Astérisque

RECENT ADVANCES
IN OPERATOR ALGEBRAS

Orléans, 1992

Classification A.M.S. de chaque exposé
(le premier numéro est le plus significatif)

S. Baaj : 46L05, 46M05
A. Bauval : 58H15, 46L87, 47C15,47D25, 81R50
M. Bekka, A. Valette : 22D25, 22E40, 46L05, 11F75
E. Blanchard : 46L05, 46M05
F. Boca : 46L55
M. Landstad : 46L55
L. Brown : 46L05
P. De La Harpe : 46L35
P. De La Harpe, D. Voiculescu : 46L35
G. Pisier : 46L99, 46M99, 46B99
S. Popa : 46L35, 46L50
F. Rădulescu : 46L10, 46L50
E. Størmer : 46L10, 46L50
L. Vainerman : 81R50, 22A30
D. Voiculescu : 46L50, 60F99, 43A99

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Introduction

This book is the collection of talks given in the conference on operator algebras held in Orléans in July 1992. Orléans has always been a privileged place for operator algebras thanks to François Combes and Claire Delaroche whose kindness and devotion to the subject played a determinant role.

The content of the book describes the recent advances and several major topics of the theory of operator algebras.

First the theory of quantum groups which after the early work of Kac, Takesaki, Enock and Schwartz and Woronowicz is undergoing rapid changes. A very simple definition of these objects was obtained by Baaj and Skandalis, simply as a unitary operator V in the tensor square of a Hilbert space, $H \otimes H$ satisfying suitable multiplicativity conditions. This very fruitful point of view is analysed by Baaj in the special case of the quantum group $E_\mu(2)$ of Woronowicz with special relevance to the modular theory. The equally important deformation aspect of quantum groups ties up (in the paper of Bauval) with continous fields of C^*-algebras. The papers of Boca and Landstad deal with actions of compact quantum groups on C^*-algebras, analysed in the ergodic case by Boca and in the case of single crossed product by Landstad. Finally Vainerman analyses double cosets of compact quantum groups with respect to subgroups and computes corresponding characters in terms of q-orthogonal polynomials.

The second topic widely covered in this book is the analysis of operator algebras associated to free groups, in which the seminal work of Voiculescu on free probability theory plays a dominant role. This work of D. Voiculescu is a major step towards the classification of type II_1 factors beyond the hyperfinite case and the theory has already provided many unexpected results. In his paper Voiculescu gives in particular an explicit way to compute the spectrum of convolution operators on the free group. Radulescu's paper gives a simple presentation of a III_λ factor whose associated II_1 factor is the II_1 factor of a free group with infinitely many generators.

The II_1 factors associated to Fuchsian groups should belong to the class of II_1 factors "next to hyperfinite" and this question is analysed in the paper of de La Harpe and Voiculescu. P. de La Harpe and his collaborators have obtained general results in particular on simplicity for C^*-algebras and von Neumann algebras of discrete groups and a general review is given in de La Harpe's paper.

Finally S. Popa analyses the free analogue of central sequences for II_1 factors and shows that certain universal commuting squares involving amalgamated free products appear asymptotically in any inclusion of finite index, a result of great interest in the theory of subfactors.

The third topic which is quite active at present is the entropy for automorphisms of II_1 factors, a subject reviewed in Størmer's paper. After my initial work with Størmer this subject has evolved slowly due to the difficulty of making explicit computations of entropy. But a number of new results involving, in particular, Narnhofer, Thirring, Størmer and Sauvageot make it quite lively at present.

Thanks to the work of Effros, Haagerup, Pisier and their collaborators, the notion of operator space has found many interesting applications and has become a bridge between operator algebras and Banach spaces. The paper of Pisier develops the notion of exactness in this context, parallel to the well-known work of E. Kirchberg.

Finally, the papers of Blanchard, Brown and Bekka-Vallette deal with questions related to the K-theory of C^*-algebras. The first analyses the notion of tensor product of two C^*-algebras over $C(X)$, the second deals with the subtle nuance between homotopy and equivalence of projections in general C^*-algebras; the third shows that the natural morphism $C^*(H) \to M(C^*(G))$ associated with a group inclusion $H \to G$ is in most cases of interest not injective.

