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Préface

Ce volume contient les contributions de certains participants du Colloque International de K-Théorie qui s'est tenu du 29 juin au 3 juillet 1992 à l'Institut de Recherche Mathématique Avancée, Strasbourg.

Ce colloque a été la première rencontre scientifique dans le cadre du projet "K-théorie" de la Communauté Européenne.

Étant liée dès ses débuts à des questions précises de topologie, géométrie, algèbre et arithmétique, la K-théorie témoigne aujourd'hui plus que jamais de cette diversité de sujets et de méthodes. Le lecteur trouvera le reflet de cette variété dans ce volume.

Le colloque a bénéficié du soutien de la C.E.E., du Ministère de l'Éducation Nationale (D.R.E.D.), du Centre National de la Recherche Scientifique (C.N.R.S.) et de l'Université Louis Pasteur (U.L.P.).

Nous remercions aussi tout le personnel de notre institut pour l'aide apportée à l'organisation matérielle et au bon déroulement de ces journées, en particulier Madame Josiane MOREAU.

Christian KASSEL, Jean-Louis LODAY, Norbert SCHAPPACHER.

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ABSTRACTS

Luca BARBIERI-VIALE, *Cicli di codimensione 2 su varietà unirazionali complesse (Codimension 2 cycles on unirational complex varieties).*

Let X be a projective complex algebraic manifold. After reviewing the main facts concerning the Zariski sheaves $\mathcal{H}^*(A)$ on X, associated to $U \mapsto H^*(U_{an}, A)$ for $A = \mathbf{Z}, \mathbf{Q}, \mathbf{Z}/n, \mathbf{C}$ we show some consequences of the vanishing of $H^0(X, \mathcal{H}^3(\mathbf{Z}))$, *e.g.*, for X unirational or a conic bundle over a surface; indeed, if X is a 3-fold and $H^0(X, \mathcal{H}^3(\mathbf{Z})) = 0$ we give a description of the global sections of $\mathcal{H}^3(\mathbf{Z}/n)$ as transcendental n-torsion 2-Hodge cycles, *i.e.*,

$$H^0(X, \mathcal{H}^3(\mathbf{Z}/n)) \cong {}_n(H^{2,2}(X_{an}, \mathbf{Z})/NS^2(X)).$$

Thus, the non-vanishing of $H^0(X, \mathcal{H}^3(\mathbf{Z}/n))$ is equivalent with the existence of a *non-algebraic* $(2,2)$-Hodge *integral* class : we show examples (of Fano 3-folds and conic bundles) for which all integral Hodge cycles are algebraic. Finally, for unirational varieties and conic bundles over surfaces we show that the cycle map $CH^2(X) \to H^4_{\mathcal{D}}(X, \mathbf{Z}(2))$ in Deligne-Beilinson cohomology is *injective.* We raise several questions and some conjectures.

Carl-Friedrich BÖDIGHEIMER, *Cyclic homology and moduli spaces of Riemann surfaces.*

We report on certain finite complexes arising as compactifications of moduli spaces of Riemann surfaces. Their cellular chain complexes resemble the Hochschild resolution of an algebra, so that Hochschild homology groups and cyclic homology groups can be defined. Using in addition a reflection operator one can also define dihedral and quaternionic homology groups.

Marcel BÖKSTEDT and Ib MADSEN, *Topological cyclic homology of the integers.*

The paper studies the topological cyclic homology functor of rings. This associates to a ring R a spectrum $TC(R)$ which turns out to be closely related to Quillen's $K(R)$, but which is better suited for algebraic topological analysis.

The homotopy groups of $TC(R)$ may be viewed as a topological refinement of Connes' cyclic homology groups.

For rings of integers in local number fields with residue characteristic $p > 0$, a recent result of R. McCarthy implies that the cyclotomic trace from $K(R)$ to $TC(R)$ becomes a homotopy equivalence after completion at p. In particular this is so when R is the ring of p-adic integers.

Our principal result evaluates the p-adic homotopy type of $TC(\mathbb{Z}_p)$ when p is odd, modulo a certain conjecture, and we give evidence to support the conjecture. It appears that S. Tsalidis has now settled the conjecture, his arguments in part being based upon the analysis presented in this paper.

