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UNIFORM ESTIMATES FOR THE CAUCHY-RIEMANN EQUATION ON q-CONCAVE WEDGES

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0. Introduction

This article is the continuation of [L-T/Le]. Both papers are preliminary works for a systematic study of the tangential Cauchy-Riemann equation on real submanifolds from the viewpoint of uniform estimates and by means of integral formulas. For this study we have to solve the Cauchy-Riemann equation with uniform estimates on q-convex and q-concave wedges in $\mathbb{C}^n$ (for historical remarks, see the introduction to [L-T/Le]). Whereas [L-T/Le] is devoted to q-convex wedges, here we study q-concave wedges.

The main result of the present paper can be formulated as follows. Let $G \subseteq \mathbb{C}^n$ be a domain, q an integer with $1 \leqslant q \leqslant n-1$, and $\varphi_1, \dots, \varphi_N$ a collection of real C^2 functions on G satisfying the following three conditions :

(i) $E := \{ z \in G : \varphi_1(z) = \cdots = \varphi_N(z) = 0 \} \neq \emptyset$;

(ii) $d\varphi_1(z) \wedge \cdots \wedge d\varphi_N(z) \neq 0$ for all $z \in G$;

(iii) If $\lambda = (\lambda_1, \dots, \lambda_N)$ is a collection of non-negative real numbers with $\lambda_1 + \cdots + \lambda_N = 1$, then, at all points in G, the Levi form of the function

$$\lambda_1\varphi_1 + \cdots + \lambda_N\varphi_N$$

has at least $q+1$ positive eigenvalues.

Set

$$D = \bigcap_{j=1}^{N} \{ z \in G : \varphi_j(z) > 0 \} \tag{0.1}$$

and

$$\Omega = \bigcup_{j=1}^{N} \{ z \in G : \varphi_j(z) > 0 \} . \tag{0.2}$$

Further, for $\xi \in \mathbf{C}^n$ and $R > 0$, we denote by $B_R(\xi)$ the open ball of radius R in $\mathbf{C}^n$ centered at ξ. Then Theorems 5.6, 5.7 and 6.6 of the present work imply the following

0.1. THEOREM. — *For each point $\xi \in E$ there exists a radius $R > 0$ such that :*

(a) If $q-N \geqslant 0$, then each holomorphic function on D extends holomorphically to $D \cup B_R(\xi)$;

(b) If $q-N \geqslant 1$ and f is a continuous $\bar\partial$-closed (n,r)-form with $1 \leqslant r \leqslant q-N$ on D, then there exists a continuous $(n,r-1)$-form u on $D \cap B_R(\xi)$ with

$$\bar\partial u = f \quad \text{on} \quad D \cap B_R(\xi) . \tag{0.3}$$

Moreover if, for some β with $0 \leqslant \beta < 1$, f satisfies the estimate

$$\| f(\zeta) \| \leqslant [\operatorname{dist}(\zeta, \partial D)]^{-\beta}, \quad \zeta \in D, \tag{0.4}$$

then the solution u of (0.3) can be given by an explicit integral operator and, for all $\varepsilon > 0$, there is a constant $C_\varepsilon > 0$ (independent of f) such that :

If $0 \leqslant \beta < 1/2$, then u is Hölder continuous with exponent $1/2-\beta-\varepsilon$ on $\overline{D \cap B_R(\xi)}$ and

$$\| u \|_{1/2-\beta-\varepsilon, \overline{D\cap B_R(\xi)}} \leqslant C_\varepsilon \sup_{\zeta \in D} \| f(\zeta) \| [\operatorname{dist}(\zeta, \partial D)]^{\beta}, \tag{0.5}$$

where $\| \cdot \|_{1/2-\beta-\varepsilon, \overline{D\cap B_R(\xi)}}$ is the Hölder norm with exponent $1/2-\beta-\varepsilon$ on $\overline{D \cap B_R(\xi)}$.

If $1/2 \leqslant \beta < 1$, then

$$\sup_{z \in D} \| u(z) \| [\operatorname{dist}(z, \partial D)]^{\beta-1/2+\varepsilon} \leqslant C_\varepsilon \sup_{\zeta \in D} \| f(\zeta) \| [\operatorname{dist}(\zeta, \partial D)]^{\beta} . \tag{0.6}$$

Note that the radius R and the constant C_ε in Theorem 0.1 depend continuously on $\varphi_1, \dots, \varphi_N$ with respect to the C^2 topology.