A. Connes

S. BAAJ : *Regular representation of the quantum $E_\mu(2)$ group of Woronowicz*

Let H be a Hilbert space. In this article, under appropriate "regularity" conditions, we associate to every multiplicative unitary $V \in \mathcal{L}(H \otimes H)$, a pair of Hopf C*-algebras in duality. We show that the regular representation of the quantum $E_\mu(2)$ group of Woronowicz is a multiplicative unitary satisfying our conditions and we calculate its covariant representations. We also calculate the Haar measures of $E_\mu(2)$ and its Pontrjagyn dual and we give their modular theory.

A. BAUVAL : *Quantum group - and Poisson - deformation of $SU(2)$*

We endow Woronowicz's family of quantum groups $(SU_\mu(2))_{\mu \in \mathbf{R}^*}$ with a structure of continuous field, and use the underlying continuous field of C*-algebras to construct a deformation of Poisson-$SU(2)$. We prove that this Poisson-deformation is, in some sense, unique. This enables us to compare it with the one constructed by Sheu.

M.E.B. BEKKA, A. VALETTE : *Lattices in semi-simple Lie groups, and multipliers of group C*-algebras*

Let Γ be a lattice in a non-compact simple Lie group G. We prove that the canonical map from the full C*-algebra $C^*(\Gamma)$ to the multiplier algebra $M(C^*(G))$ is not injective in general (it is never injective if G has Kazhdan's property (T), and not injective for many lattices either in $SO(n,1)$ or $SU(n,1)$). For a locally compact group G, Fell introduced a property $(WF3)$, stating that for any closed subgroup H of G, the canonical map from $C^*(H)$ to $M(C^*(G))$ is injective. We prove that, for an almost connected G, property $(WF3)$ is equivalent to amenability.

E. BLANCHARD : *Tensor products of $C(X)$-algebras over $C(X)$*

Given a Hausdorff compact space X, we study the C*-(semi)-norms on the algebraic tensor product $A \otimes_{alg,C(X)} B$ of two $C(X)$-algebras A and B over $C(X)$. In particular, if one of the two $C(X)$-algebras defines a continuous field of C*-algebras over X, there exist minimal and maximal C*-norms on $A \otimes_{alg,C(X)} B$, but there does not exist any C*-norm on $A \otimes_{alg,C(X)} B$ in general.

F.P. BOCA : *Ergodic actions of compact matrix pseudogroups on C*-algebras*

A generalization of the classical finiteness theorem of Høegh-Krohn, Landstad and Størmer for ergodic actions of compact groups on operator algebras is proved for actions of compact matrix pseudogroups on C*-algebras. This, together with the Takesaki-Takai type duality result of Baaj and Skandalis, show that the reduced C*-crossed product of a unital C*-algebra by an ergodic action of a compact matrix pseudogroup is a direct sum of C*-algebras of compact operators.

L.G. BROWN : *Homotopy of projections in C*-algebras of stable rank one*

S. Zhang has suggested the study of the following question for a particular projection

p in a C^*-algebra A: Is every projection which is unitarily equivalent to p necessarily homotopic to p? It was shown by Effros, Kaminker and Zhang that the answer is yes if A is a unital or non-unital purely infinite simple C^*-algebra, and by Zhang that the answer is yes if A has real rank zero and (topological) stable rank one. We show that the answer is yes whenever A has stable rank one. We also give an example where A is extremally rich and of real rank zero and the answer is no. A second theorem makes an additional hypothesis which rules out such examples. In addition the paper discusses the concept of extremal richness and its K–theoretic consequences.

P. DE LA HARPE : *Operator algebras, free groups and other groups*

The operator algebras associated to non commutative free groups have received a lot of attention, by F.J. Murray and J. von Neumann, and by later workers. We review some properties of these algebras, both for free groups and for other groups such as lattices in Lie groups and Gromov hyperbolic groups. We have also collected a list of open problems.

P. DE LA HARPE, D. VOICULESCU : *A problem on the II_1-factors of Fuchsian groups*

We discuss a problem concerning the von Neumann algebra $W^*_\lambda(\Gamma)$ of a Fuchsian group Γ which is finitely generated and non elementary. The problem is to find how such an algebra is related to the factors in the Dykema-Radulescu family $(L(F_r))_{1 < r \leq \infty}$ interpolating continuously the non abelian free group factors.