The p-completion of $TC(\mathbb{Z}_p)$ turns out to be the product of three spectra, namely the p-completion of the special unitary group SU, the p-completion of Quillen's $F\psi^k$, also called the image of J-space, and the p-completion of it classifying space. Here k is a generator of the p-adic units. This then determines the homotopy groups with p-adic coefficients of $K(\mathbb{Z}_p)$.

The methods of the paper are homotopy theoretical, and even involve equivariant homotopy theory with respect to certain natural circle actions which go back to Connes' theory of cyclic sets. The main tools are spectral sequences and other methods from algebraic topology.

Jean-Luc BRYLINSKI, *Holomorphic gerbes and the Beilinson regulator.*

This paper gives a geometric interpretation of the Beilinson regulator $c_{2,1} : K_1(X) \longrightarrow H^3(X, \mathbb{Z}(2)_D)$ for a complex projective algebraic manifold X. This interpretation rests on a theory of holomorphic gerbes and of their differentiable stuctures, in the spirit of a recent book of the author. We give a direct geometric construction of a holomorphic gerbe associated to an element of $K_1(X)$. We also present an l-adic analog of the construction, the consistency of which depends on an étale covering of group $\mu_m^{\otimes 2}$ of the Fermat curve with affine equation $x^m + y^m = 1$, which gives a ramified Galois covering of the projective line.

Lars HESSELHOLT, *Stable topological cyclic homology is topological Hochschild homology.*

This paper defines the stable topological cyclic homology $TC^S(R; M)$ of a simplicial ring R with coefficients in an R-bimodule M. Moreover, it is proved that after profinite completion this construction equals the topological Hochschild homology $T(R; M)$ of R with coefficient in M. This result should be compared to a recent result by B. Dundas and R. McCarthy which states that also the stable K-theory $K^S(R;M)$ equals topological Hochschild homology.

The proof is based on a splitting of the topological Hochschild homology of the split extension of R by the square zero ideal M as a wedge of certain new functors in R and M, one for each $n \geq 0$. It is shown that, in order to evaluate $TC^S(R;M)$, it suffices to consider the summands $n = 0, 1$. These are then identified as $T(R)$, resp. $S^1_+ \wedge T(L;R)$.

Dale HUSEMÖLLER, *Algebraic K-theory of operator ideals (after Mariusz Wodzicki).*

Among the proper two-sided ideals in the ring of bounded operators $\mathcal{B}(H)$ on a separable Hilbert space, the algebraic K-theory of the largest ideal $\mathcal{K}$ of compact operators was determined by Suslin and Wodzicki in terms of topological K-theory. Similar results were obtained by Wodzicki for a large class of ideals in terms of topological K-theory and cyclic homology, see the main index sequence (2.8).

Andreas LANGER, *On a specialization map in K_2-cohomology.*

Let S be the spectrum of a discrete valuation ring R with generic point $\eta = \operatorname{Spec} K$ and closed point $s = \operatorname{Spec} k$. Let $\mathcal{X}$ be a smooth S-scheme with generic fiber X_η and closed fiber X_s. We construct a specialization map of Zariski-K-cohomology groups $f : H^1(X_\eta, \mathcal{K}_2) \to H^1(X_s, \mathcal{K}_2)$, which depends on the choice of a uniformizing element $\pi \in R$. Then we show that f is compatible with the natural reduction map from $H^1(\mathcal{X}, \mathcal{K}_2)$ to $H^1(X_s, \mathcal{K}_2)$. This observation is exploited to prove the following

THEOREM. — *The notations are as above, in particular let K be a local field of characteristic zero, which is unramified over $\mathbb{Q}_p$; let X_η be a smooth projective variety with ordinary good reduction, such that* $\dim X_\eta \leq p-2$ *and X_s is ordinary. Then, if the condition* $\operatorname{Pic}(X_{\bar{s}})(p) \equiv 0$ *is satisfied, the map*

$$\bar{f} : H^1(X_{\bar{\eta}}, \mathcal{K}_2) \longrightarrow H^1(X_{\bar{s}}, \mathcal{K}_2),$$

induced by f by passing to the geometric fibers, is surjective on the p-primary torsion groups.

The proof combines results of Bloch and Kato on p-adic étale cohomology in the case of ordinary reduction with assertions of Suslin, Lichtenbaum, Colliot-Thélène and Raskind on Zariski-K-cohomology, especially in characteristic p.

