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1 Introduction and Results

Let $Q = Q(z,\bar{z})$ be a subharmonic and non-harmonic polynomial on the complex plane $\mathbf{C}$ with real values. Then the degree the non-harmonic part Q^N of Q is an even positive number $2k \in \mathbf{N}^*$. In their paper [1], F. Berteloot and G. Cœuré proved that the domain $\Omega_Q = \{(w,z) \in \mathbf{C}^2 \mid \operatorname{Re} w + Q(z,\bar{z}) < 0\}$ is **hyperbolic** for every Q like above. In this note, we consider the positive cone M of all such polynomials and the associated domains $\Omega_Q \subset \mathbf{C}^2$.

Let $Q_1, Q_2 \in M$ and $\Omega_{Q_1}, \Omega_{Q_2}$ be the associated domains. In what follows, we use also Ω, Ω_1, Ω_2 instead of Ω_Q, Ω_{Q_1}, Ω_{Q_2} if there is no confusion possible. First, we introduce an equivalence relation on the cone M.

Definition 1.1 *Let $Q_1, Q_2 \in M$. We say that Q_1 and Q_2 are equivalent $Q_1 \sim Q_2$, if there is a real number $\rho > 0$, a holomorphic polynomial $p(z)$ and an automorphism $g(z)$ of $\mathbf{C}$ such that*

$$Q_1(z,\bar{z}) = \rho \operatorname{Re}(p(z)) + \rho Q_2(g(z), \overline{g(z)}). \tag{1.1}$$

On the other hand, there is another equivalence relation on M given by the biholomorphy of the domains Ω_{Q_1} and Ω_{Q_2}. The first results states that these two equivalence relations are the same.

Theorem 1.2 *Let Q_1, $Q_2 \in M$. Then Ω_1 and Ω_2 are biholomorphic, if and only if the two polynomials Q_1 and Q_2 are equivalent in the sense of definition 1.1. In particular the degrees of the non-harmonic parts Q_1^N and Q_2^N are equal, if the domains Ω_1 and Ω_2 are biholomorphic.*

The fact that Ω is hyperbolic implies that the holomorphic automorphism group $\operatorname{Aut}_{\mathcal{O}}(\Omega)$ is a real Lie group and that all isotropy groups of the action of $\mathrm{Aut}_{\mathcal{O}}(\Omega)$ on Ω are compact [3]. We denote by G, G_1, G_2 the connected identity components of $\mathrm{Aut}_{\mathcal{O}}(\Omega)$, $\mathrm{Aut}_{\mathcal{O}}(\Omega_1)$, $\mathrm{Aut}_{\mathcal{O}}(\Omega_2)$. Clearly, if Ω_1 and Ω_2 are biholomorphic, then G_1 and G_2 are isomorphic.

Let $\mathcal{G}, \mathcal{G}_1, \mathcal{G}_2$ denote the Lie algebras of G, G_1, G_2.

Let J, J_1, J_2 denote the subgroups of G, G_1, G_2 generated by the translation $\{(w,z) \mapsto (w+it, z) \mid t \in \mathbf{R}\}$ and j, j_1, j_2 their Lie algebras. Hence the dimension of G, G_1, G_2 is at least one.

The second result gives a "canonical" defining polynomial for the domain Ω if $\dim_{\mathbf{R}} \mathcal{G} \geq 2$.

Theorem 1.3 *Let $\Omega = \{\mathrm{Re}\, w + Q(z) < 0\}$ as above. Assume that $\dim_{\mathbf{R}} G \geq 2$. Then there are the following cases :*

a) Ω is homogeneous. Then $\Omega \simeq \mathbf{B}_2 = \{|w|^2 + |z|^2 < 1\}$ and $Q \sim P_1 \sim P_2$, where $P_1(z,\bar{z}) = (\mathrm{Re}\, z)^2$ and $P_2(z,\bar{z}) = |z|^2$.

b) Ω is not homogeneous.

1) $\dim_{\mathbf{R}} G = 2$. Then $\deg Q^N \geq 4$ and either i) $Q \sim P_1$ or ii) $Q \sim P_2$, or iii) $Q \sim P_3$, where

i) $P_1(z,\bar{z}) = P_1(\mathrm{Re}\, z)$ is an element of M depending only on $\mathrm{Re}\, z$ and $G \simeq (\mathbf{R}^2, +)$,

ii) $P_2(z,\bar{z}) = P_2(|z|^2)$ is an element of M depending only on $|z|^2$, and $G \simeq \mathbf{R} \times S^1$,

iii) $P_3(z,\bar{z})$ is a homogeneous polynomial of degree $2k$, $k \geq 2$, i.e. $P_3(\lambda z, \lambda \bar{z}) = \lambda^{2k} P_3(z,\bar{z})$ for all $\lambda \in \mathbf{R}$ and G is the non-abelian two dimensional real Lie group.

2) $\dim_{\mathbf{R}} G \geq 3$. Then $\deg Q^N \geq 4$ and either i) $Q \sim P_1$ or ii) $Q \sim P_2$ where

i) $P_1(z,\bar{z}) = (\mathrm{Re}\, z)^{2k}$ and G is 3-dimensional and solvable,

ii) $P_2(z,\bar{z}) = |z|^{2k}$ and G is 4-dimensional and contains a finite covering of $SL_2(\mathbf{R})$.

We are going to prove the two theorems simultaneously by distinguishing the dimension of G. First we handle the one and two-dimensional cases, then the homogeneous case and we finish with the three and higher dimensional cases.

Before doing so, we prove the easy direction of theorem1.1.

Lemma 1.4 *If $Q_1 \sim Q_2$, then Ω_1 and Ω_2 are biholomorphic.*

Proof : Assume (1.1). Let $\Psi = (\Psi_1, \Psi_2)$ be the biholomorphic map of $\mathbf{C}^2$ defined by

$$(*) \qquad \begin{cases} \Psi_1(w,z) = \frac{1}{\rho}w + p(z) \\ \Psi_2(w,z) = g(z) \end{cases}$$

Then $\Psi(\Omega_1) = \Omega_2$. ∎

Remark 1.5 *In what follows we will often make a global coordinate change in $\mathbf{C}^2$ like $(*)$, which is coherent with the equivalence of the defining polynomials. In the following, we take the notation from above.*

2 The one-dimensional case

Let $\Psi : \Omega_1 \to \Omega_2$ be a biholomorphic map. For a subgroup $N \subset G_2$ let $\Psi^*(N)$ be the group $\Psi^{-1} \circ N \circ \Psi \subset G_1$.

Lemma 2.1 *Assume that $\Psi^*(J_2) = J_1$. Then $Q_1 \sim Q_2$.*

Proof : From our hypothesis it follows that there is a non-zero real number ρ such that

$$\Psi^{-1} \circ T_t \circ \Psi = T_{\rho t}, \ (T_t(w,z) = (w + it, z)),$$

since Ψ^* is a continuous group isomorphism of two copies of $\mathbf{R}$.

So we get with $\Psi = (\Psi_1, \Psi_2)$

$$\Psi_1(w,z) + it = \Psi_1(w + i\rho t, z)$$
$$\Psi_2(w,z) = \Psi_2(w + i\rho t, z)$$

which implies :

$$\begin{aligned} \Psi_1(w,z) &= \frac{1}{\rho}w + p(z) \\ \Psi_2(w,z) &= g(z) \end{aligned}$$

with $p \in \mathcal{O}(\mathbf{C})$ and $g \in \mathrm{Aut}_{\mathcal{O}}(\mathbf{C})$, since the projection $\pi : \mathbf{C}^2 \to \mathbf{C}$, $(w,z) \mapsto z$ is surjective on Ω_1 and Ω_2.

Therefore Ψ is a biholomorphic map of $\mathbf{C}^2$ which maps Ω_1 to Ω_2 and so we have

$$\begin{aligned} \Omega_1 &= \{\mathrm{Re}\, w + Q_1(z,\bar{z}) < 0\} = \Psi^{-1}(\Omega_2) \\ &= \{\mathrm{Re}(\frac{1}{\rho}w + p(z)) + Q_2(g(z), \overline{g(z)}) < 0\} \\ &= \{\mathrm{Re}\, w + \rho\, \mathrm{Re}\, p(z) + \rho Q_2(g(z), \overline{g(z)}) < 0\}. \end{aligned}$$

It follows that

$$Q_1(z,\bar{z}) = \rho \operatorname{Re} p(z) + \rho Q_2(g(z), \overline{g(z)}).$$

This equality implies the positivity of ρ and the fact that the holomorphic function $p(z)$ is already a polynomial. Hence $Q_1 \sim Q_2$. ■

We mention the following direct consequence, which is the statement of theorem 1.2 in the case $\dim_{\mathbf{R}} G_1 = 1$.

Corollary 2.2 *If* $\dim_{\mathbf{R}} G_1 = 1$, *then* Q_1 *and* Q_2 *are equivalent.*

Proof : Here we have $G_1 = J_1$ and $G_2 = J_2$, hence $\Psi^*(J_2) = J_1$. ■

3 The two-dimensional case

We are going to handle this case in a sequence of lemmas. We always assume that there is a two-dimensional subgroup $H \subset G$ such that $J \subset H$. Since $J \subset G$ is a closed subgroup isomorphic to $\mathbf{R}$ there are two possibilities for H :

i) H is abelian and non-compact.

ii) H is the solvable two dimensional non-abelian Lie group.