M.B. LANDSTAD : *Simplicity of crossed products from ergodic actions of compact matrix pseudogroups*

The result that, for an ergodic covariant system $(\mathcal{M}, \rho, G)$ over a compact group G, the crossed product $\mathcal{M} \times_\rho G$ is a simple C^*–algebra iff the multiplicity of each $\pi \in \widehat{G}$ in ρ equals $\dim(\pi)$, is generalised to ergodic actions of the compact matrix pseudogroups defined by S. L. Woronowicz. The crossed product turns out to be simple iff the *quantum* dimension equals the *quantum* multiplicity for each irreducible representation of the pseudogroup. As in the group case, the crossed product is then isomorphic to the algebra of compact operators.

G. PISIER : *Exact operator spaces*

We study the notion of exactness in the category of operator spaces, in analogy with Kirchberg's work for C^*-algebras. As for C^*-algebras, exactness can be characterized either by the exactness of certain sequences, or by the property that the finite dimensional subspaces embed almost completely isometrically into a nuclear C^*-algebra. Let E be an n-dimensional operator space. We define $d_{SK}(E) = \inf\{\|u\|_{cb}\|u^{-1}\|_{cb}\}$ where the infimum runs over all isomorphims u between E and an arbitrary n-dimensional subspace of the algebra of all compact operators on ℓ_2. An operator space X is exact iff $d_{SK}(E)$ remains bounded when E runs over all possible finite dimensional subspaces of X. In the general case, it can be shown that $d_{SK}(E) \leq \sqrt{n}$ (here again $n = \dim(E)$), and we give examples showing that this cannot be improved at least asymptotically. We show that $d_{SK}(E) \leq C$ iff for all ultraproducts $\widehat{F} = \Pi F_i/\mathcal{U}$ (of operator spaces) the

canonical isomorphism (which has norm ≤ 1) v_E: $\Pi(E \otimes_{\min} F_i)/\mathcal{U} \to E \otimes_{\min} (\Pi F_i/\mathcal{U})$ satisfies $\|v_E^{-1}\| \leq C$. Finally, we show that $d_{SK}(E) = d_{SK}(E^*) = 1$ holds iff E is a point of continuity with respect to two natural topologies on the set of all n-dimensional operator spaces.

S. POPA : *Free-independent sequences in type II_1 factors and related problems*
    We prove that, unlike central sequences (i.e., commuting-independent sequences) which in general may or may not exist, free-independent sequences exist in any separable type II_1 factor. More generally, we prove that certain universal commuting squares involving amalgamated free products appear asymptotically in any inclusion of finite index.

F. RĂDULESCU *A type III_λ factor with core isomorphic to the von Neumann algebra of a free group, tensor $B(H)$*
    We construct a type III factor by using the free product construction introduced by Voiculescu and show that its core is $L(F_\infty) \otimes B(H)$. We prove that $M_2(C) * L^\infty[0, 1]$ is a type III_λ factor if $M_2(C)$ is endowed with a nontracial state (depending on λ).

E. STØRMER : *Entropy in operator algebras*
    We give a survey of the theory of dynamical entropy in operator algebras as it was by the end of 1992. Since then Problems 4.2 and 6.6 in the article have been solved, the first positively by D.Voiculescu and the second negatively by Narnhofer, Thirring and the author.

L. VAINERMAN : *Hypergroups structures associated with Gel'fand pairs of compact quantum groups*
    Double cosets of compact quantum groups with respect to their subgroups are considered and cases of a Gel'fand pair and a strict Gel'fand pair are distinguished. It is shown that every strict Gel'fand pair of compact quantum groups generates a normal commutative hypercomplex system with a compact basis and a commutative discrete hypergroup which are in duality to each other. The examples of strict Gel'fand pairs of compact quantum groups are considered and characters of the corresponding hypergroups are described in terms of q-orthogonal polynomials.

D. VOICULESCU : *Operations on certain non-commutative operator-valued random variables*
    In the context of free products with amalgamation over an algebra B, additive and multiplicative free convolution are studied. Analogues of the R- and S- transforms are obtained. Applications to the B-free central limit theorem and to the spectra of convolution operators on free groups are considered.