Theorem 0.1 implies the following corollary for the domain Ω defined by (0.2) :

0.2. COROLLARY. — *For each point $\xi \in E$ there exists a radius $R > 0$ such that :*

(i) If $q \geqslant 1$, then each holomorphic function on Ω extends holomorphically to $\Omega \cup B_R(\xi)$;

(ii) If $q \geqslant 2$ and f is a continuous $\bar\partial$-closed (n,r)-form with $1 \leqslant r \leqslant q-1$ on Ω, then there is a continuous $(n,r-1)$-form u on $\Omega \cap B_r(\xi)$ with

$$\bar\partial u = f \quad \text{on} \quad \Omega \cap B_r(\xi) . \tag{0.7}$$

It is easy to see that, for $r = 1$, estimates (0.5) and (0.6) (with Ω instead of D) hold also in this corollary. We do not know whether this is true for $r \geqslant 2$.

For the smooth case ($N = 1$) Theorem 0.1 was obtained by Lieb [Li]. We prove Theorem 0.1 by means of integral formulas which are obtained combining the construction of Lieb [Li] with the construction of Range and Siu [R/S]. The main problem then consists in the proof of the estimates. Fortunately, in large parts, this proof is parallel to the corresponding proof in the q-convex case which is carried out in [L-T/Le]. Note that, in both proofs, an idea of Henkin plays a very important role (see the introduction to [L-T/Le]). Note also that in the survey article [He] of Henkin a global result, corresponding to the important special case $\beta = 0$, $\varepsilon = \frac{1}{2}$ of Theorem 0.1 is formulated (see [He] th. 8-12 d)).

Finally we want to compare our results with the work [G] of Grauert. He studied domains of type Ω defined by (0.2), where instead of condition *(iii)* the following stronger hypothesis is used :

(iii)' There is a fixed $(q+1)$-dimensional subspace T of $\mathbb{C}^n$ such that, for all $j = 1, \dots, N$ and $z \in G$, the Levi form φ_j is positive definite on T.

Under this hypothesis, Corollary 0.2 follows from Satz 1 in [G]. Note that the conclusion of Satz 1 in [G] is essentially stronger than the conclusion of our Corollary 0.2 : we can solve $\bar\partial u = f$ only on the smaller set $\Omega \cap B_r(\xi)$ if f is given on Ω, whereas Grauert proves the existence of a basis of Stein neighborhoods U of ξ such that, if f is given on $\Omega \cap U$, the equation $\bar\partial u = f$ can be solved on the same set $\Omega \cap U$. In the smooth case ($N = 1$) such a solution without shrinking of the domain is possible also with estimates as in Theorem 0.1 (see Theorem 14.1 in [He/Le 2]). On the other hand, it is not clear whether one can solve (even without estimates) the $\bar\partial$-equation without shrinking of the domain in the situation of Theorem 0.1 if $N \geqslant 2$. Note also that the statement of Theorem 0.1 under the stronger condition *(iii)'* and without estimates and with shrinking of the domain can be obtained also from Satz 1 in [G].

1. Preliminaries

1.1. — For $z \in \mathbb{C}^n$ we denote by $z_1, \dots, z_n$ the canonical complex coordinates of z. We write $\langle z, w\rangle = z_1 w_1 + \dots + z_n w_n$ and $|z| = \langle z, z\rangle^{1/2}$ for $z, w \in \mathbb{C}^n$.

1.2. — Let M be a closed real C^1 submanifold of a domain $\Omega \subseteq \mathbb{C}^n$, and let $\zeta \in M$. Then we denote by $T_\zeta^{\mathbb{C}}(M)$ the *complex*, and by $T_\zeta^{\mathbb{R}}(M)$ the *real* tangent space of M at ζ. We identify these spaces with subspaces of $\mathbb{C}^n$ as follows : if $\rho_1, \dots, \rho_N$ are real C^1 functions in a neighborhood U_ζ of ζ such that $M \cap U = \{\rho_1 = \dots = \rho_N = 0\}$ and

$d\rho_1(\zeta) \wedge \cdots \wedge d\rho_N(\zeta) \neq 0$, then

$$T_\zeta^{\mathbf{C}}(M) = \{\, t \in \mathbf{C}^n : \sum_{\nu=1}^{n} \frac{\partial \rho_j(\zeta)}{\partial \zeta_\nu} t_\nu = 0 \ \text{ for } \ j = 1, \dots, n \,\}$$

and

$$T_\zeta^{\mathbf{R}}(M) = \{\, t \in \mathbf{C}^n : \sum_{\nu=1}^{2n} \frac{\partial \rho_j(\zeta)}{\partial x_\nu} x_\nu(t) = 0 \ \text{ for } \ j = 1, \dots, n \,\},$$

where $x_1, \dots, x_{2n}$ are the real coordinates on $\mathbf{C}^n$ with $t_\nu = x_\nu(t) + i x_{\nu+n}(t)$ for $t \in \mathbf{C}^n$ and $\nu = 1, \dots, n$.

