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A CAUCHY PROBLEM FOR A SEMI-AXIALLY SYMMETRIC WAVE EQUATION *

by

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1. Introduction

This paper is concerned with an explicit solution of the Cauchy problem

$$(1) \qquad u(\bar{x}, 0) = f(\bar{x}), \quad u_t(\bar{x}, 0) = g(\bar{x}), \quad \bar{x} = (x, x_1, \cdots, x_{m-2})$$

for the differential equation

$$(2) \qquad u_{tt} - u_{xx} - \frac{k}{x} u_x - \sum_{i=1}^{m-2} u_{x_i x_i} = F(\bar{x}, t), x > t > 0,$$

where k is a real parameter. When k is a positive integer, equation (2) is simply the wave equation in the variables $x_1, \cdots, x_{m-2}, t, x_m, \cdots, x_{m+k}$ with its solution being axially symmetric in the last $(k+1)$ variables, that is,

$$u(x_1, \cdots, x_{m-2}, t, x_m, \cdots, x_{m+k}) \equiv u(x, x_1, \cdots, x_{m-2}, t)$$

where $x = (x_m^2 + \cdots + x_{m+k}^2)^{\frac{1}{2}}$.

When $g = 0$ and $F = 0$, (1), (2) can be transformed to a particular case of a problem treated by Fox [1] in which a device due to Bureau was employed. In the present problem where g and F are not necessarily zero, we shall solve the problem by a method developed by Riesz [2]. Riesz' method has also been used by Davis [3] and Young [4], [5], among others. The uniqueness of our solution of (1), (2) will follow from the use of Green's formula.

For convenience we shall replace the differential operator in (2) by the self-adjoint operator L

$$(3) \qquad Lv = v_{tt} - \Delta v - \frac{\nu(\nu-1)}{x^2} v$$

which is derived from (2) by the substitution $u = x^\nu v$ with $\nu = k/2$. Here Δ denotes the Laplacian in the variables $x, x_1, \cdots, x_{m-2}$. Morover, it suffices to consider the problem

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$$\text{(4)} \qquad \begin{aligned} Lu &= 0, \quad x > t > 0 \\ u(\bar{x}, 0) &= 0, \quad u_t(\bar{x}, 0) = g(\bar{x}) \end{aligned}$$

since the solution of the general problem $Lu = F$ with the initial conditions (1) can be obtained from the solution of (4) by the use of the Stokes' rule and Duhamel's principle [6, p. 370].

2. The kernel function and Green's formula

The Riesz method consists in determining a kernel function $V^\alpha(\bar{x}, t; \bar{\xi}, \tau)$, depending on two points $(\bar{x}, t)$ and $(\bar{\xi}, \tau)$ and on a parameter α, which vanishes together with its first derivatives on the characteristic cone

$$\text{(5)} \qquad \Gamma \equiv (t-\tau)^2 - (\bar{x}-\bar{\xi})^2$$

and satisfies the relation

$$LV^{\alpha+2} = V^\alpha(\bar{x}, t; \bar{\xi}, \tau).$$

Here we have written

$$(\bar{x}-\bar{\xi})^2 = \sum_{i=1}^{m-2} (x_i - \xi_i)^2 + (x-\xi)^2.$$

Now from [4] we readily deduced that for sufficiently large α, the function

$$\text{(6)} \qquad \begin{aligned} &V_*^\alpha(x_1, \cdots, x_{m-1}, t; \xi_1, \cdots, \xi_{m-1}, \tau) \\ &\qquad = \frac{\Gamma_*^{(\alpha-m)/2}}{H_m(\alpha)} F\left(\nu, 1-\nu; \frac{\alpha+2-m}{2}; \frac{-\Gamma_*}{4t\tau}\right) \end{aligned}$$

where F is a hypergeometric function,

$$\Gamma_* = (t-\tau)^2 - \sum_{i=1}^{m-1} (x_i - \xi_i)^2$$

and

$$H_m(\alpha) = 2^{\alpha-1}\pi^{(m-2)/2}\Gamma(\alpha/2)\Gamma[(\alpha+2-m)/2]$$

satisfies the equation

$$\text{(7)} \qquad L^* V_*^{\alpha+2} = V_*^\alpha$$

with L^* being the differential operator

$$L^* = \left(\frac{\partial}{\partial t}\right)^2 + \frac{\nu(1-\nu)}{t^2} - \sum_{i=1}^{m-1} \left(\frac{\partial}{\partial x_i}\right)^2.$$