Lemma 3.1 *Suppose that* H *is abelian. Then* $Q \sim P_1$ *or* $Q \sim P_2$, *where* $P_1(z,\bar{z}) = P_1(\operatorname{Re} z)$ *is an element of* M *which depends only on* $\operatorname{Re} z$, *or* $P_2(z,\bar{z}) = P_2(|z|^2)$ *is an element of* M *which depends only on* $|z|^2$.

In the first case, the domain $\{\operatorname{Re} w + P_1(\operatorname{Re} z) < 0\}$ *realizes the domain* Ω *as a tube domain.*

Proof : Let $L = \{\sigma^t = (\sigma_1^t, \sigma_2^t) \mid t \in \mathbf{R}\}$ be a one parameter group of H such that L and J generate H. The group H being abelian implies that L and J commute and so we get for all $s, t \in \mathbf{R}$:

$$\begin{aligned}
\sigma_1^t(w+is,z) &= \sigma_1^t(w,z) + is \\
\sigma_1^t(w+is,z) &= \sigma_1^t(w,z).
\end{aligned}$$

The restriction of the projection $\pi : (w,z) \to z$ from $\mathbf{C}^2$ to Ω being surjective and the second equality imply that

$$\sigma_2^t(w,z) = \sigma_2^t(z)$$

is a non-trivial one-parameter subgroup of $\mathrm{Aut}_{\mathcal{O}}(\mathbf{C}) \simeq \mathbf{C}^* \ltimes \mathbf{C}$. Furthermore $\sigma_1^t(w,z) = w + f(t,z)$, where $f(t,\cdot) \in \mathcal{O}(\mathbf{C})$. Since $\sigma^t \in \mathrm{Aut}_{\mathcal{O}}(\mathbf{C}^2)$ and stabilises Ω, it follows that $f(t,\cdot)$ is a holomorphic polynomial for all $t \in \mathbf{R}$.

After a holomorphic change of coordinates in $\{z \in \mathbf{C}\}$, which is in fact polynomial and therefore coherent with the equivalence of defining polynomials, we have that

a) $\sigma_2^t(z) = z + it$ or

b) $\sigma_2^t(z) = e^{\alpha \cdot t} \cdot z$ for $\alpha \in \mathbf{C}^*$ fixed.

ad (a) : Here we have

$$\begin{aligned} \sigma_1^t(w,z) &= w + f(t,w) \\ \sigma_2^t(w,z) &= z + it \quad \text{for all } t \in \mathbf{R}. \end{aligned}$$

It follows that

$$f(t_1 + t_2, z) = f(t_1, z + it_2) + f(t_2, z) \text{ for all } t_1, t_2 \in \mathbf{R}. \tag{3.1}$$

and therefore there is a *holomorphic polynomial* $\tilde{f}$ such that

$$f(t,z) = \tilde{f}(z + it) - \tilde{f}(z). \tag{3.2}$$

After the change of coordinates in $\mathbf{C}^2$

$$\begin{pmatrix} \tilde{w} \\ \tilde{z} \end{pmatrix} = \begin{pmatrix} w - \tilde{f}(z) \\ z \end{pmatrix},$$

we have that Ω is given by $\{\mathrm{Re}\,\tilde{w} + \tilde{Q}(\tilde{z},\bar{\tilde{z}}) < 0\}$, with a polynomial $\tilde{Q}$ equivalent to Q. The action of L is then given by

$$\begin{aligned} \sigma_1^t(\tilde{w},\tilde{z}) &= \tilde{w} \\ \sigma_2^t(\tilde{w},\tilde{z}) &= \tilde{z} + it. \end{aligned}$$

This means that $\tilde{Q}(\tilde{z},\bar{\tilde{z}})$ is invariant under translations of the form $\{\tilde{z} \mapsto \tilde{z} + it \mid t \in \mathbf{R}\}$, which implies that $\tilde{Q}(\tilde{z},\bar{\tilde{z}}) = \tilde{Q}(\mathrm{Re}\,\tilde{z})$ and that Ω is realized as a tube domain. The group H is isomorphic to $(\mathbf{R}^2, +)$.

ad (b) : In this case, we have

$$\begin{aligned} \sigma_1^t(w,z) &= w + f(t,z) \\ \sigma_2^t(w,z) &= e^{\alpha t} \cdot z \end{aligned}$$

for all $t \in \mathbf{R}$ with fixed $\alpha = a + ib \in \mathbf{C}^*$. By the same argument as in case (a), we see that $f(t, \cdot)$ is a holomorphic polynomial and that $\sigma^t \in \mathrm{Aut}_{\mathcal{O}}(\mathbf{C}^2)$ for all $t \in \mathbf{R}$. So we have :

$$\begin{aligned} \Omega &= \{(w,z) \in \mathbf{C}^2 \mid \operatorname{Re} w + Q(z,\bar{z}) < 0\} \\ &= \{(w,z) \in \mathbf{C}^2 \mid \operatorname{Re} w + \operatorname{Re} f(t,z) + Q(e^{\alpha t} \cdot z, e^{\bar{\alpha} t} \cdot \bar{z}) < 0\} \text{ for all } t \in \mathbf{R}, \end{aligned}$$

i.e. $Q(z, \bar{z}) = \operatorname{Re} f(t,z) + Q(e^{\alpha t} \cdot z, e^{\bar{\alpha} t} \cdot \bar{z})$. Without loss of generality, we may assume that the harmonic part of Q is trivial, which implies that $\operatorname{Re} f(t,z) \equiv 0$ for all $t \in \mathbf{R}$, i.e. $f(t,z) = f(t) \in i\mathbf{R}$ for all $t \in \mathbf{R}$. Hence $f(t) = i\beta t$ with $\beta \in \mathbf{R}$. Then we have that $Q(z,\bar{z}) = Q(e^{\alpha t} \cdot z, e^{\bar{\alpha} t} \cdot \bar{z})$ for all $t \in \mathbf{R}$. This implies that $\alpha \in i\mathbf{R}^*$ and that $Q(z,\bar{z}) = Q(|z|^2)$, i.e. the polynomial Q depends only on $|z|^2$.

The action of L then is given by

$$\begin{aligned} \sigma_1^t(w,z) &= w + i\beta t \\ \sigma_2^t(w,z) &= e^{\alpha t} \cdot z, \quad \text{for all } t \in \mathbf{R}. \end{aligned}$$

The group H is isomorphic to $\mathbf{R} \times S^1$. ∎

Lemma 3.2 *Suppose that H is the two dimensional solvable non-abelian Lie group. Then the polynomial Q is equivalent to a polynomial P_{2k}, which is homogeneous of degree $2k$, i.e. $P_{2k}(\lambda z, \lambda \bar{z}) = \lambda^{2k} P_{2k}(z,\bar{z})$ for all $\lambda \in \mathbf{R}$ and J is a normal subgroup of H.*

Proof : Let $L = \{\sigma^t = (\sigma_1^t, \sigma_2^t) \mid t \in \mathbf{R}\}$ be a one parameter subgroup of H such that L and J generate H. Then there are two cases :

a) J is not the normal subgroup of dimension one in H.

b) J is normal in H.

ad(a) : We may assume that L is normal in H.

Let $X = i\frac{\partial}{\partial w} - i\frac{\partial}{\partial \bar{w}}$ and $Y = f\frac{\partial}{\partial w} + g\frac{\partial}{\partial z} + \bar{f}\frac{\partial}{\partial \bar{w}} + \bar{g}\frac{\partial}{\partial \bar{z}}$ be the two holomorphic infinitesimal transformations induced by J and L on Ω. By our assumption there is a $\lambda \in \mathbf{R}^*$ such that $[X,Y] = \lambda \cdot Y$. This equation yields $f(w,z) = e^{-i\lambda w} h_1(z)$ and $g(w,z) = e^{-i\lambda w} h_2(z)$, $h_1, h_2 \in \mathcal{O}(\mathbf{C})$. It follows that Y is a global infinitesimal holomorphic transformation of $\mathbf{C}^2$, since $\pi : \Omega \to \mathbf{C}$, $(w,z) \mapsto z$ is surjective.

Furthermore h_2 vanishes nowhere, since $h_2(z_0) = 0$ implies that the set $\{(w, z_0) \mid \operatorname{Re} w + Q(z_0, \bar{z}_0) < 0\}$ is stabilized by H with J as a non-normal subgroup which is impossible. Now we have $Y(\operatorname{Re} w + Q(z,\bar{z})) \mid_{\{\operatorname{Re} w + Q(z,\bar{z}) = 0\}} \equiv 0$.