1.3. — Let $\Omega \subseteq \mathbf{C}^n$ be a domain and ρ a real C^2 function on Ω. Then we denote by $L_\rho(\zeta)$ the Levi form of ρ at $\zeta \in \Omega$, and by $F_\rho(\,\cdot\,, \zeta)$ the Levi polynomial of ρ at $\zeta \in \Omega$, i.e.

$$L\rho(\zeta)t = \sum_{j,k=1}^{n} \frac{\partial^2 \rho(\zeta)}{\partial \overline{\zeta}_j \partial \zeta_k} \overline{t}_j t_k$$

$\zeta \in \Omega$, $t \in \mathbf{C}^n$, and

$$F_\rho(z, \zeta) = 2 \sum_{j=1}^{n} \frac{\partial \rho(\zeta)}{\partial \zeta_j} (\zeta_j - z_j) - \sum_{j,k=1}^{n} \frac{\partial^2 \rho(\zeta)}{\partial \zeta_j \partial \zeta_k} (\zeta_j - z_j)(\zeta_k - z_k)$$

$\zeta \in \Omega$, $z \in \mathbf{C}^n$. Recall that by Taylor's theorem (see, e.g., Lemma 1.4.13 in [He/Le 1])

$$\operatorname{Re} F_\rho(z, \zeta) = \rho(\zeta) - \rho(z) + L_\rho(\zeta)(\zeta - z) + o(|\zeta - z|^2)\,. \tag{1.1}$$

1.4. — Let $J = (j_1, \dots, j_\ell)$, $1 \leqslant \ell < \infty$, be an ordered collection of elements in $\mathbf{N} \cup \{*\}$. Then we write $|J| = \ell$, $J(\hat{\nu}) = (j_1, \dots, j_{\nu-1}, j_{\nu+1}, \dots, j_\ell)$ for $\nu = 1, \dots, \ell$, and $j \in J$ if $j \in \{\, j_1, \dots, j_\ell \,\}$.

1.5. — Let $N \geqslant 1$ be an integer. Then we denote by $P(N)$ the set of all ordered collections $K = (k_1, \dots, k_\ell)$, $\ell \geqslant 1$, of integers with $1 \leqslant k_1, \dots, k_\ell \leqslant N$, and by $P(N, *)$ the set of all ordered collections $K = (k_1, \dots, k_\ell)$, $\ell \geqslant 1$ such that either $K \in P(N)$ or for a $\nu \in \{\, 1, \dots, \ell \,\}$, $k_\nu = *$ and $K(\hat{\nu}) \in P(N)$ as well as $K = (*)$. We call $P'(N)$ the subset of all $K = (k_1, \dots, k_\ell) \in P(N)$ with $k_1 < \cdots < k_\ell$ and $P'(N, *)$ the subset of all $K = (k_1, \dots, k_\ell)$ where either $K \in P'(N)$ or $1 \leqslant k_1 < \cdots < k_{\ell-1} \leqslant N$ and $k_\ell = *$, i.e. $K_{(\hat{\ell})} \in P'(N)$ and $K = K_{(\hat{\ell})}*$, as well as $K = (*)$.

1.6. — Let $J = (j_1, \dots, j_\ell)$, $1 \leqslant \ell < \infty$, be an ordered collection of integers with $0 \leqslant j_1 < \cdots < j_\ell$. Then we denote by Δ_J (or $\Delta_{j_1 \cdots j_\ell}$) the simplex of all sequences $\{\, \lambda_j \,\}_{j=0}^{\infty}$ of numbers $0 \leqslant \lambda_j \leqslant 1$ such that $\lambda_j = 0$ if $j \notin J$ and $\Sigma \lambda_j = 1$. We orient Δ_J by the form $d\lambda_{j_2} \wedge \cdots \wedge d\lambda_{j_\ell}$ if $\ell \geqslant 2$, and by +1 if $\ell = 1$.

Further Δ_{J*} (or $\Delta_{j_1 \cdots j_\ell *}$) will be the simplex of all sequences $\{\, \lambda_j \,\}_{j=0}^{\infty} \cup \{\, \lambda_* \,\}$ of numbers $0 \leqslant \lambda_j \leqslant 1$, $0 \leqslant \lambda_* \leqslant 1$ such that $\lambda_j = 0$ if $j \notin J$ and $\sum_{j=0}^{\infty} \lambda_j + \lambda_* = 1$. We orient Δ_{J*} by the form $d\lambda_{j_2} \wedge \cdots \wedge d\lambda_{j_\ell} \wedge d\lambda_*$.

We set also $\Delta_\emptyset = \emptyset$.

1.7. — We denote by $\overset{\circ}{\chi}$ a fixed C^∞ function

$$\overset{\circ}{\chi}\colon [0,1] \longrightarrow [0,1]$$

with $\overset{\circ}{\chi}(\lambda) = 0$ if $0 \leqslant \lambda \leqslant 1/4$ and $\overset{\circ}{\chi}(\lambda)=1$ if $1/2 \leqslant \lambda \leqslant 1$.