Clearly the function (6) is also defined for complex values of the variables x_i, t, ξ_i, τ. In particular, if we replace the variables x_{m-1} and t in

(6) by *it* and *ix*, respectively, and make corresponding replacement for the variables ξ_{m-1} and τ, we obtain

$$(8)\qquad V^\alpha(\bar{x}, t; \bar{\xi}, \tau) = \frac{\Gamma^{(\alpha-m)/2}}{H_m(\alpha)} F\left(\nu, 1-\nu; \frac{\alpha+2-m}{2}; \frac{\Gamma}{4x\xi}\right)$$

where Γ is as defined in (5). Under this change of variables it is clear that the operator L^* becomes the operator L given in (3). It therefore follows from (7) that the function (8) satisfies the equation

$$(9)\qquad LV^{\alpha+2}(\bar{x}, t; \bar{\xi}, \tau) = V^\alpha(\bar{x}, t; \bar{\xi}, \tau)$$

and vanishes together with its first derivatives on the characteristic cone $\Gamma = 0$, provided α is sufficiently large. Thus (8) is the kernel function for our operator L.

Consider now the Green's formula

$$(10)\qquad \int_D (uLv - vLu)\, d\bar{x}\, dt = \int_{C+S} \left(u \frac{\partial v}{\partial n} - v \frac{\partial u}{\partial n}\right) dS$$

taken over the domain D bounded by the retrogade characteristic cone $\Gamma = 0$, $\tau - t > 0$ for $x > t$ and the hyperplane $t = 0$. Here C denotes the surface on the characteristic cone that is cut off by the hyperplane $t = 0$ and S is the part on $t = 0$ that is intercepted by the characteristic cone. In fact S is an $(m-1)$ dimensional sphere with radius τ and center at $(\xi, \xi_1, \cdots, \xi_{m-2})$ in the hyperplane $t = 0$. The derivative $\partial u/\partial n$ indicates the conormal derivative on C and S. If we substitute the kernel function (8) for v in (10) and use the fact that for $\alpha > m+2$ V^α vanishes together with its derivatives on C, then in view of (4) and (9) we have

$$(11)\qquad \int_D u(\bar{x}, t) V^\alpha(\bar{x}, t; \bar{\xi}, \tau)\, d\bar{x}\, dt = \int_S g(\bar{x}) V^{\alpha+2}(\bar{x}, 0; \bar{\xi}, \tau)\, dS.$$

Setting

$$(12)\qquad I^\alpha u(\bar{\xi}, \tau) = \int_D u(\bar{x}, t) V^\alpha(\bar{x}, t; \bar{\xi}, \tau)\, d\bar{x}\, dt$$

and

$$(13)\qquad G^\alpha(\bar{\xi}, \tau) = \int_S g(\bar{x}) V^\alpha(\bar{x}, 0; \bar{\xi}, \tau)\, dS$$

we can write equation (11) in the compact form

$$(14)\qquad I^\alpha u(\bar{\xi}, \tau) = G^{\alpha+2}(\bar{\xi}, \tau).$$

It is clear that for $\alpha > m$ the integrals (12) and (13) both converge in the domain $x > t > 0$. Under the same condition it follows from (9) that in the variables $(\bar{\xi}, \tau)$

(15) $$LI^{\alpha}u(\bar{\xi}, \tau) = LG^{\alpha+2}(\bar{\xi}, \tau) = G^{\alpha}(\bar{\xi}, \tau).$$

As will be shown in the next section, under sufficient differentiability requirement it is possible to perform analytic continuation with respect to α for $\alpha \leqq m$ such that

(16) $$\begin{aligned} &(a)\ \ I^0 u(\bar{\xi}, \tau) = u(\bar{\xi}, \tau) \\ &(b)\ \ G^0(\bar{\xi}, \tau) = 0. \end{aligned}$$

Thus (15) will yield the result

$$Lu(\bar{\xi}, \tau) = 0$$

which verifies that $I^0 u(\bar{\xi}, \tau)$ satisfies the differential equation in (4). Further, the analytic continuation of (14) will yield the explicit solution

(17) $$u(\bar{\xi}, \tau) = G^{0+2}(\bar{\xi}, \tau)$$

which will be shown to satisfy the initial conditions in (4). It is in this sense that formula (14) provides the solution of the problem (4).