This yields the equation

$$h_1(z) + h_2(z)\frac{\partial Q}{\partial z}(z,\bar{z}) + e^{2i\lambda Q(z,\bar{z})}\left(\overline{h_1(z)} + \overline{h_2(z)}\frac{\partial Q}{\partial \bar{z}}(z,\bar{z})\right) \equiv 0.$$

The expression $h_1(z) + h_2(z)\frac{\partial Q}{\partial z}(z,\bar{z})$ being a polynomial in $\bar{z}$ implies that the expression $e^{2i\lambda Q(z,\bar{z})}\left(\overline{h_1(z)} + \overline{h_2(z)}\frac{\partial Q}{\partial \bar{z}}(z,\bar{z})\right)$ is also a polynomial in $\bar{z}$. By differentiating n times, $n \in \mathbf{N}$ with respect to $\bar{z}$ this yields that $\overline{h_2(z)} = 0$ for all $z \in \mathbf{C}$, a contradiction to the fact mentioned above.

ad (b) : Assume that J is normal in H. We get

$$\begin{aligned} \sigma_1^t(w+is,z) &= \sigma_1^t(w,z) + ie^{\alpha t}\cdot s \\ \sigma_2^t(w+is,z) &= \sigma_2^t(w,z),\ \alpha \in \mathbf{R}^* \text{ fixed.} \end{aligned}$$

So we have again $\sigma_2^t(w,z) = \sigma_2^t(z)$ and $\sigma_2^t \in \mathrm{Aut}_{\mathcal{O}}(\mathbf{C})$ for all $t \in \mathbf{C}$.

Furthermore $\sigma_1^t(w,z) = e^{\alpha\cdot t}w + f(t,z)$ with $f(t,\cdot) \in \mathcal{O}(\mathbf{C})$ for all $t \in \mathbf{R}$. Hence $\sigma^t \in \mathrm{Aut}_{\mathcal{O}}(\mathbf{C}^2)$ and $f(t,z)$ is a holomorphic polynomial for all $t \in \mathbf{R}$.

Since $\dim_{\mathbf{R}} H = 2$, the one parameter group $\{\sigma_2^t(z) \mid t \in \mathbf{R}\} \subset \mathrm{Aut}_{\mathcal{O}}(\mathbf{C})$ cannot be trivial. So after a change of coordinates in the z-variable, we have

(i) $\sigma_2^t(z) = z + it$ or

(ii) $\sigma_2^t(z) = e^{\beta t}\cdot z,\ \beta \in \mathbf{C}^*$ fixed.

If (i) $\sigma_2^t = z + it$, we get

$$\begin{aligned} \sigma_1^t(w,z) &= e^{\alpha t}w + f(t,z) \\ \sigma_2^t(w,z) &= z + it \ \text{ and } \ \sigma^t \in \mathrm{Aut}_{\mathcal{O}}(\mathbf{C}^2) \end{aligned}$$

This yields

$$Q(z,\bar{z}) = e^{-\alpha t}\operatorname{Re} f(t,z) + e^{-\alpha t}Q(z+it,\bar{z}-it).$$

It is easy to see that this is not possible by considering the highest degree homogeneous summand of the non-harmonic part Q^N of Q.

So we may assume (ii), $\sigma_2^t(z) = e^{\beta t}\cdot z$, $\beta \in \mathbf{C}^*$ fixed.

Hence

$$\begin{aligned} \sigma_1^t(w,z) &= e^{\alpha t}\cdot w + f(t,z) \\ \sigma_2^t(w,z) &= e^{\beta t}\cdot z, \quad \alpha \in \mathbf{R}^*, \quad \beta = a + ib \in \mathbf{C}^* \text{ fixed} \end{aligned}$$

and it follows that

$$Q(z,\bar{z}) = e^{-\alpha t}\operatorname{Re} f(t,z) + e^{-\alpha t}Q(e^{\beta t}z, e^{\bar{\beta}t}\bar{z}).$$

We may assume that Q has no harmonic summands and therefore

$$Q(z,\bar{z}) = e^{-\alpha t} Q(e^{\beta t}\cdot z, e^{\bar{\beta}t}\cdot\bar{z}), \text{ for all } t\in\mathbf{R}.$$

The highest degree of Q is an even number $2k$, $k\in\mathbf{N}^*$. Let $Q_{2k}(z,\bar{z}) = \sum_{j=1}^{2k-1} a_j z^j \bar{z}^{2k-j}$, $(a_j = \bar{a}_{2k-j})$ be the highest degree homogeneous summand of Q. We get

$$\begin{aligned} Q_{2k}(z,\bar{z}) &= e^{-\alpha t} Q_{2k}(e^{\beta t}\cdot z, e^{\bar{\beta}t}\cdot\bar{z}), \text{ i.e.} \\ a_j &= a_j e^{-\alpha t}\cdot e^{j\beta t+(2k-j)\bar{\beta}t}, 1\le j\le 2k-1. \end{aligned}$$

A necessary condition for this is

$$\alpha = 2k\cdot \operatorname{Re}\beta$$

and that there are no summands in Q of degree smaller then $2k$.

Hence $Q = Q_{2k} = P_{2k}$ and the lemma is proved. ■

Remark 3.3 *Lemma 3.1 and Lemma 3.2 give the proof of theorem 1.3 in the case* $\dim_{\mathbf{R}} G = 2$.

Lemma 3.4 *Let* $\Omega_1 = \{\operatorname{Re} w + Q_1(z,\bar{z}) < 0\}$ *and* $\Omega_2 = \{\operatorname{Re} w + Q_2(z,\bar{z}) < 0\}$ *like above. Assume that* $\Psi : \Omega_1 \to \Omega_2$ *is biholomorphic and that* J_1 *and* $\Psi^*(J_2)$ *are both contained in a two-dimensional subgroup* $H \subset G_1$. *Then* $J_1 = \Psi^*(J_2)$ *and* $Q_1 \sim Q_2$.

Proof : We have again to consider the following two cases :

a) H is abelian,

b) H is not abelian.

In both cases, we assume $J_1 \neq \Psi^*(J_2)$ and produce a contradiction. Let $\Psi^*(J_2) = \{\sigma^t = (\sigma_1^t, \sigma_2^t) \mid t\in\mathbf{R}\}$.

ad(a) : i) Assume that $H = (\mathbf{R}^2, +)$. Then by Lemma 3.1, we may suppose that $\Omega_1 = \{\operatorname{Re} w + Q_1(\operatorname{Re} w) < 0\}$ and $\Omega_2 = \{\operatorname{Re} w + Q_2(\operatorname{Re} z) < 0\}$ are already realized as tube domains and that the biholomorphism Ψ is equivariant with respect to the action of $H \simeq i\mathbf{R}^2$ as imaginary translations on both domains. Hence Ψ is an affine linear automorphism of $\mathbf{C}^2$, i.e. $\Psi_1(w,z) = aw + bz + e$, $\Psi_2(w,z) = cw + dz + f$ with

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in GL_2(\mathbf{R}) \quad \text{and} \quad \begin{pmatrix} e \\ f \end{pmatrix} \in \mathbf{R}^2$$

We get

$\Omega_1 = \{\operatorname{Re} w + Q_1(\operatorname{Re} z) < 0\} = \{a \operatorname{Re} w + b \operatorname{Re} z + e + Q_2(c \operatorname{Re} w + d \operatorname{Re} z + f) < 0\}$,

which implies $c = 0$, i.e. $J_1 = \Psi^*(J_2)$ and that the two polynomials are equivalent.

ii) Assume that $H = \mathbf{R} \times S^1$. Then by Lemma 3.1, we may assume that $\Omega_1 = \{\operatorname{Re} w + Q_1(|z|^2) < 0\}$ and $\Omega_2 = \{\operatorname{Re} w + Q_2(|z|^2) < 0\}$ where Q_1 and Q_2 depend only on $|z|^2$. Furthermore, the action of S^1 on both domains is given by rotations in the z-variable. Hence there is an $\alpha \in \mathbf{R}^*$ such that

$$\begin{aligned} \Psi_1(w, e^{i\alpha t} \cdot z) &= \Psi_1(w, z) \\ \Psi_2(w, e^{i\alpha t} \cdot z) &= e^{it} \cdot \Psi_2(w, z), \quad \text{for all } t \in \mathbf{R}. \end{aligned}$$

We get $\alpha = 1$ and

$$(1) \quad \begin{cases} \Psi_1(w, z) = \Psi_1(w) \\ \Psi_2(w, z) = z \cdot g(w). \end{cases}$$

Furthermore there exist $b \in \mathbf{R}$, $\beta \in \mathbf{R}^*$ such that $\Psi^*(J_2) = \{\sigma^t \mid t \in \mathbf{R}\}$ looks like :

$$\begin{aligned} \sigma_1^t(w, z) &= w + i\beta t \\ \sigma_2^t(w, z) &= e^{ibt} z \end{aligned}$$

We get

$$\begin{aligned} \Psi_1(w + i\beta t, e^{ibt} \cdot z) = \Psi_1(w, z) + it \\ \Psi_2(w + i\beta t, e^{ibt} \cdot z) = \Psi_2(w, z). \end{aligned}$$

Now the above expression (1) yields

$$\begin{aligned} &\Psi_1(w, z) = \Psi_1(w) = \frac{1}{\beta} w \\ &\Psi_2(w, z) = z \cdot g(w) = e^{ibt} \cdot z \cdot g(w + i\beta t), \quad \text{for all } t \in \mathbf{R}, \end{aligned}$$

i.e. $e^{-ibt} g(w) = g(w + i\beta t)$, for all $t \in \mathbf{R}$.