1.8. — Let $N \geqslant 1$ be an integer and $K = (k_1, \dots, k_\ell) \in P'(N,*)$. Then, for $\lambda \in \Delta_{OK}$ with $\lambda_0 \neq 1$, we denote by $\overset{\circ}{\lambda}$ the point in Δ_K defined by

$$\overset{\circ}{\lambda}_{k_\nu} = \frac{\lambda_{k_\nu}}{1-\lambda_0} \quad (\nu = 1, \dots, \ell)$$

and for $\lambda \in \Delta_{K*}$ with $\lambda_* \neq 1$, we set $\overset{*}{\lambda}$ the point in Δ_K defined by

$$\overset{*}{\lambda}_{k_\nu} = \frac{\lambda_{k_\nu}}{1-\lambda_*} \quad (\nu = 1, \dots, \ell) \ .$$

If $\lambda \in \Delta_{OK*}$ with $\lambda_0 \neq 1$ we set $\overset{\circ}{\lambda}_* = \frac{\lambda_*}{1-\lambda_0}$ and if moreover $\lambda_* \neq 1$ we define $\overset{\circ *}{\lambda} \in \Delta_K$ by

$$\overset{\circ *}{\lambda}_{k_\nu} = \frac{\overset{*}{\lambda}_{k_\nu}}{1-\lambda_0} \ .$$

1.9. — Let $D \subset\subset \mathbb{C}^n$ be a domain. D will be called a C^k *intersection*, $k = 1, 2, \dots, \infty$, if there exist a neighborhood $U_{\overline{D}}$ of $\overline{D}$ and a finite number of real C^k functions $\rho_1, \dots, \rho_N, \rho_*$ in a neighborhood of $\overline{U_{\overline{D}}}$ such that

$$D = \{\, z \in U_{\overline{D}} \colon \rho_j(z) < 0 \ \text{ for } \ j = 1, \dots, N, * \,\}$$

and

$$d\rho_{k_1}(z) \wedge \cdots \wedge d\rho_{k_\ell}(z) \neq 0$$

for all $(k_1, \dots, k_\ell) \in P'(N,*)$ and $z \in \partial D$ with $\rho_{k_1}(z) = \cdots = \rho_{k_\ell}(z) = 0$. In this case, the collection $(U_{\overline{D}}, \rho_1, \dots, \rho_N, \rho_*)$ will be called a C^k *frame* for D.

1.10. — Let $D \subset\subset \mathbb{C}^n$ be a C^1 intersection and $(U_{\overline{D}}, \rho_1, \dots, \rho_N, \rho_*)$ a frame for D. Then, for $K = (k_1, \dots, k_\ell) \in P(N,*)$, we set

$$S_K = \{\, z \in \partial D \colon \rho_{k_1}(z) = \cdots = \rho_{k_\ell}(z) = 0 \,\}$$

if $k_1, \dots, k_\ell$ are different in pairs, and

$$S_K = \emptyset$$

otherwise. We orient the manifolds S_K so that the orientation is skew symmetric in $k_1, \dots, k_\ell$, and

$$\partial D = \sum_{j=1}^{N} S_j + S_* \tag{1.2}$$

and

$$\partial S_K = \sum_{j=1}^{N} S_{Kj} + S_{K*} \tag{1.3}$$

for all $K \in P(N,*)$.

1.11. — Let f be a differential form on a domain $D \subseteq \mathbb{C}^N$. Then we denote by $\|f(z)\|$, $z \in D$, the Riemannian norm of f at z (see, e.g., Sect. 0.4 in [He/Le 2]).

1.12. — If M is an oriented real C^1 manifold and f is a differential form of maximal degree, then we denote by $|f|$ the absolute value of f (see, e.g., Sect. 0.3 in [He/Le 2]).

1.13. — Let $D \subset\subset \mathbb{C}^n$ be a domain. Then we shall use the following spaces and norms of differential forms :

$C^0_*(D)$ is the set of continuous forms on D. Set

$$\|f\|_0 = \|f\|_{O,D} = \sup_{z\in D} \|f(z)\| \tag{1.4}$$

for $f \in C^0_*(D)$.

$C^\alpha_*(\overline{D})$, $0 \leqslant \alpha \leqslant 1$, is the set of forms $f \in C^0_*(D)$ whose coefficients admit a continuous extension to $\overline{D}$ which are, if $\alpha > 0$, even Hölder continuous with exponent α on $\overline{D}$. Set

$$\|f\|_\alpha = \|f\|_{\alpha,D} = \|f\|_{O,D} + \sup_{\substack{z,\zeta\in D\\ z\neq\zeta}} \frac{\|f(z)-f(\zeta)\|}{|\zeta - z|^\alpha} \tag{1.5}$$

for $0 < \alpha \leqslant 1$ and $f \in C^\alpha_*(\overline{D})$.