3. The analytic continuation of $I^{\alpha} u(\bar{\xi}, \tau)$ and $G^{\alpha}(\bar{\xi}, \tau)$

In establishing (16) we need to distinguish the cases m even and m odd. Here we shall concern ourselves only with the case m even as the same procedure applies to the case m odd. Let us first establish (16a). We expand the hypergeometric function appearing in V^{α} of (12) in infinite series and consider the leading term

(18) $$I_0^{\alpha} u(\bar{\xi}, \tau) = \frac{1}{H_m(\alpha)} \int_D u(\bar{x}, t) \Gamma^{(\alpha-m)/2}\, d\bar{x}\, dt.$$

This is the generalized Riemann-Liouville integral considered by Riesz in [2] for which he proved that

$$\lim_{\alpha \to 0} I^{\alpha} u(\bar{\xi}, \tau) = u(\bar{\xi}, \tau)$$

under the assumption that u is $m/2$ times continuously differentiable. Hence we need only show that the remaining terms in the expansion of $I^{\alpha}u$ vanish as α is continued analytically to zero. But this can be carried out by the same procedure used in [3].

To establish (16b) we consider the general term of the first $(m-2)/2$ terms in the expansion of G^{α}, namely

(19) $$G_j^{\alpha}(\bar{\xi}, \tau) = \frac{C_j(\alpha)}{\Gamma\left(\frac{\alpha+2-m}{2} + j\right)} \int_S \frac{g(\bar{x}) \Gamma_0^{(\alpha-m+2j)/2}}{(x\xi)^j}\, dS$$

where

$$C_j(\alpha) = \frac{\Gamma(\nu+j)\Gamma(1-\nu+j)\Gamma\left(\dfrac{\alpha+2-m}{2}\right)}{4^j H_m(\alpha)\Gamma(\nu)\Gamma(1-\nu)j!}$$

$0 \leqq j \leqq (m-2)/2$. If we introduce polar coordinates

$$\bar{x} = \xi + \rho\bar{\beta}, \qquad 0 \leqq \rho \leqq \tau \tag{20}$$

where $\bar{\beta} = (\beta, \beta_1, \cdots, \beta_{m-2})$ denotes the outward normal vector on S, then (19) becomes

$$G_j^\alpha(\xi, \tau) = \frac{C_j(\alpha)}{\Gamma\left(\dfrac{\alpha+2-m}{2}+j\right)} \int_0^\tau M(g)(\tau^2-\rho^2)^{(\alpha-m+2j)/2}\rho^{m-2}\,d\rho \tag{21}$$

where

$$M(g) = \int_{\omega_{m-1}} \frac{g(\xi+\rho\bar{\beta})}{(\xi+\rho\beta)^j\xi^j}\,d\omega_{m-1}. \tag{22}$$

We now assume that g is at least $m/2$ times continuously differentiable. Then we may integrate (21) by parts $(m-2j-2)/2$ times, $0 \leqq j \leqq (m-2)/2$, to obtain

$$\begin{aligned} G_j^\alpha(\xi, \tau) &= \frac{C_j(\alpha)}{2^{(m-2j-2)/2}\Gamma(\alpha/2)} \\ &\quad \times \int_0^\tau (\tau^2-\rho^2)^{(\alpha-2)/2}(A\rho^{2j}M+\rho G^*)\,d\rho \end{aligned} \tag{23}$$

where

$$G^* = \left[a_1\rho^{2j}M_\rho + a_2\rho^{2j+1}M_{\rho\rho} + \cdots + \rho^{(m-4+2j)/2}\left(\frac{\partial}{\partial\rho}\right)^{(m-2-2j)/2} M\right]$$

with A and the a_i denoting numerical constants. In order that we may set $\alpha = 0$ we need to integrate (23) by parts one more time. We find that