It follows :

$$\begin{aligned} -ibg(w) &= g'(w) i\beta, \text{ i.e.} \\ g'(w) &= -\frac{b}{\beta} g(w), \text{ hence} \\ g(w) &= c \cdot e^{-\frac{b}{\beta} \cdot w} \end{aligned}$$

and Ψ is a global automorphism of $\mathbf{C}^2$. This yields easily that $b = 0$ and $c \neq 0$, i.e. $\Psi_2(w, z) = c \cdot z$. But then $\Psi^*(J_2) = J_1$ and $Q_1 \sim Q_2$.

ad (b) : Assume that H is not abelian. By lemma 3.2, we have $J_1 \lhd H$. Suppose that $\Psi^*(J_2) \neq J_1$. Let $\Sigma = \Psi^{-1}$ the inverse of Ψ. Then we have that $J_2 = \Sigma^*(\Psi^*(J_2))$ is not normal in H. But lemma 3.2 applied to the domain Ω_2 gives a contradiction. Hence $\Psi^*(J_2) = J_1$ and Lemma 3.4 is proved. ∎

Remark 3.5 : *Lemma 3.4 gives the proof of Theorem 1.2 in the case* $\dim_{\mathbf{R}} G_1 = \dim_{\mathbf{R}} G_2 = 2$.

4 The homogeneous case

Now we are going to handle the case when the domains in question are homogeneous, i.e. the group G acts transitively on them.

Assume that $\Omega = \{\operatorname{Re} w + Q(z, \bar{z}) < 0\}$ is a homogeneous complex manifold.

Then by a theorem of Rosay [5] the domain Ω is biholomorphic to the unit ball $\mathbf{B}_2 = \{|w|^2 + |z|^2 < 1\}$. As other "canonical" models for $\mathbf{B}^2$ we mention the two realisations $\{\operatorname{Re} w + (Rez)^2 < 0\}$ and $\{\operatorname{Re} w + |z|^2 < 0\}$, which we use in the sequel. Here the polynomials $(\operatorname{Re} z)^2$ and $|z|^2$ are obviously equivalent.

So we assume that $\Omega_1 = \{\operatorname{Re} w + (\operatorname{Re} z)^2 < 0\}$ and $\Omega_2 = \{\operatorname{Re} w + Q_2(z, \bar{z}) < 0\}$.

Lemma 4.1 *Suppose that Ω_1 and Ω_2 are biholomorphic. Then $Q_2(z, z) \sim (Rez)^2$.*

Proof : Let $\Psi : \Omega_1 \to \Omega_2$ denote a biholomorphism. The group G_1 is isomorphic to $SU(2,1)$ and J_1 and $\Psi^*(J_2)$ are two closed one-dimensional non-compact subgroups of $SU(2,1)$. By investigating the structure of $SU(2,1)$ one can show that the normaliser $N_{G_1}(J_1)$ of J_1 in G_1 is five-dimensional and closed and that there is an element $g \in G_1$ such that $g\Psi^*(J_2)g^{-1} \subset N_{G_1}(J_1)$. So one can replace the map Ψ by another biholomorphism $\tilde{\Psi}$, such that J_1 and $\tilde{\Psi}^*(J_2)$ are contained in a two dimensional subgroup H of G_1. But then by lemma 3.4 $\Psi^*(J_2) = J_1$ and $Q_2(z, \bar{z}) = (\operatorname{Re} z)^2$. ∎

Remark 4.2 : *The above mentioned theorem of Rosay and lemma 4.1 prove theorem 1.2 and theorem 1.3 in the homogeneous case.*

5 The three-dimensional case

We start with the following two useful lemmas.

Lemma 5.1 *Let $H \subset G$ be an at least three-dimensional subgroup of* $G = \operatorname{Aut}^0_{\mathcal{O}}(\Omega)$. *Then H is not abelian.*

Proof : By assumption G and therefore H act effectively on Ω. The lemma follows from the fact that Ω is a two-dimensional hyperbolic complex manifold. ∎

Lemma 5.2 *Assume that* $G = \mathrm{Aut}^0_{\mathcal{O}}(\Omega)$ *is not solvable and that* Ω *is not homogeneous. Let* $\mathcal{G} = s \ltimes r$ *be a Levi-Malcev decomposition of* $\mathcal{G} = Lie(G)$*. Then the semisimple part* s *is isomorphic to* $sl_2(\mathbf{R})$*, the Lie algebra of* $SL_2(\mathbf{R})$*.*

Proof : Let $\tilde{s}$ be a complex simple Lie algebra admitting a one or two codimensional complex subalgebra. Then $\tilde{s} \simeq sl_2(\mathbf{C})$ or $\tilde{s} = sl_3(\mathbf{C})$.

Hence our real semi-simple algebra s is isomorphic to $sl_2(\mathbf{R})$, $su(2)$, $sl_3(\mathbf{R})$, $su(2,1)$ or $su(3)$.

In the last four cases, s admits a subalgebra, which is isomorphic to $su(2)$. This means that we have an almost effective action of $SU(2,\mathbf{C})$ on Ω. Then the generic orbit of this action is a compact 3-dimensional CR-hypersurface isomorphic to a finite quotient of S^3. But we have also the non-compact closed subgroup $J \subset G$, which shows that G has an open orbit in Ω. This orbit is isomorphic to the unit ball $\mathbf{B}_2$ and for a point p in this orbit the isotropy group $I_G(p)$ is a maximal compact subgroup K. Assume that there is a point $q \in \Omega$ such that $\dim_{\mathbf{R}} G(q) < 4$. The Ω being hyperbolic implies that $I_G(q)$ is compact and of greater dimension that K. This is impossible. Hence Ω is already homogeneous. But this contradicts our assumption. Hence $s \simeq sl_2(\mathbf{R})$ and the lemma is proved. ∎

Now we assume that $\dim_{\mathbf{R}} G \geq 3$ and that there is a three-dimensional subgroup $H \subset G$ with Lie algebra $\mathfrak{h}$ such that $J \subset H$. In view of Lemmas 5.1 and 5.2, we have the following cases :
I. $\mathfrak{h}$ is solvable and not abelian.
II. $\mathfrak{h} \simeq sL_2(\mathbf{R})$.

5.1 Case I :

The Lie algebra $\mathfrak{h}$ is solvable and $\dim_{\mathbf{R}} h = 3$.

In view of lemma 5.1 $\mathfrak{h}$ cannot be abelian.

We use the classification of three-dimensional solvable Lie algebras given in [2]. Let $\mathfrak{h} = < a, b, c >_{\mathbf{R}}$. Then there are the following cases :

(1) $[a,b] = b$, $[a,c] = [b,c] = 0$;

(2) $[a,c] = b$, $[a,b] = [c,b] = 0$, i.e. $\mathfrak{h}$ is nilpotent.

(3) $[c,b] = 0$, $[a,b] = \alpha b + \beta c$, $[a,c] = \gamma b + \delta c$, where

$$D := \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \in GL_2(\mathbf{R})$$

Lemma 5.3 *Assume that the structure of* $\mathfrak{h}$ *is given by (1) above. Then* $j = \mathfrak{h}'$, *the commutator of* $\mathfrak{h}$ *and* $Q \sim P$, *where* $P(z, \bar{z}) = |z|^{2k}$, $k \geq 2$.

Proof : In view of lemma 3.2, we have that $j \subset < b, c >_{\mathbf{R}} \subset \mathfrak{h}$. Our first step of the proof will be to prove that the group H cannot be simply connected. So we assume this and produce a contradiction.

Then the group L associated to the Lie algebra $l = < b, c >_{\mathbf{R}}$ is isomorphic to $(\mathbf{R}^2, +)$ and contains J.

Hence $\Omega = \{\mathrm{Re}\, w + Q(\mathrm{Re}\, z) < 0\}$ by lemma 3.1. Since L is normal H we have by [4] that the group H acts as a subgroup of $GL_2(\mathbf{R}) \ltimes \mathbf{R}^2$ on $\mathbf{C}^2$ and hence on Ω. So we have a one parameter subgroup $\{(A(t), v(t)) \in H \mid t \in \mathbf{R}\}$ with $\{A(t) \mid t \in \mathbf{R}\} \subset GL_2(\mathbf{R})$ being a non-trivial one parameter subgroup of $GL_2(\mathbf{R})$. By considering the Lie algebra structure of $\mathfrak{h}$ and the shape of Ω, it is an easy calculation to see that this is impossible.

Hence H is not simply connected and isomorphic to $N \times S^1$ where N is the non-abelian two-dimensional Lie group. The group J is contained in $N' \times S^1 \simeq \mathbf{R} \times S^1$ and therefore we have that $\Omega = \{\mathrm{Re}\, w + Q(|z|^2) < 0\}$, the action of S^1 being given as the rotations in the z-variable.