$B^\beta_*(D)$, $\beta \geqslant 0$, is the set of forms $f \in C^0_*(D)$ such that, for some constant $C > 0$,

$$\|f(z)\| \leqslant C[\mathrm{dist}(z,\partial D)]^{-\beta}, \quad z \in D\ ,$$

where $\mathrm{dist}(z,\partial D)$ is the Euclidean distance between z and ∂D. Set

$$\|f\|_{-\beta} = \|f\|_{-\beta,D} = \sup_{z\in D} \|f(z)\|\, [\mathrm{dist}(z,\partial D)]^\beta \tag{1.6}$$

for $\beta \geqslant 0$ and $f \in B^\beta_*(D)$.

If $\Lambda_{p,r}(D)$ is the space of forms of bidegree (p,r) on D, then we set

$$C^0_{p,r}(D) = C^0_*(D) \cap \Lambda_{p,r}(D),$$

$$C^\alpha_{p,r}(\overline{D}) = C^\alpha_*(\overline{D}) \cap \Lambda_{p,r}(D),$$

$$B^\beta_{p,r}(D) = B^\beta_*(D) \cap \Lambda_{p,r}(D),$$

and

$$C^0_{p,*}(D) = \cup_{0\leqslant r\leqslant n} C^0_{p,r}(D),$$

$$C^\alpha_{p,*}(\overline{D}) = \cup_{0\leqslant r\leqslant n} C^\alpha_{p,r}(\overline{D}),$$

$$B^\beta_{p,*}(D) = \cup_{0\leqslant r\leqslant n} B^\beta_{p,r}(D).$$

2. Local q-concave wedges

In this section n and q are fixed integers with $0 \leqslant q \leqslant n-1$. Denote by $MO(n,q)$ the complex manifold of all complex $n \times n$-matrices which define an orthogonal projection from $\mathbb{C}^n$ onto some q-dimensional subspace of $\mathbb{C}^n$.

2.1. DEFINITION. — A collection $(U, \rho_1, \dots, \rho_N)$ will be called a *q-configuration in $\mathbb{C}^n$* if $U \subseteq \mathbb{C}^n$ is a convex domain, and $\rho_1, \dots, \rho_N$ are real C^3 functions on U satisfying the following conditions :

(i) $\{ z \in U : \rho_1(z) = \cdots = \rho_N(z) = 0 \} \neq \emptyset$;

(ii) $d\rho_1(z) \wedge \cdots \wedge d\rho_N(z) \neq 0$ for all $z \in U$;

(iii) If $\lambda \in \Delta_{1 \dots N}$ (see Sect. 1.6) and

$$\rho_\lambda := \lambda_1 \rho_1 + \cdots + \lambda_N \rho_N \ ,$$

then the Levi form $L_{\rho_\lambda}(z)$ (see Sect. 1.3) has at least $q+1$ positive eigenvalues.

2.2. DEFINITION. — A *local q-concave wedge* (E, D), $0 \leqslant q \leqslant n-1$, is a C^3 intersection D such that one can find a frame $(U_{\overline{D}}, \rho_1, \dots, \rho_N, \rho_*)$ (see Sect. 1.9) with $E = \{ z \in U_{\overline{D}} \colon \rho_1(z) = \cdots = \rho_N(z) = 0, \rho_*(z) < 0 \}$ satisfying

(i) if $K = (k_1, \dots, k_\ell) \in P'(N)$ and $U_{\overline{D}}^K = \{ z \in U_{\overline{D}} \colon \rho_{k_1}(z) = \cdots = \rho_{k_\ell}(z) \}$ then $d\rho_{k_1}(z) \wedge \cdots \wedge d\rho_{k_\ell}(z) \neq 0$ for all $z \in U_{\overline{D}}^K$;

(ii) ρ_* is convex and if $U_{\overline{D}}^{K*} = \{ z \in U_{\overline{D}} \colon \rho_{k_1}(z) = \cdots = \rho_{k_\ell}(z) = \rho_*(z) \}$ then $d\rho_{k_1}(z) \wedge \cdots \wedge d\rho_{k_\ell}(z) \wedge d\rho_*(z) \neq 0$ for all $z \in U_{\overline{D}}^{K*}$;

(iii) there exist a C^∞ map $Q \colon \Delta_{1 \dots N} \to MO(n, n-q-1)$ and constants $\alpha, A > 0$ such that

$$-\operatorname{Re} F_{\rho_\lambda}(z, \zeta) \geqslant \rho_\lambda(z) - \rho_\lambda(\zeta) + \alpha |\zeta - z|^2 - A |Q(\lambda)(\zeta - z)|^2$$

for all $\lambda \in \Delta_{1 \dots N}$ and $z, \zeta \in U_{\overline{D}}$.

