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ASTÉRISQUE

2009

FROM PROBABILITY TO GEOMETRY (II)
VOLUME IN HONOR OF THE 60th BIRTHDAY
OF JEAN-MICHEL BISMUT

Xianzhe DAI, Rémi LÉANDRE, Xiaonan MA and Weiping ZHANG, editors

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FROM PROBABILITY TO GEOMETRY (II)
VOLUME IN HONOR OF THE 60th BIRTHDAY
OF JEAN-MICHEL BISMUT

Xianzhe Dai, Rémi Léandre, Xiaonan Ma and Weiping Zhang, editors

Abstract. — These two volumes contain original research articles submitted by colleagues and friends to celebrate the 60th birthday of Jean-Michel Bismut.

These articles cover a wide range of subjects in probability theory, in global analysis and in arithmetic geometry, to which Jean-Michel Bismut has made fundamental contributions.

Résumé (De Probabilité à Géométrie, volume en l'honneur du 60^e anniversaire de Jean-Michel Bismut)

Ces deux volumes regroupent des articles originaux soumis par des collègues et amis à l'occasion des 60 ans de Jean-Michel Bismut.

Ces articles portent sur la théorie des probabilités, sur l'analyse sur les variétés et sur la géométrie arithmétique, domaines où Jean-Michel Bismut a fait des contributions fondamentales.

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We replace $F = \int_0^1 f dt$ by $E = \frac{1}{2} \int_0^1 |\dot{x}|^2 dt$

- ▶ Note that $\nabla E = -\ddot{x}$.
- ▶ Then one would have

$$\chi(X) = \int_{LX} \exp\left(-\frac{1}{2} \int_0^1 |\dot{x}|^2 dt - \frac{T^2}{2} \int_0^1 |\ddot{x}|^2 dt \dots\right).$$

- ▶ Interpretation of the path integral: $t \rightarrow x_t$ is a path in X whose speed \dot{x} is a Brownian motion, i.e. x is a physical Brownian motion.

$$\dot{x} = p,$$

$$\dot{p} = -\frac{\partial}{\partial x} V(x) - p + \dots,$$

$$\ddot{x} = \frac{1}{T} (-\dot{x} + \dot{w}).$$

Jean-Michel Bismut The Exponential

This volume is dedicated to Jean-Michel Bismut.

The editors : Xianzhe Dai, Rémi Léandre, Xiaonan Ma and Weiping Zhang.