$$\begin{aligned} G_j^\alpha(\xi, \tau) = \frac{C_j(\alpha)}{2^{(m-2j-2)/2}\Gamma\left(\dfrac{\alpha+2}{2}\right)} &\left\{ - \left[\frac{(\tau^2-\rho^2)^{\alpha/2}}{(\tau+\rho)}(A\rho^{2j}M+\rho G^*)\right]_{\rho=0}^{\rho=\tau} \right. \\ &\left. + \int_0^\tau (\tau-\rho)^{\alpha/2}\frac{\partial}{\partial\rho}\left[(\tau+\rho)^{(\alpha-2)/2}(A\rho^{2j}M+\rho G^*)\right]d\rho \right\} \end{aligned}$$

is now convergent for $\alpha > -2$. Thus letting $\alpha \to 0$ we see that $G_j^0(\xi, \tau) = 0$, $0 \leqq j \leqq (m-2)/2$, since $C_j(0) = 0$ in view of the factor $\Gamma(\alpha/2)$ in $H_m(\alpha)$.

The remaining terms in the expansion of G^α can be expressed as

$$R^\alpha(\xi, \tau) = \frac{B(\alpha)}{H_m(\alpha)} \int_S \frac{g(\bar{x})\Gamma_0^{\alpha/2}}{(4x\xi)^{m/2}} F^*(z_0)\, dS \tag{24}$$

where $B(\alpha)$ denotes constant factor, $\Gamma_0 = \Gamma$ at $t = 0$, and F^* is a generalized hypergeometric function with $z_0 = \Gamma_0/4x\xi$. It is clear that (24) converges for $\alpha > -2$ so that as α tends to zero $R^0(\xi, \tau) = 0$.

By what we have just proved we can therefore conclude from (15) that (14) indeed satisfies the differential equation in (4).

4. The explicit solution

We now perform the analytic continuation with respect to α in (14) to arrive at an explicit solution of the problem (4). For this purpose we require the function g to have continuous derivatives up to the order $[(m+2)/2]$. In the cases $m = 2, 3$, we readily obtain the solution of (4) by simply setting $\alpha = 0$ in (14). We have for $m = 2$

$$u(\xi, \tau) = \tfrac{1}{2} \int_{\xi-\tau}^{\xi+\tau} g(x) F(\nu, 1-\nu; 1; z_0)\, dx \tag{25}$$

where $z_0 = [\tau^2 - (x-\xi)^2]/4x\xi$, and for $m = 3$

$$u(\xi, \eta, \tau) = \frac{1}{2\pi} \iint_S \frac{g(x, y)}{\Gamma_0^{\frac{1}{2}}} F(\nu, 1-\nu; \tfrac{1}{2}; z_0)\, dx\, dy \tag{26}$$

where $\Gamma_0 = \tau^2 - (x-\xi)^2 - (y-\eta)^2$, $z_0 = \Gamma_0/4x\xi$, and S is the disk $(x-\xi)^2 + (y-\eta)^2 \leqq \tau^2$. It is easily shown that (25) and (26) indeed satisfy the initial conditions in (4) for $m = 2$ and $m = 3$ respectively.

Hence we need to perform the analytic continuation of (14) only when $m \geqq 4$. We shall carry out here the continuation in the case when m is even, since the case m odd, $m \geqq 5$, can be done in the same fashion. As in the preceding section we expand $G^{\alpha+2}$ in infinite series and consider the first $(m-4)/2$ terms given by

$$U_j(\alpha) = \frac{B_j(\alpha)}{\Gamma\left(\dfrac{\alpha+4-m}{2} + j\right)} \int_S \frac{g(\bar{x})}{x^j \xi^j} \Gamma_0^{(\alpha+2-m+2j)/2}\, dS \tag{27}$$

$0 \leqq j \leqq (m-4)/2$. Here $\Gamma_0 = \tau^2 - (\bar{x} - \bar{\xi})^2$ and

$$B_j(\alpha) = \frac{\Gamma(\nu+j)\Gamma(1-\nu+j)\Gamma\left(\dfrac{\alpha+4-m}{2}\right)}{4^j j! H_m(\alpha+2)\Gamma(\nu)\Gamma(1-\nu)}.$$