2.3. LEMMA. — *Let $(U, \varphi_1, \dots, \varphi_N)$ be a q-configuration in $\mathbb{C}^n$, $0 \leqslant q \leqslant n-1$. Then for each $\xi \in U$ with $\varphi_1(\xi) = \cdots = \varphi_N(\xi) = 0$, there exists a number $R_\xi > 0$ such that for all R with $0 < R < R_\xi$, if*

$$D = \{ z \in U : \varphi_j(z) > 0, \ j = 1, \dots, N \} \cap \{ z \in \mathbb{C}^n : |z - \xi| < R \}$$

and

$$E = \{ z \in U : \varphi_1(z) = \cdots = \varphi_N(z) = 0 \} \cap \{ z \in \mathbb{C}^n : |z - \xi| < R \}$$

then (E, D) is a local q-concave wedge.

If $U_{\overline{D}} = \{ z \in \mathbb{C}^n : |z - \xi| < R_\xi \}$, $\rho_j = -\varphi_j$ for $j = 1, \dots, N$, $\rho_*(z) = |z - \xi|^2 - R^2$ then $(U_{\overline{D}}, \rho_1, \dots, \rho_N, \rho_*)$ is a frame for D.

Proof. — It is sufficient to repeat the proof of Lemma 2.4 in [L-T/Le] using $-\rho_\lambda = -(\lambda_1 \rho_1 + \cdots + \lambda_N \rho_N) = \lambda_1 \varphi_1 + \cdots + \lambda_N \varphi_N$ at the place of ρ_λ^R. ■

2.4. Definition. — We shall say that a local q-concave wedge (E, D) is defined by a q-configuration if there exists a frame $(U_{\overline{D}}, \rho_1, \ldots, \rho_N, \rho_*)$ for (E, D) such that $(U_{\overline{D}}, -\rho_1, \ldots, -\rho_N)$ is a q-configuration.

2.5. *Remark.* — It is easy to see, using Lemma 2.3 and Lemma 2.2 in [L-T/Le], that if $\xi \in \mathbb{C}^n$ is a fixed point and $\varphi_1, \ldots, \varphi_N$ are real C^3 functions in a neighborhood V of ξ such that the following conditions are fulfilled

(i) $d\varphi_1(\xi) \wedge \cdots \wedge d\varphi_N(\xi) \neq 0$;

(ii) $\varphi_1(\xi) = \cdots = \varphi_N(\xi) = 0$;

(iii) set $Y_j = \{ z \in V : \varphi_j(z) = 0 \}$ for $j = 1, \ldots, N$ and $\varphi_\lambda = \lambda_1\varphi_1 + \cdots + \lambda_N\varphi_N$ for $\lambda \in \Delta_{1\ldots N}$, then for all $K = (k_1, \ldots, k_\ell) \in P'(N)$ and $\lambda \in \Delta_K$ (see sects 1.5 and 1.6), the Levi form $L_{\rho_\lambda}(\xi)$ restricted to $T_\xi^{\mathbb{C}}(Y_{k_1} \cap \cdots \cap Y_{k_\ell})$ (see Sect. 1.2) has at least

$$\dim_{\mathbb{C}} T_\xi^{\mathbb{C}}(Y_{k_1} \cap \cdots \cap Y_{k_\ell}) - n + q + 1$$

negative eigenvalues ;

then there exists a number $R_\xi > 0$ such that, for all R with $0 < R \leqslant R_\xi$, (E, D), where $E = Y_1 \cap \cdots \cap Y_N \cap \{ z \in \mathbb{C}^n : |z - \xi| < R \}$ and $D = \{ z \in V : \varphi_j(z) < 0 \} \cap \{ z \in \mathbb{C}^n : |z - \xi| < R \}$, is a local q-concave wedge defined by a q configuration.

2.6. *Remark.* — It is clear that in the case of a local q-concave wedge defined by a q-configuration we can choose the constant α of Definition 2.2 *(iii)* such that for each $\lambda \in \Delta_{1\ldots N}$, $z \in U_{\overline{D}}$, the Levi form $L_{\tilde{\rho}_\lambda}(\zeta)$ of $\tilde{\rho}_\lambda(\zeta) = \rho_\lambda(\zeta) - \rho_\lambda(z) + \frac{\alpha}{2}|\zeta - z|^2$ has at least $(q+1)$ negative eigenvalues on $U_{\overline{D}}$.

3. A Leray map for local q-concave wedges

Let $D \subset\subset \mathbb{C}^n$ be a C^3 intersection, $(U_{\overline{D}}, \rho_1, \ldots, \rho_N, \rho_*)$ a frame for D, and let S_K be the corresponding manifolds introduced in Sect. 1.10.