Marc LEVINE, *Bloch's higher Chow groups revisited.*

Bloch has defined higher Chow groups $CH^q(X,p)$ of a scheme X over a field k by constructing a complex out of the codimension q algebraic cycles on $X \times \mathbb{A}^n_k$, $n = 0,1,2\ldots$. We show that the $\mathbb{Q}$-vector space $CH^q(X,p)_{\mathbb{Q}}$ is naturally isomorphic to the weight q portion of the pth K-group of X, $K_p(X)^{(q)}$, for X a smooth quasi-projective variety over k, generalizing the classical isomorphism $CH^q(X)_{\mathbb{Q}} \to K_0(X)^{(q)}$. We also show that the functors $CH^q(-,*)_{\mathbb{Q}}$ satisfy most of the properties of a Bloch-Ogus twisted duality theory. Finally, we show that the alternating cycle groups defined by Bloch agree with the rational higher Chow groups.

Ruth I. MICHLER, *Hodge-components of cyclic homology for affine quasi-homogeneous hypersurfaces.*

In this paper, we prove that the Hodge-components of Hochschild homology of a reduced affine hypersurface are given by torsion modules of Kaehler differentials. Using results of T. Goodwillie, J.-L. Loday and U. Vetter we prove a new vanishing result for the Hodge-components of cyclic homology of affine hypersurfaces and give an explicit computation of these Hodge-components of cyclic homology in the case of an hypersurface defined by a quasi-homogeneous polynomial.

Alexandre NENASHEV, *Comparison theorem for λ-operations in higher algebraic K-theory.*

We prove that the two constructions of Λ-maps given by D. Grayson and the author by means of certain (multi) simplicial sets provide the same definition for λ-operations on higher K-theory of the underlying exact category with operations. The equivalence is proved on the simplicial level. Our main technical result is that, under certain assumptions, the multidimensional mapping cone construction of a cube of exact categories may be computed as a homotopy fiber of the map of multidimensional S-constructions applied to faces of codimension one, which is a generalization of Grayson's Theorem C.

Claudio PEDRINI and Charles WEIBEL, *Divisibility in the Chow group of zero-cycles on a singular surface.*

In this paper we study the divisibility of the group $CH^2(X)$ of zero-cycles on a singular surface X over a field k. Our results generalize known facts about the Chow group of a nonsingular surface. When k is algebraically closed, we show that the subgroup of cycles of "degree" zero is divisible. When k is the real numbers $\mathbb{R}$, the non-divisible torsion of $CH^2(X)$ is detected by the topology of the stratified manifold $X(\mathbb{R})$.

Ravi RAO and Wilberd van der KALLEN, *Improved stability for SK_1 and WMS_d of a non-singular affine algebra.*

We establish some stability results for K_1 of algebras over algebraically closed fields, whose analogues fail for algebras over the reals. More technically, let k be a perfect C_1 field and A a non-singular affine algebra of Krull dimension $d \geq 3$ over k. Then we show $SL_r(A)/E_r(A) \to SK_1(A)$ is an isomorphism for $r \geq d+1$ and we find a homomorphism $SL_d(A) \to WMS_d(A)$ whose kernel is $SL_{d-1}(A)E_d(A)$, which is thus shown to be a normal subgroup.

Ulrike TILLMANN, *Hopf structure on the Van Est spectral sequence in K-theory.*

This paper explores the duality relationship between the multiplicative K-theory of a Banach algebra A and the indecomposables in the smooth cohomology of the associated general linear group. The main technical result is that a spectral sequence of coprimitive Hopf algebras induces a long exact sequence of indecomposables. This is applied to the Van Est spectral sequence yielding the above formal duality relationship. In the last section we also reinterpret the Van Est spectral sequence as a Serre spectral sequence in continuous cohomology.

Paul ZUSMANOVICH, *The second homology group of current Lie algebras.*

We calculate the second homology group of the current Lie algebra $L \otimes A$ for arbitrary Lie algebra L and associative commutative unital algebra A. The main tool is the Hopf formula, expressing the second homology of a Lie algebra in terms of its presentation. We also give the formulas for the second homology group of the Lie algebra associated with algebra $A \otimes B$ for arbitrary associative unital A and B.