AST

Cocycles over partially hyperbolic maps - Pages préliminaires

Astérisque, tome 358 (2013), p. III-IX
<http://www.numdam.org/item?id=AST_2013__358__R3_0>

358

ASTÉRISQUE

2013

COCYCLES OVER PARTIALLY HYPERBOLIC MAPS

Artur AVILA, Jimmy SANTAMARIA, Marcelo VIANA & Amie WILKINSON

SOCIÉTÉ MATHÉMATIQUE DE FRANCE

Publié avec le concours du CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

Artur Avila
CNRS UMR 7586, Institut de Mathématiques de Jussieu - Paris Rive Gauche, Bâtiment Sophie Germain, Case 7012, 75205 Paris Cedex 13, France / IMPA – Estrada D. Castorina 110, Jardim Botânico, 22460-320 Rio de Janeiro – Brazil
artur@math.jussieu.fr

Jimmy Santamaria
IMPA – Estrada D. Castorina 110, Jardim Botânico, 22460-320 Rio de Janeiro – Brazil
jimmath@impa.br

Marcelo Viana
IMPA – Estrada D. Castorina 110, Jardim Botânico, 22460-320 Rio de Janeiro – Brazil
viana@impa.br

Amie Wilkinson
Department of Mathematics, University of Chicago, 5734 S. University Avenue, Chicago, Illinois 60637. USA
wilkinso@math.uchicago.edu

***Classification mathématique par sujet* (2000).** — 37A20, 37D25, 37D30; 37A50, 37C40.

Mots-clefs. — Abelian cocycle, cohomological equation, holonomy invariance, invariance principle, linear cocycle, Livšic theory, Lyapunov exponent, partial hyperbolicity, rigidity, smooth cocycle.

COCYCLES OVER PARTIALLY HYPERBOLIC MAPS

Artur AVILA, Jimmy SANTAMARIA, Marcelo VIANA & Amie WILKINSON

Abstract. — The works collected in this volume, while addressing quite different goals, are focused on the same type of mathematical object: cocycles over partially hyperbolic diffeomorphisms. We begin with a preliminary overview giving background on the history and applications of the study of dynamical cocycles and partially hyperbolic theory and elucidating the connections between the two main articles. The first one investigates effective conditions which ensure that the Lyapunov spectrum of a (possibly non-linear) cocycle over a partially hyperbolic dynamical system is non-trivial. In the second one, the classical Livšic theory of the cohomological equation for Anosov diffeomorphisms is extended to accessible partially hyperbolic diffeomorphisms.

***Résumé* (Des cocycles au-dessus d'applications partiellement hyperboliques.)** — Les travaux réunis dans ce volume, ayant pourtant des objectifs très différents, sont axés sur le même genre d'objet mathématique : les cocycles au dessus d'un difféomorphisme partiellement hyperbolique. On commence par des rappels sur l'histoire et les applications de l'étude des cocycles dynamiques et la théorie partiellement hyperbolique; ils mettent en évidence des liens entre l deux articles principaux. Dans le premier, on étudie des conditions efficaces qui garantissent la non-trivialité du spectre de Lyapunov d'un cocycle (éventuellement non-linéaire) au dessus d'un difféomorphisme partiellement hyperbolique. Dans le second, la théorie classique de Livšic sur l'équation cohomologique pour les difféomorphismes d'Anosov est étendue au cas d'un difféomorphisme partiellement hyperbolique et accessible.

TABLE DES MATIÈRES

Artur Avila, Jimmy Santamaria, Marcelo Viana & Amie Wilkinson — *Cocycles over partially hyperbolic maps* 1

1. Partially hyperbolic diffeomorphisms 1
2. Cocycles 3
   Abelian cocycles 3
   Linear cocycles 5
3. The central problems 5
   Cohomological equation 5
   The role of Lyapunov exponents 7
4. The general theory 7
5. Fibered systems 9

References 10

Artur Avila, Jimmy Santamaria & Marcelo Viana — *Holonomy invariance: rough regularity and applications to Lyapunov exponents* 13

1. Introduction 13
2. Preliminaries and statements 14
   2.1. Partially hyperbolic diffeomorphisms 14
   2.2. Fiber bundles 16
   2.3. Linear cocycles 17
   2.4. Smooth cocycles - invariant holonomies 19
   2.5. Lyapunov exponents and rigidity 21
   2.6. Sections of continuous fiber bundles 23
   Acknowledgements 26
3. Cocycles with holonomies 26
   3.1. Fiber bunched linear cocycles 26
   3.2. Differentiability of holonomies 29
   3.3. Dominated smooth cocycles 33
4. Invariant measures of smooth cocycles 36
   4.1. Abstract invariance principle 37
   4.2. Global essential invariance 38

4.3. A local Markov construction 39
4.4. Local essential invariance 41
5. Density points 41
5.1. Density sequences 42
5.2. Fake foliations and juliennes 44
5.2.1. Fake foliations 44
5.2.2. Juliennes 45
5.3. Lebesgue and julienne density points 46
6. Bi-essential invariance implies essential bi-invariance 49
6.1. Lebesgue densities 50
6.2. Proof of bi-invariance 51
7. Accessibility and continuity 53
7.1. Non-injective parametrizations 54
7.1.1. Exhaustion of accessibility classes 55
7.1.2. Fiber bundles induced by local strong leaves 56
7.1.3. Construction of non-injective parametrizations 57
7.2. Selection of nearby access sequences 58
8. Generic linear cocycles over partially hyperbolic maps 59
8.1. Accessibility with slow recurrence 60
8.2. Holonomies on loops with slow recurrence 63
8.3. Invariant measures of generic matrices 67
8.3.1. Complex case 68
8.3.2. Real case 69
References 73

Amie Wilkinson — ***The cohomological equation for partially hyperbolic diffeomorphisms*** 75
Introduction 75
1. Techniques in the proof of Theorem A 80
2. Partial hyperbolicity and bunching conditions 83
2.1. Notation 84
3. The partially hyperbolic skew product associated to a cocycle 85
4. Saturated sections of admissible bundles 87
4.1. Saturated cocycles: proof of Theorem A, parts I and III 91
5. Hölder regularity: proof of Theorem A, part II. 93
6. Jets 101
6.1. Prolongations 102
6.2. Isomorphism of jet bundles 102
6.3. The graph transform on jets 103
7. Proof of Theorem B 105
8. Journé's theorem, re(re)visited. 109
9. Saturated sections of partially hyperbolic extensions 121
9.1. Proof of Theorem A, Part IV from Theorem C 122

10. Tools for the proof of Theorem C 122
10.1. Fake invariant foliations 122
10.2. Further consequences of r-bunching 130
10.3. Fake holonomy 139
10.4. Central jets 145
10.5. Coordinates on the central jet bundle 146
10.6. Holonomy on central jets 148
10.7. E^c curves 151
11. Proof of Theorem C 154
12. Final remarks and further questions 163
Acknowledgments 163
References 163