3.1. Definition. — A *Leray map* for D or, more precisely, for the frame $(U_{\overline{D}}, \rho_1, \ldots, \rho_N, \rho_*)$ is a map ψ which attaches to each $K \in P'(N, *)$ a $\mathbb{C}^n$-valued map

$$\psi_K(z, \zeta, \lambda) = \big(\psi_K^1(z, \zeta, \lambda), \ldots, \psi_K^n(z, \zeta, \lambda) \big)$$

defined for $(z, \zeta, \lambda) \in D \times S_K \times \Delta_K$ such that $\langle \psi_K(z, \zeta, \lambda), \zeta - z \rangle = 1$.

Now let (E, D) be a local q-concave wedge and $(U_{\overline{D}}, \rho_1, \ldots, \rho_N, \rho_*)$ the associated frame.

Since ρ_* is a convex function, if we set

$$w^*(\zeta) := 2\Big(\frac{\partial \rho_*}{\partial \zeta_1}(\zeta), \ldots, \frac{\partial \rho_*}{\partial \zeta_n}(\zeta) \Big)$$

for $\zeta \in U_{\overline{D}}$ and

$$\psi^*(z,\zeta) = \langle w^*(\zeta), \zeta - z\rangle$$

for $(z,\zeta) \in \mathbb{C}^n \times U_{\overline{D}}$, then there exists $\varepsilon, \gamma > 0$ such that

$$\operatorname{Re} \psi^*(z,\zeta) \geqslant \rho_*(\zeta) - \rho_*(z) + \gamma|\zeta - z|^2 \tag{3.1}$$

for all $(z,\zeta) \in \mathbb{C}^n \times U_{\overline{D}}$ with $|\zeta - z| \leqslant \varepsilon$.

It follows that $\psi^*(z,\zeta) \neq 0$ for all $(z,\zeta) \in D \times S_*$.

Since $\rho_1, \dots, \rho_N$ are defined and of class C^3 in a neighborhood of $\overline{U}_{\overline{D}}$, we can find C^∞ functions a_ν^{kj} $(\nu = 1, \dots, N;\ k,j = 1, \dots, n)$ on $U_{\overline{D}}$ such that

$$\left| a_\nu^{kj}(\zeta) - \frac{\partial^2 \rho_\nu(\zeta)}{\partial \zeta_k \partial \zeta_j} \right| < \frac{\alpha}{2n^2}$$

for all $\zeta \in U_{\overline{D}}$, where α is as in Definition 2.2.

Set $\rho_\lambda = \lambda_1 \rho_1 + \dots + \lambda_N \rho_N$ and $a_\lambda^{kj} = \lambda_1 a_1^{kj} + \dots + \lambda_N a_N^{kj}$ for $\lambda \in \Delta_{1\dots N}$. Then

$$\left| \sum_{k,j=1}^{n} \left(a_\lambda^{kj}(\zeta) - \frac{\partial^2 \rho_\lambda}{\partial \zeta_k \partial \zeta_j}(\zeta) \right) t_k t_j \right| \leqslant \frac{\alpha}{2}|t|^2 \tag{3.2}$$

for all $\zeta \in U_{\overline{D}}$, $t \in \mathbb{C}^n$ and $\lambda \in \Delta_{1\dots N}$. Set

$$\widetilde{F}_{\rho_\lambda}(z,\zeta) = 2\sum_{j=1}^{n} \frac{\partial \rho_\lambda}{\partial \zeta_j}(\zeta_j - z_j) - \sum_{k,j=1}^{n} a_\lambda^{kj}(\zeta)(\zeta_k - z_k)(\zeta_j - z_j)$$

for $(z,\zeta,\lambda) \in \mathbb{C}^n \times U_{\overline{D}} \times \Delta_{1\dots N}$. Then it follows from (3.2) and condition *(iii)* in Definition 2.2 that

$$-\operatorname{Re} \widetilde{F}_{\rho_\lambda}(z,\zeta) \geqslant \rho_\lambda(z) - \rho_\lambda(\zeta) + \frac{\alpha}{2}|\zeta - z|^2 - A|Q(\lambda)(\zeta - z)|^2 \tag{3.3}$$

for all $(z,\zeta,\lambda) \in U_{\overline{D}} \times U_{\overline{D}} \times \Delta_{1\dots N}$.