Now let $\{\sigma^t \mid t \in \mathbf{R}\}$ be the one parameter subgroup of H with Lie algebra $< a >_{\mathbf{R}}$. Since S^1 is central in H, it follows

$$\begin{aligned} \sigma_1^t(w, e^{is} \cdot z) &= \sigma_1^t(w, z) \\ \sigma_2^t(w, e^{is} \cdot z) &= e^{is} \sigma_2^t(w, z) \quad \text{for all} \quad s, t \in \mathbf{R}, \end{aligned}$$

i.e.

$$\begin{aligned} \sigma_1^t(w, z) &= \sigma_1^t(w) \\ \sigma_2^t(w, z) &= g(t, w) \cdot z \text{ with } \quad g(t, \cdot) \text{ holomorphic in } w. \end{aligned}$$

Furthermore there is a non-compact one parameter group of the form

$$\{ \begin{pmatrix} w \\ z \end{pmatrix} \mapsto \begin{pmatrix} w + is \\ e^{i\alpha s} \cdot z \end{pmatrix} \mid \alpha \in \mathbf{R} \text{ fixed}, t \in \mathbf{R}\} \triangleleft N$$

which generates together with $\{\sigma^t\}$ the group N i.e. there is a $\rho \in \mathbf{R}^*$ such that

$$\sigma_1^t(w + is, e^{i\alpha s} \cdot z) = \sigma_1^t(w, z) + ie^{\rho t} \cdot s$$

$$\sigma_2^t(w + is, e^{i\alpha s} \cdot z) = \sigma_1^t(w, z) \cdot e^{i\alpha e^{\rho t} \cdot s}$$

and so

$$\sigma_1^t(w, z) = e^{\rho t} \cdot w$$

$$\sigma_2^t(w, z) = g(t, w) \cdot z$$

with $g(t,w)\cdot e^{i\alpha e^{\rho t}\cdot s} = g(t,w+is)\cdot e^{i\alpha s}$ for all $s,t\in\mathbf{R}$ i.e. $g(t,w+is) = e^{i\alpha s(e^{\rho t}-1)}\cdot g(t,w)$ and so

$$\frac{\partial g}{\partial w}(t,w) = \alpha(e^{\rho t}-1)\cdot g(t,w)$$

$$g(t,w) = c(t)e^{(\alpha(e^{\rho t}-1))\cdot w}.$$

Hence is a global automorphism of $\mathbf{C}^2$ stabilizing Ω. But this is only possible if $g(t,w)$ does not depend on w, i.e. $g(t,w)=g(t)$ and then

$$\sigma_1^t(w,z) = e^{\rho t}\cdot w, \quad \text{and} \quad \sigma_2^t(w,z) = g(t)\cdot z, \text{ with } g(t+\tilde{t}) = g(t)\cdot g(\tilde{t}).$$

This implies $g(t)=c{\cdot}e^{\nu\cdot t}$, $\nu\in\mathbf{R}$. Then it is easy to conclude that $Q(z,\bar{z})\sim|z|^{2k}$ and it is obvious that $J=N'\lhd H$. The lemma is proved. ∎

Remark 5.4 : *In the setting of lemma 5.3, i.e.* $\Omega=\{\operatorname{Re}w+|z|^{2k}<0\}$, *the automorphism group* G *of* Ω *is* $S\cdot T$, *where* S *is a finite covering of* $SL_2(\mathbf{R})$ *and* T *is a central subgroup isomorphic to* S^1, *i.e.* $\dim G=4$. *This case will also appear below.*

Lemma 5.5 *Assume that the structure of* $\mathfrak{h}$ *is given by (2) above. Then* Ω *is biholomorphic to the unit ball* $\mathbf{B}_2$.

Proof : Here $\mathfrak{h}$ is isomorphic to the Lie algebra of the three-dimensional Heisenberg group H_3. First we consider the case that H is not simply- connected. Then $H=H_3/\Gamma$, where Γ is a discrete subgroup of H_3 isomorphic to $\mathbf{Z}$ lying in the center C of H_3. Hence H contains a central subgroup $L=C/\Gamma\simeq S^1$. Then J and L generate a two- dimensional subgroup isomorphic to $\mathbf{R}\times S^1$ and by lemma 3.1 we may assume that $\Omega=\{\operatorname{Re}w+Q(|z|^2)<0\}$ with the natural $\mathbf{R}\times S^1$ action. The polynomial Q depends only on $|z|^2$, is subharmonic and can be assumed to satisfy $Q(0)=0$ and $Q\geq 0$. Then $\tau(\Omega)=\{\operatorname{Re}w<0\}$, where $\tau:(w,z)\to z$ from $\mathbf{C}^2$ to $\mathbf{C}$ denotes the projection on the first component.

This map is an equivariant H-map since $L\simeq S^1$ is central in H and the L-action is given by rotations in the Z-variable. Therefore the two-dimensional group H/L acts on $\{\operatorname{Re}w<0\}=\tau(\Omega)$. But this action cannot be effective, since there is no two-dimensional abelian subgroup in the automorphism group of the half-plane. Hence a two-dimensional subgroup of H containing L stabilizes all fibers of τ and acts effectively on the fibers. But the τ-fibers in Ω are also half-planes and every two-dimensional subgroup of H is abelian. This is again not possible. So we have proven that H is isomorphic to the simply-connected Heisenberg group H_3. Then there is a two-dimensional subgroup A containing

J which is isomorphic to $(\mathbf{R}^2,+)$. By lemma 3.1, the domain Ω is given by $\{\operatorname{Re} w + Q(\operatorname{Re} z) < 0\}$ a tube domain.

Let $\{\sigma^t = (\sigma_1^t, \sigma_2^t) \mid t \in \mathbf{R}\}$ be a one-parameter group in H which together with A generates H. Since $A \subset H$ is normal, we have by [4] that $\{\sigma^t \mid t \in \mathbf{R}\}$ is a subgroup of the affine linear group $GL_2(\mathbf{R}) \ltimes \mathbf{R}^2$.

So let $\{\begin{pmatrix} a(t) & b(t) \\ c(t) & d(t) \end{pmatrix}, \begin{pmatrix} e(t) \\ f(t) \end{pmatrix} = \sigma^t\} \subset GL_2(\mathbf{R}) \ltimes \mathbf{R}^2\}$ denote this group. The group $\{A(t) = \begin{pmatrix} a(t) & b(t) \\ c(t) & d(t) \end{pmatrix} \mid t \in \mathbf{R}\}$ is not trivial in $GL_2(\mathbf{R})$. We have

$$\sigma^t(w,z) = \begin{pmatrix} a(t)w + b(t)z + e(t) \\ c(t)w + d(t)z + f(t) \end{pmatrix}$$

and σ^t stabilizing Ω implies :

$$\begin{aligned} \Omega &= \{\operatorname{Re} w + Q(\operatorname{Re} z) < 0\} \\ &= \{a(t)\operatorname{Re} w + b(t)\operatorname{Re} z + e(t) + Q(c(t)\operatorname{Re} w + d(t)\operatorname{Re} z + f(t)) < 0\} \end{aligned}$$

It follows immediately that $c(t) = 0$ for all $t \in \mathbf{R}$ and that

$$Q(\operatorname{Re} z) = \frac{b(t)}{a(t)}\operatorname{Re} z + \frac{e(t)}{a(t)} + \frac{1}{a(t)}Q(d(t)\operatorname{Re} z + f(t))$$

The group H being nilpotent implies that $a(t) = d(t) = 1$ for all $t \in \mathbf{R}$, i.e.

$$Q(\operatorname{Re} z) = b(t)\operatorname{Re} z + e(t) + Q(\operatorname{Re} z + f(t)).$$

Since $b(t)$ is not identically zero, this equation implies that $\deg Q = 2$ and that Ω is biholomorphic to $\mathbf{B}_2$. ∎

Lemma 5.6 *Assume that the structure of $\mathfrak{h}$ is given by (3) above and that Ω is not homogeneous. Then $\Omega = \{\operatorname{Re} w + (\operatorname{Re} w)^{2k} < 0\}$, $k \geq 2$ and $G = H$.*

Proof : The structure of h implies that $\dim_{\mathbf{R}} h' = 2$ and that the associated group $H' \subset H$ is isomorphic to $(\mathbf{R}^2,+)$. So Ω as a simply-connected hyperbolic Stein manifold of dimension two with an action of $(\mathbf{R}^2,+)$, therefore it is biholomorphic to a tube domain $\Omega' = F + i\mathbf{R}^2$, where F is a convex domain in $\mathbf{R}^2$ containing no complex lines (see [7]). The group $H' \simeq (\mathbf{R}^2,+)$ being normal in H implies that H acts on Ω' as a subgroup of $GL_2(\mathbf{R}) \ltimes \mathbf{C}^2$ (see [4]).

Let $\{\sigma^t = (\sigma_1^t, \sigma_2^t)\}$ be a one-parameter subgroup of H generating together with H' the group H. Then

$$\sigma^t = \left(\begin{pmatrix} a(t) & b(t) \\ c(t) & d(t) \end{pmatrix}, \begin{pmatrix} e(t) \\ f(t) \end{pmatrix}\right) \in GL_2(\mathbf{R}) \ltimes \mathbf{R}^2$$

$$= (A(t), \vec{v}(t)),$$

with $A(t) = e^{t\cdot D}$, where

$$D = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}.$$

Since $D \in GL_2(\mathbf{R})$, after a conjugation with an element of

$$\{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \ltimes \mathbf{R}^2\},$$

we have that $\vec{v}(t) = 0$ for all $t \in \mathbf{R}$, i.e.

$$\sigma^t = \left(\begin{pmatrix} a(t) & b(t) \\ c(t) & d(t) \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right) \text{ for all } t \in \mathbf{R}$$

Now assume that D is not triangulisable over $\mathbf{R}$. Then $\{\sigma^t \mid t \in \mathbf{R}\}$ is isomorphic to S^1, since any one dimensional subgroup of $GL_2(\mathbf{R})$, which is not compact, is triangulisable over $\mathbf{R}$. So the domain $F \subset \mathbf{R}^2$ is invariant by a linear S^1-action and must therefore be bounded.

