + iA_1)|\overline{\partial K}$ is the largest sublattice of $C_{\mathbf{R}}(\overline{\partial K})$ which is contained in $A(K)|\overline{\partial K}$. This space coincides with the restriction of the centre of $A(K)$ if and only if $A = C_{\mathbf{C}}(X)$.*

PROOF: Let L be a maximal sublattice of $C_{\mathbf{R}}(\overline{\partial K})$ which is contained in $A(K)|\overline{\partial K}$, and let $f = \theta(u+iv) \in L$ with $\|f\| \leqq 1$. Since L is a closed subalgebra of $C_{\mathbf{R}}(\overline{\partial K})$ containing the constants, the function

$$\varphi(f) = \theta(\varphi(u)+i\varphi(v))$$

belongs to L for every continuous real-valued function φ on $[-1, 1]$. It follows that $\varphi(u) \in \mathrm{re}\, A$, and a result of Arenson [2] gives $u \in A_1$. Similarly, we have $v \in A_1$ so that L is contained in, and hence equal to, the sublattice $\theta(A_1 + iA_1)|\overline{\partial K}$.

It was shown [7] that the centre Z of $A(K)$ consists of the functions $\theta(u+iv)$, where $u, v \in A$, and $(u-v)$ belongs to the essential ideal I of A. The function $\theta(1)$ belongs to L, but it does not belong to Z unless $1 \in I$, that is $A = C_{\mathbf{C}}(X)$.

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