Introducing the polar coordinates (20), equation (27) becomes

$$(28)\quad U_j(\alpha) = \frac{B_j(\alpha)}{\Gamma\left(\dfrac{\alpha+4-m}{2}+j\right)}\int_0^\tau M(g)(\tau^2-\rho^2)^{(\alpha+2-m+2j)/2}\rho^{m-2}\,d\rho$$

$$= \frac{B_j(\alpha)}{2\Gamma\left(\dfrac{\alpha+4-m}{2}+j\right)}\int_0^s M(g)(s-r)^{(\alpha+2-m+2j)/2}r^{(m-3)/2}\,dr$$

where we have set $s = \tau^2$, $\rho^2 = r$, and M is as defined in (22). In view of the differentiability condition imposed on g, we may integrate (28) by parts with respect to $r(m-2-2j)/2$ times to obtain

$$U_j(\alpha) = \frac{B_j(\alpha)}{2\Gamma\left(\dfrac{\alpha+2}{2}\right)}\int_0^s (s-r)^{\alpha/2}\left(\frac{\partial}{\partial r}\right)^{(m-2-2j)/2}[r^{(m-3)/2}M]\,dr$$

which converges for $\alpha > -2$. Thus letting $\alpha \to 0$, we find

$$(29)\qquad U_j(0) = \frac{B_j(0)}{2}\left(\frac{\partial}{\partial s}\right)^{(m-4-2j)/2}[s^{(m-3)/2}M(g)]$$

with

$$B_j(0) = \frac{\Gamma(\nu+j)\Gamma(1-\nu+j)}{2\cdot 4^j j!\pi^{(m-2)/2}\Gamma(\nu)\Gamma(1-\nu)}$$

$0 \leqq j \leqq (m-4)/2$.

Next, let $a = \nu$, $b = 1-\nu$, $c = (\alpha+4-m)/2$, $z = \Gamma/4x\xi$, and $n = (m-2)/2$. Then the remaining terms in the expansion of the hypergeometric function appearing in $V^{\alpha+2}$ can be expressed as

$$\frac{(a)_n(b)_n}{(c)_n}\frac{z^n}{n!}\left[1+\frac{(a+n)(b+n)}{(c+n)(n+1)}z+\cdots\right]$$

$$= \frac{(a)_n(b)_n}{(c)_n}\frac{z^n}{n!}F^*(a+n,\, b+n,\, 1;\, c+n;\, n+1;\, z)$$

where $(\lambda)_n = \Gamma(\lambda+n)/\Gamma(\lambda)$ and F^* is a generalized hypergeometric function. Since for $x > t \geqq 0$, $z < 1$, the function F^* converges uniformly in z for $z \leqq \sigma$ with any $\sigma < 1$. Hence the remaining terms in the expansion of $G^{\alpha+2}$ can be written in the form

$$R(\alpha) = A(\alpha)\int_S \frac{g(\bar{x})\Gamma_0^{\alpha/2}}{(x\xi)^{(m-2)/2}}F^*\left(\nu+\frac{m-2}{2},\frac{m}{2}-\nu;1;\frac{\alpha+2}{2};\frac{m}{2};z_0\right)dS$$

with A denoting all constant factors. This integral converges for $\alpha > -2$.

Therefore, as α is allowed to approach zero, we have

$$(30)\quad R(0) = A(0)\int_S \frac{g(\bar{x})}{(x\xi)^{(m-2)/2}} F\left(v+\frac{m-2}{2}, \frac{m}{2}-v; \frac{m}{2}; z_0\right) dS$$

with

$$A(0) = \frac{\Gamma\left(v+\frac{m-2}{2}\right)\Gamma\left(\frac{m}{2}-v\right)}{2^{m-1}\pi^{(m-2)/2}\Gamma(v)\Gamma(1-v)\Gamma(m/2)}.$$

Thus for m even, $m \geqq 4$, our explicit solution is given by

$$(31)\quad u(\xi, \tau) = \frac{1}{4\pi^{(m-2)/2}} \sum_{j=0}^{(m-4)/2} \frac{\Gamma(v+j)\Gamma(1-v+j)}{4^j j!\,\Gamma(v)\Gamma(1-v)} \times \left(\frac{\partial}{\partial s}\right)^{(m-4-2j)/2} [s^{(m-3)/2}M(g)]$$
$$+ \frac{\Gamma\left(v+\frac{m-2}{2}\right)\Gamma\left(1-v+\frac{m-2}{2}\right)}{2^{m-1}\pi^{(m-2)/2}\Gamma(v)\Gamma(1-v)\Gamma\left(\frac{m}{2}\right)} \int_S \frac{g(\bar{x})}{(x\xi)^{(m-2)/2}} \times F\left(v+\frac{m-2}{2}, \frac{m}{2}-v; \frac{m}{2}; z_0\right) dS$$

where $z_0 = [\tau^2-(\bar{x}-\bar{\xi})^2]/4x\xi$ and $M(g)$ as defined in (22) with ρ replaced by $s^{\frac{1}{2}}$.