On the other hand we have that J has to lie in H' because otherwise it would be a compact group. Then $\Omega = \{\operatorname{Re} w + Q(\operatorname{Re} z) < 0\}$ and H acts affinely on Ω. Since the set $\{(y, x) \in \mathbf{R}^2 \mid y + Q(x) < 0\}$ is not bounded we get a contradiction.

So we can assume that the matrix D is triangulisable over $\mathbf{R}$. Hence H' contains a one-dimensional normal subgroup of H. If $J \not\subset H'$, then this group and J generate a two-dimensional non-abelian group, which is impossible by lemma 3.2.

So we have that $J \subset H' \simeq (\mathbf{R}^2, +)$, $\Omega = \{\operatorname{Re} w + Q(\operatorname{Re} z) < 0\}$ a tube domain and that H acts affinely on $\mathbf{C}^2$ and on Ω with $H' \subset H$ the group of imaginary translations as a normal subgroup.

We have that

$$\begin{aligned} \sigma^t = A(t) &= \begin{pmatrix} a(t) & b(t) \\ c(t) & d(t) \end{pmatrix}, \in GL_2(\mathbf{R}) \\ &= e^{t\cdot D}, D = \begin{pmatrix} \alpha & \beta \\ \mu & \delta \end{pmatrix}, t \in \mathbf{R}. \end{aligned}$$

Then

$$\begin{aligned} \Omega &= \{\operatorname{Re} w + Q(\operatorname{Re} z) < 0\} \\ &= \{a(t) \operatorname{Re} w + b(t) \operatorname{Re} z + Q(c(t) \operatorname{Re} w + d(t) \operatorname{Re} z) < 0\} \end{aligned}$$

which shows that $c(t) \equiv 0$, i.e.

$$Q(\operatorname{Re} z) = \frac{b(t)}{a(t)} \operatorname{Re} z + \frac{1}{a(t)} Q(d(t) \operatorname{Re} z) \quad \text{for all } t \in \mathbf{R}.$$

Since we may assume that Q has no harmonic summands we get

$$Q(\operatorname{Re} z) = \frac{1}{a(t)} Q(d(t) \operatorname{Re} z).$$

This implies that $Q(\operatorname{Re} z) = (\operatorname{Re} z)^{2k}$, $k \geq 2$ and that the action of σ^t is given by

$$\sigma^t(w, z) = (e^{2kt} \cdot w, e^t \cdot z), t \in \mathbf{R}.$$

Now we prove that $G = H$. First we show that G is solvable. Assume to the contrary that G is not solvable. Then, since Ω is not homogeneous, the semisimple part of G is isomorphic to a covering of $SL_2(\mathbf{R})$. Then by checking the possibilities for G as an automorphism group of a 2-dimensional hyperbolic manifold (see Case II) it is easy to see that G' does not contain a two-dimensional abelian subgroup. So G is solvable and $\dim_{\mathbf{R}} G' \geq 2$. Furthermore G' is nilpotent and contains $H' \simeq (\mathbf{R}^2, +)$. Then it is easy to see (by checking the possibilities for G') that $H' \triangleleft G'$, which implies that $H' = G'$ (lemma 5.1 and lemma 5.5). Then $H' \triangleleft G$ and by applying again [4] one concludes that $G = H$. ∎

5.2 Case II : $\mathfrak{h} \sim sl_2(\mathbf{R})$

Here we are going to handle completely the situation where Ω is not homogeneous and G is not solvable.

By lemma 5.2, there is a three-dimensional subgroup H of G such that the Lie algebra $\mathfrak{h}$ is isomorphic to $sl_2(\mathbf{R})$.

Since Ω is not homogeneous we have that $3 \leq \dim_{\mathbf{R}} \mathcal{G} \leq 5$, in view of the possibilities of a maximal compact subgroup K : $K = (e)$, $K = S^1$, $K = (S^1)^2$.

Let $\mathcal{G} = \mathfrak{h} \ltimes r$ be a Levi-Malcev decomposition of $\mathcal{G}$. Here r denotes the radical of $\mathcal{G}$. Hence $\dim_{\mathbf{R}} r = 1$ or 2. If $\dim_{\mathbf{R}} r = 2$, then r is abelian, because otherwise the center of $SL_2(\mathbf{R}) \ltimes R$ is too small to admit a discrete central quotient with maximal compact subgroup $(S^1)^2$. But then $\mathcal{G} = \mathfrak{h} \times r$ is a direct product again because otherwise there is no central subgroup with quotient $(S^1)^2$. The existence of a three-dimensional abelian subgroup excludes this case (Lemma 5.1). If $\dim_{\mathbf{R}} r = 1$, then $\mathcal{G} = h \times r$ a direct product.

Hence we have only two possibilities for $\mathcal{G}$:

$$\mathcal{G} = \mathfrak{h} = sl_2(\mathbf{R}) \text{ or } \mathcal{G} = \mathfrak{h} \times \mathbf{R} = sl_2(\mathbf{R}) \times \mathbf{R}.$$

We consider these cases in the following lemmas.

Lemma 5.7 *Assume that $j \subset \mathfrak{h} \subset \mathcal{G}$. Then J is contained in a two-dimensional subgroup of H.*

Proof : If H is modulo a finite covering isomorphic to $SL_2(\mathbf{R})$, then J as a non-compact subgroup of H is contained in a two-dimensional subgroup of H. So assume that $H \simeq \widetilde{SL_2(\mathbf{R})}$, the universal covering of $SL_2(\mathbf{R})$, and let C denote the center of H which is isomorphic to $\mathbf{Z}$. If $J \cap C = (e)$, then J is also contained in a two-dimensional subgroup of H. So assume that $J \cap C \neq (e)$, i.e. $J \cap C \simeq \mathbf{Z}$.

First this implies that H is a closed subgroup of G. (If $H \simeq \widetilde{SL_2(\mathbf{R})}$ is not closed in G, then the maximal compact subgroup K of G is $(S^1)^2$ and contains C. But $J \subset G$ is a closed, non-compact subgroup of G and therefore $J \cap C = (e)$, which is a contradiction.)

Hence H acts freely on Ω and all orbits are closed and isomorphic to $\mathbf{R}^3$. We may assume that $J \cap C = \{(w,z) \mapsto (w + 2\pi i k, z) \mid k \in \mathbf{Z}\}$. This group acts freely and properly discontinuous on Ω and we can consider the quotient

$$\Omega = \{\operatorname{Re} w + Q(z,\bar{z}) < 0\} \overset{(e^w,z)}{\rightarrow} \{0 < |w|^2 e^{2Q(z,\bar{z})} < 1\} = \Omega'.$$

Then there is an action of a group $S = SL_2(\mathbf{R})/J \cap C$ on Ω' and the group $J/J \cap C$ acts as rotations in the w-variable. Furthermore the S-action is free and all orbits are closed.

Now let (X_1, X_2, X_3) be a basis of the three-dimensional vector space of holomorphic vector fields induced by the S-action on Ω'. We take the exterior products $\sigma_1 = X_1 \wedge X_2$, $\sigma_2 = X_1 \wedge X_3$, $\sigma_3 = X_2 \wedge X_3$. The σ_i are sections in the anticanonical bundle $\det(T^{1,0}_{\mathcal{O}}\Omega') = \kappa^{-1}$ and generate an S-invariant subspace of $\Gamma_{\mathcal{O}}(\Omega', \kappa^{-1})$. For every point $p \in \Omega'$, there is σ_i such that $\sigma_i(p) \neq 0$. Hence we get an S-equivariant holomorphic mapping $\alpha : \Omega' \to \mathbf{P}_2(\mathbf{C})$ defined by

$$\alpha(p) = \big(\sigma_1(p) : \sigma_2(p) : \sigma_3(p)\big),$$

where the S-action on $\mathbf{P}_2(\mathbf{C})$ is given by the natural $S/C(S) \simeq PSL_2(\mathbf{R})$-action which is of course projective-linear.

Since there is no $PSL_2(\mathbf{R})$-fix-point in $\mathbf{P}_2(\mathbf{C})$ the map α cannot be trivial.

Hence the map α is either locally biholomorphic or the dimension of the fibers is one.

In the latter case, the restriction of α to every S-orbit is an S^1-principal Cauchy-Riemann bundle (see [5]) and this fact yields that there is an additional holomorphic S^1-action on Ω' which commutes with the S-action. Hence $\dim_{\mathbf{R}} G = 4$ and we get a 2-dimensional abelian subgroup of G containing J, i.e. by Lemma 3.1, $\Omega = \{\operatorname{Re} w + Q(|z|^2) < 0\}$ or $\Omega = \{\operatorname{Re} w + Q(\operatorname{Re} z) < 0\}$. In both cases, one can assume that $Q(z,\bar{z}) \geq 0$ for all $z \in \mathbf{C}$.

But then an automorphism of Ω' extends to an automorphism of $\Omega' \cup \{w = 0\}$ and we get an S-action on $\mathbf{C} \simeq \{w = 0\}$. This is impossible.

So we have to consider the case where the map α is locally biholomorphic. By considering the $PSL_2(\mathbf{R})$-invariant domains in $\mathbf{P}_2$, with the property that all $PSL_2(\mathbf{R})$-orbits are 3-dimensional, one sees that the image of Ω' by α is contained in a domain biholomorphic to $\Delta \times \Delta \setminus \mathrm{Diag}(\Delta \times \Delta)$ with the diagonal $PSL_2(\mathbf{R})$-action. (Here $\Delta = \{y \in \mathbf{C} \mid |y| < 1\}$).

Furthermore the associated map of S resp. $PSL_2(\mathbf{R})$-orbits is injective, since they are 3-dimensional in a 2-dimensional complex manifold and α is locally biholomorphic.

So we have a locally biholomorphic, S-equivariant map

$$\tilde{\alpha} : \Omega' \to \Delta \times \Delta \setminus \mathrm{Diag}(\Delta \times \Delta).$$

Using the S-equivariance and the concrete description of $PSL_2(\mathbf{R})$- orbits in $\Delta \times \Delta \setminus \mathrm{Diag}(\Delta \times \Delta)$, one can see that this is impossible. The lemma is proved. ∎

Lemma 5.8 *Assume that $j \subset \mathfrak{h} \subset \mathcal{G}$ and that J is contained in a two-dimensional subgroup of H. Then H is a finite covering of $SL_2(\mathbf{R})$ and $Q \sim P$, with $P(z, \bar{z}) = |z|^{2k}$, $k \geq 2$.*

Proof : We assume that J is contained in a two dimensional subgroup of H. We are going to prove $Q \sim P$, with $P(z, \bar{z}) = |z|^{2k}$ directly. Then is follows that H is modulo a finite covering isomorphic to $SL_2(\mathbf{R})$, by an investigation of the automorphism group of $\{\mathrm{Re}\, w + |z|^{2k} < 0\}$.

By lemma 3.2, we have the two holomorphic vector fields $X = i\frac{\partial}{\partial w}$ and $Z = -2w\frac{\partial}{\partial w} - \frac{z}{k}\frac{\partial}{\partial z}$ induced by J and the group $\{(w, z) \mapsto (e^{2kt} \cdot w, e^t \cdot z) \mid t \in \mathbf{R}\}$. In view of structure of H there is a third holomorphic vector field Y induced by a one parameter subgroup of H such that

$$\begin{aligned} [Z, X] &= 2X \\ [X, Y] &= Z \\ [Z, Y] &= -2Y. \end{aligned}$$

Furthermore $< \mathrm{Re}\, X, \mathrm{Re}\, Y, \mathrm{Re}\, Z >_{\mathbf{R}}$ is the Lie algebra of real infinitesimal holomorphic transformations induced by H on Ω.

Now let $Y(w, z) = f(w, z)\frac{\partial}{\partial w} + g(w, z)\frac{\partial}{\partial z}$. Using the commutator relations we calculate f and g :

$$\begin{aligned} [X, Y] &= [i\frac{\partial}{\partial w}, f\frac{\partial}{\partial w} + g\frac{\partial}{\partial z}] \\ &= i\frac{\partial f}{\partial w}\frac{\partial}{\partial w} + i\frac{\partial g}{\partial w}\frac{\partial}{\partial z} \\ &= -2w\frac{\partial}{\partial w} - \frac{z}{k}\frac{\partial}{\partial z} = Z \end{aligned}$$

Hence $\frac{\partial f}{\partial w} = -2iw$, $\frac{\partial g}{\partial w} = -\frac{iz}{k}$ and so

$$f(w,z) = -iw^2 + f_1(z) \text{ and } g(w,z) = -\frac{izw}{k} + g_1(z).$$

Furthermore :

$$\begin{aligned}
[Z,Y] &= [-2w\frac{\partial}{\partial w} - \frac{z}{k}\frac{\partial}{\partial z}, f\frac{\partial}{\partial w} + g\frac{\partial}{\partial z}] \\
&= -2w\frac{\partial f}{\partial w}\frac{\partial}{\partial w} - 2w\frac{\partial g}{\partial w}\frac{\partial}{\partial z} - \frac{z}{k}\frac{\partial f}{\partial z}\frac{\partial}{\partial w} \\
&- \frac{z}{k}\frac{\partial g}{\partial z}\frac{\partial}{\partial z} + 2f\frac{\partial}{\partial w} + \frac{g}{k}\frac{\partial}{\partial z} \\
&= -2f\frac{\partial}{\partial w} - 2g\frac{\partial}{\partial z} = -2Y.
\end{aligned}$$

and therefore

$$\begin{aligned}
-2f &= -2w\frac{\partial f}{\partial w} + 2f - \frac{z}{k}\frac{\partial f}{\partial z} \\
-2g &= -2w\frac{\partial g}{\partial w} - \frac{z}{k}\frac{\partial g}{\partial z} + \frac{g}{k}
\end{aligned}$$

and finally

$$4f = 2w\frac{\partial f}{\partial w} + \frac{z}{k}\frac{\partial f}{\partial z}, \; (2k+1)g = 2kw\frac{\partial g}{\partial w} + z\frac{\partial g}{\partial z}.$$

It follows that :

$$\begin{aligned}
4(-iw^2 + f_1(z)) &= -4iw^2 + \frac{z}{k}f_1'(z) \\
(2k+1)(-\frac{izw}{k} + g_1(z)) &= -2izw - \frac{izw}{k} + zg_1'(z), \quad \text{i.e.}
\end{aligned}$$

$4f_1(z) = \frac{z}{k}f_1'(z)$ and $(2k+1)g_1(z) = zg_1'(z)$, which implies

$$\begin{aligned}
f_1(z) &= c \cdot z^{4k} \\
g_1(z) &= d \cdot z^{2k+1}, \quad c, d \in \mathbf{C}.
\end{aligned}$$

The vector field Y is therefore given by

$$Y = (-iw^2 + cz^{4k})\frac{\partial}{\partial w} + (-\frac{izw}{k} + d \cdot z^{2k+1})\frac{\partial}{\partial z}.$$

In particular Y is a global holomorphic vector field on $\mathbf{C}^2$ and $\operatorname{Re} Y$ stabilizes the CR-hypersurface $M = \{\operatorname{Re} w + P_{2k}(z,\bar{z}) = 0\}$, which means that

$$(Y + \bar{Y})(\operatorname{Re} w + P_{2k}(z,\bar{z})) \,|_M \equiv 0.$$

We will compute this expression now :

$$\begin{aligned}
(Y \quad + \quad & \bar{Y})(\operatorname{Re} w + P_{2k}(z,\bar{z})) = \frac{1}{2}(-iw^2 + cz^{4k}) + \frac{1}{2}(i\bar{w}^2 + \bar{c}\bar{z}^{4k}) \\
+ \quad & (-\frac{izw}{k} + dz^{2k+1})\frac{\partial P_{2k}}{\partial z} + (\frac{i\bar{z}\bar{w}}{k} + \bar{d}\bar{z}^{2k+1})\frac{\partial P_{2k}}{\partial \bar{z}} \\
= \quad & \frac{1}{2}(cz^{4k} + \bar{c}\bar{z}^{4k}) + \frac{1}{2}i(-(\operatorname{Re} w + i\operatorname{Im} w)^2 + (\operatorname{Re} w - i\operatorname{Im} w)^2) \\
+ \quad & (dz^{2k+1}\frac{\partial P_{2k}}{\partial z} + \bar{d}\bar{z}^{2k+1}\frac{\partial P_{2k}}{\partial \bar{z}}) - \frac{iz}{k}(\operatorname{Re} w + i\operatorname{Im} w)\frac{\partial P_{2k}}{\partial z} \\
+ \quad & \frac{i\bar{z}}{k}(\operatorname{Re} w - i\operatorname{Im} w)\frac{\partial P_{2k}}{\partial \bar{z}} \\
= \quad & \frac{1}{2}(cz^{4k} + \bar{c}\bar{z}^{4k}) + (dz^{2k+1}\frac{\partial P_{2k}}{\partial z} + \bar{d}\bar{z}^{2k+1}\frac{\partial P_{2k}}{\partial \bar{z}}) \\
+ \quad & (2\operatorname{Re} w \operatorname{Im} w + \frac{z}{k}\operatorname{Im} w\frac{\partial P_{2k}}{\partial z} + \frac{\bar{z}}{k}\operatorname{Im} w\frac{\partial P_{2k}}{\partial \bar{z}}) \\
+ \quad & (-\frac{iz}{k}\operatorname{Re} w\frac{\partial P_{2k}}{\partial z} + \frac{i\bar{z}}{k}\operatorname{Re} w\frac{\partial P_{2k}}{\partial \bar{z}}).
\end{aligned}$$