The case m odd, $m \geqq 5$, can be continued analytically with respect to α in the same manner and we obtain the result

$$(32)\quad u(\xi, \tau) = \frac{1}{4\pi^{(m-1)/2}} \sum_{j=0}^{(m-5)/2} \frac{\Gamma(v+j)\Gamma(1-v+j)}{4^j j\; \Gamma(v)\Gamma(1-v)} \times \int_0^s (s-r)^{-\frac{1}{2}} \left(\frac{\partial}{\partial r}\right)^{(m-3-2j)/2} [r^{(m-3)/2}M(g)]\, dr$$
$$+ \frac{\Gamma\left(v+\frac{m-3}{2}\right)\Gamma\left(\frac{m-1}{2}-v\right)}{2^{m-2}\pi^{(m-1)/2}\Gamma\left(\frac{m-1}{2}\right)\Gamma(v)\Gamma(1-v)} \times \int_S \frac{g(\bar{x})\Gamma_0^{-\frac{1}{2}}}{(x\xi)^{(m-3)/2}} F\left(v+\frac{m-3}{2}, \frac{m-1}{2}-v; 1; \tfrac{1}{2}; \frac{m-1}{2}; z_0\right) dS$$

where the notations z_0 and $M(g)$ have the same definition as for (31).

5. Verification of initial data and remarks

That (31) or (32) satisfies the initial data in (4) can be readily verified. We shall demonstrate this for (31) when m is even, $m \geqq 4$. By introducing polar coordinates the second term in (31) involving an integral is easily seen to vanish together with its derivative with respect to τ at $\tau = 0$. This is also true of each of the terms in the finite sum for $1 \leqq j \leqq (m-4)/2$ since each summand is at least of order $0(s^{\frac{3}{2}})$ where we recall $s = \tau^2$. The term corresponding to $j = 0$ is

$$\text{(33)} \qquad \frac{1}{4\pi^{(m-2)/2}} \left(\frac{\partial}{\partial s}\right)^{(m-4)/2} [s^{(m-3)/2} M(g)]$$

which obviously tends to zero as $s \to 0$ since each term after differentiation is at least of order $0(s^{\frac{1}{2}})$. The derivative with respect to s of each term in (33) also tends to zero with s except for the term involving the lowest power of s, namely,

$$\text{(34)} \qquad w = \frac{1}{2\pi^{(m-2)/2}} \Gamma\left(\frac{m-1}{2}\right) s^{\frac{1}{2}} \int_{\omega_{m-1}} g(\xi + s^{\frac{1}{2}}\beta)\, d\omega_{m-1}.$$

Since $\partial w/\partial \tau = 2s^{\frac{1}{2}} \partial w/\partial s$, we see that as $s \to 0$

$$\frac{\partial w}{\partial \tau} = \frac{1}{2\pi^{(m-2)/2}} \Gamma\left(\frac{m-1}{2}\right) \int_{\omega_{m-1}} g(\xi + s^{\frac{1}{2}}\beta)\, d\omega_{m-1}|_{s=0}$$

which yields

$$\lim_{\tau \to 0} \frac{\partial w}{\partial \tau} = \frac{1}{2\pi^{(m-2)/2}} \Gamma\left(\frac{m-1}{2}\right) \omega_{m-1} g(\xi) = g(\xi).$$

Thus (31) indeed satisfies the initial conditions in (4).

An inspection of (31) also reveals that for ν an integer, $-(m-4)/2 \leqq \nu \leqq (m-2)/2$, Huygens' principle holds, that is, the solution depends only on the initial values on the surface of S. But for m odd there is no Huygens' principle as (32) shows that the solution depends always on all initial values in the domain S. However, when ν is an integer $-[(m-4)/2] \leqq \nu \leqq [(m-2)/2]$ the second term involving an integral in (32) drops out so that in this case (31) or (32) is just a generalization of a well known formula of Poisson for the wave equation.

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