We put $\operatorname{Re} w = -P_{2k}$ and observe that P_{2k} being homogeneous implies that $P_{2k} = \frac{1}{2k}(z\frac{\partial P_{2k}}{\partial z} + \bar{z}\frac{\partial P_{2k}}{\partial \bar{z}})$ to get that the expression

$$\begin{aligned}
\frac{1}{2}(cz^{4k} \quad + \quad & \bar{c}\bar{z}^{4k}) + (dz^{2k+1}\frac{\partial P_{2k}}{\partial z} + \bar{d}\bar{z}^{2k+1}\frac{\partial P_{2k}}{\partial \bar{z}}) \\
+ \quad & (\frac{iz}{k}P_{2k}\frac{\partial P_{2k}}{\partial z} - \frac{i\bar{z}}{k}P_{2k}\frac{\partial P_{2k}}{\partial \bar{z}}) = 0 \quad \text{for all } z \in \mathbf{C}.
\end{aligned}$$

We may assume that P_{2k} has no harmonic summands and reduce to

$$dz^{2k+1}\frac{\partial P_{2k}}{\partial z} + \bar{d}\bar{z}^{2k+1}\frac{\partial P_{2k}}{\partial \bar{z}} + \frac{iz}{k}\frac{\partial P_{2k}}{\partial z} \cdot P_{2k} - \frac{i\bar{z}}{k}\frac{\partial P_{2k}}{\partial \bar{z}}P_{2k} = 0,$$

for all $z \in \mathbf{C}$, with $P_{2k}(z,\bar{z}) = \sum_{j=1}^{2k-1} a_j z^j \bar{z}^{2k-j}$, $a_j = \overline{a_{2k-j}}$ and $k \geq 2$.

If the constant $d = 0$, then it follows that

$$\bar{z}\frac{\partial P_{2k}}{\partial \bar{z}} = z\frac{\partial P_{2k}}{\partial z}, \text{ which forces } P_{2k}(\bar{z},\bar{z}) = a_k|z|^{2k}, a_k \in \mathbf{R}^{>0}.$$

So assume that $d \neq 0$. Then we have

$$d \cdot \sum_{j=1}^{2k-1} ja_j z^{2k+j}\bar{z}^{2k-j} + \bar{d}\sum_{j=1}^{2k-1} a_j(2k-j)z^j\bar{z}^{4k-j}$$

$$+ \frac{2i}{k}\left[(\sum_{j=1}^{2k-1} a_j z^j \bar{z}^{2k-j})(\sum_{j=1}^{2k-1} a_j (j-k) z^j \bar{z}^{2k-j})\right]$$

$$= \bar{d}\sum_{j=1}^{2k-1} a_j(2k-j)z^j\bar{z}^{4k-j} + d\sum_{j=2k+1}^{4k-1} a_{j-2k}(j-2k)z^j\bar{z}^{4k-j}$$

$$+ \frac{2i}{k}\left[\sum_{j=2}^{4k-2}(\sum_{l+n=j} a_l a_n (n-k))z^j\bar{z}^{k-j}\right] = 0 \text{ for all } z \in \mathbf{C}.$$

Let $\tau \in \{1, \ldots, k\}$ be the smallest number such that $a_\tau \neq 0$. Then our expression becomes

$$\bar{d}\sum_{j=\tau}^{2k-\tau} a_j(2k-j)z^j\bar{z}^{4k-j} + d\sum_{j=2k+\tau}^{4k-\tau} a_{j-2k}(j-2k)z^j\bar{z}^{4k-j}$$

$$+\frac{2i}{k}[\sum_{j=2\tau}^{4k-2\tau}(\sum_{l+n=j} a_l a_n(n-k))z^j\bar{z}^{4k-j}] = 0.$$

But then $a_\tau = 0$, which is a contradiction.

So we have that $\mathcal{P}(z,\bar{z}) = |z|^{2k}$, $k \geq 2$ and the lemma is proved. ∎

Lemma 5.9 *Assume that $\mathcal{G} = \mathfrak{h} \times r$, $\dim r = 1$. Then $j \subset \mathfrak{h}$.*

Proof : Assume that $\mathcal{G} = \mathfrak{h} \times r$ and $j \not\subset \mathfrak{h}$. In view of lemma 5.3, we have $j \neq r$. Let $\pi : \mathcal{G} \to \mathfrak{h}$ be the projection of $\mathcal{G}$ onto h with kernel r. Again in view of lemma 5.3, we have that $\pi(j)$ is the Lie algebra of a maximal compact subgroup of $SL_2(\mathbf{R})$. Let L be the two-dimensional subgroup of G whose Lie algebra l is generated by r and $\pi(j)$. It is clear that L is a two-dimensional Lie group containing J and the center C of G. Therefore $L = S^1 \times \mathbf{R}$, since otherwise $G = SL_2(\tilde{\mathbf{R}}) \times \mathbf{R}$, which is impossible. Hence $\Omega = \{\mathrm{Re}\, w + Q(|z|^2) < 0\}$, where we may assume that $Q(|z|^2) \geq 0$ for all $z \in \mathbf{C}$. The action of the connected component of C^0 the center of G is given by

$$(w,z) \mapsto (w + it, e^{i\rho t}\cdot z), t \in \mathbf{R}, \rho \in \mathbf{R}^* \text{ fixed.}$$

We consider the function $(w,z) \xrightarrow{f} z \cdot e^{-\rho w} \in \mathbf{C}$, which is invariant under this action. We have

$$|z \cdot e^{-\rho w}|^2 = |z|^2 \cdot e^{-\rho 2\,\mathrm{Re}\, w} \geq |z|^2 e^{\rho 2 Q(|z|^2)}.$$

The expression on the right side tends to $+\infty$ when $|z| \to +\infty$ and the image of f is S^1-invariant. Hence $f : \Omega \to \mathbf{C}$ is surjective and has maximal rank everywhere. Hence we get an G/C^0 action on $\mathbf{C}$ which is impossible. The lemma is proved. ■

Remark 5.10 *a) The automorphism group of a domain* $\Omega = \{\mathrm{Re}\, w + |z|^{2k} < 0\}$, $k \geq 2$ *is a product* $S \cdot S^1$, *where* S *is modulo a finite group isomorphic to* $SL_2(\mathbf{R})$ *and* S^1 *is a central one-dimensional group. Hence* G *is four-dimensional.*

b) In the case $\dim_{\mathbf{R}} G = 3$ *the lemmas 5.3 to 5.9 prove theorem 1 and theorem 2.*

c) We mention that from now on we may assume that G *is solvable since the non-solvable case is completely handled by the lemmas 5.2 to 5.9 .*

6 The case $\dim_{\mathbf{R}} G \geq 4$

Lemma 6.1 *Let* $\Omega = \{\mathrm{Re}\, w + Q(z, \bar{z}) < 0\}$ *and assume that* $G = \mathrm{Aut}^0_{\mathcal{O}}(\Omega)$ *is solvable. Then* $\dim_{\mathbf{R}} G \leq 3$.

Proof : We assume that $\dim_{\mathbf{R}} G \geq 4$ and that Ω is not homogeneous. So we have that $\dim G = 4$ or 5, since the highest dimensional compact subgroup of G is $(S^1)^2$.

Let $N \subset G$ be the largest nilpotent normal connected subgroup of G. Clearly, N contains $(G')^0$, the connected component of the commutator G' of G.

We first show that $\dim_{\mathbf{R}} N \leq 3$. Assume the contrary, i.e. $\dim N \geq 4$. Then the maximal compact subgroup of N is not trivial, i.e. isomorphic to S^1 or $(S^1)^2$. But compact subgroups of nilpotent Lie groups are always central, in view of the bijectivity of the exponential map. Then N as a subgroup of G doesnot act effectively, a contradiction. So $\dim_{\mathbf{R}} N \leq 3$. So we have to consider three cases :

i) $n = h_3$ the three-dimensional Heisenberg algebra ;

ii) $\dim N = 2$ and N is abelian ;

iii) $\dim N = 1$.

Cas i) : $n = h_3$. By similar arguments as above and using the fact that all maximal compact subgroups are conjugate one sees that N is simply connected. Hence all N and therefore all G-orbits in Ω are closed CR-hypersurfaces isomorphic to $\mathbf{R}^3$. Using the results of [4], [7], it is not hard to check that a simply connected hyperbolic Stein manifold acted on by H_3 is biholomorphic to the ball ; this contradicts our assumption.

Cas ii) : $\dim_{\mathbf{R}} N = 2$ and N is abelian.

If $J \not\subset N$ then J and N generate a three-dimensional solvable group. Using the lemmas of Section **V**, we see that G cannot be solvable and of dimension four or greater, if Ω is not homogeneous.

So we have $J \subset N$ and we can find a 3-dimensional solvable group containing J. Using again the lemmas of Section **V** we conclude like above.

Case iii) : $\dim_{\mathbf{R}} N = 1$. Then either $J = N$ or J and N generate a two dimensional abelian group. In both cases we can take the complex-analytic quotient of Ω by N, which is either the upper half plane or $\mathbf{C}$. But G/N is at least 3-dimensional and abelian. This is impossible. ∎

Remark 6.2 *Using the same methods as above it can be shown that the number of connected components of* $\mathrm{Aut}_{\mathcal{O}}(\Omega)$ *is always finite.*

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