Denote by $Q_{kj}(\lambda)$ the entires of the matrix $Q(\lambda)$, i.e.

$$Q(\lambda) = \big(Q_{kj}(\lambda)\big)_{k,j=1}^{n} \quad (k = \text{ column index }).$$

If $(z,\zeta,\lambda) \in \mathbb{C}^n \times U_{\overline{D}} \times \Delta_{1\dots N}$, then we set

$$\begin{cases} v^j(z,\zeta,\lambda) = 2\dfrac{\partial \rho_\lambda}{\partial \zeta_j}(\zeta) - \displaystyle\sum_{k=1}^{n} a_\lambda^{kj}(\zeta)(\zeta_k - z_k) - A\sum_{k=1}^{n} \overline{Q_{kj}(\lambda)(\zeta_k - z_k)} \\ v = (v^1, \dots, v^n) \\ \varphi = \langle v(z,\zeta,\lambda), \zeta - z\rangle \end{cases} \tag{3.4}$$

Since $Q(\lambda)$ is an orthogonal projection, we have

$$\varphi(z,\zeta,\lambda) = \widetilde{F}_{\rho_\lambda}(z,\zeta) - A|Q(\lambda)(\zeta - z)|^2 \tag{3.5}$$

for all $(z,\zeta,\lambda) \in \mathbb{C}^n \times U_{\overline{D}} \times \Delta_{1\dots N}$ and it follows from estimates (3.3) that

$$-\operatorname{Re} \varphi(z,\zeta,\lambda) \geqslant \rho_\lambda(z) - \rho_\lambda(\zeta) + \frac{\alpha}{2}|\zeta - z|^2 \tag{3.6}$$

for all $(z, \zeta, \lambda) \in U_{\overline{D}} \times U_{\overline{D}} \times \Delta_{1 \ldots N}$.

Now we set for $(z, \zeta, \lambda) \in U_{\overline{D}} \times \mathbb{C}^n \times \Delta_{1 \ldots N}$.

$$\left.\begin{aligned} w^j(z, \zeta, \lambda) &= v^j(\zeta, z, \lambda) \\ \psi(z, \zeta, \lambda) &= \varphi(\zeta, z, \lambda) \end{aligned}\right\} \tag{3.7}$$

It follows from estimate (3.6) that $\psi(z, \zeta, \lambda) \neq 0$ if $(z, \zeta, \lambda) \in D \times S_K \times \Delta_K$ for some $K \in P'(N)$.

Therefore, by setting

$$\psi_K(z, \zeta, \lambda) = \frac{w(z, \zeta, \lambda)}{\psi(z, \zeta, \lambda)} \tag{3.8}$$

for $(z, \zeta, \lambda) \in D \times S_K \times \Delta_K, K \in P'(N)$ and

$$\psi_{K*}(z, \zeta, \lambda) = \overset{\circ}{\chi}(\lambda_*) \frac{w^*(\zeta)}{\psi^*(z, \zeta)} + \left(1 - \overset{\circ}{\chi}(\lambda_*)\right) \frac{w(z, \zeta, \overset{*}{\lambda})}{\psi(z, \zeta, \overset{*}{\lambda})} \tag{3.9}$$

for $(z, \zeta, \lambda) \in D \times S_{K*} \times \Delta_{K*}$, $K \in P'(N)$, we obtain a family $\psi = \{\psi_K, \psi_{K*}\}_{K \in P'(N)}$ of $\mathbb{C}^n$-valued C^1 maps. Obviously, ψ is a Leray map for the frame $(U_{\overline{D}}, \rho_1, \ldots, \rho_N, \rho_*)$.

3.2. DEFINITION. — A map f defined on some complex manifold X will be called *k-holomorphic* if, for each point $\xi \in X$, there exist holomorphic coordinates $h_1, \ldots, h_n$ in a neighborhood of ξ such that f is holomorphic with respect to $h_1, \ldots, h_k$.

We deduce immediately from (3.4), (3.7) and Lemma 3.3 in [L-T/Le] that :

3.3. LEMMA. — *For every fixed $(z, \lambda) \in U_{\overline{D}} \times \Delta_{1 \ldots N}$ the map $w(z, \zeta, \lambda)$ and the function $\psi(z, \zeta, \lambda)$ are (q+1)-holomorphic in $\zeta \in \mathbb{C}^n$.*

4. An integral formula in local q-concave wedges

We denote by $\widehat{B}(z, \zeta)$ the Martinelli-Bochner kernel for (n, r)-forms, *i.e.*

$$\widehat{B}(z, \zeta) = \frac{1}{(2\pi i)^n} \det \left(\overbrace{\frac{\overline{\zeta} - \overline{z}}{|\zeta - z|^2}}^{1}, \overbrace{d \frac{\overline{\zeta} - \overline{z}}{|\zeta - z|^2}}^{n-1} \right) \wedge dz_1 \wedge \cdots \wedge dz_n$$

for all $z, \zeta \in \mathbb{C}^n$ with $z \neq \zeta$ (for the definition of determinants of matrices of differential forms, see, e.g., Sect. 0.7 in [He/Le 2]). If $D \subset\subset \mathbb{C}^n$ is a domain and f is a continuous differential form with integrable coefficients on D, then we set

$$B_D f(z) = \int_{\zeta \in D} f(\zeta) \wedge \widehat{B}(z, \zeta), \quad z \in D$$

(for the definition of integration with respect to a part of the variables, see, e.g., Sect. 0.2 in [He/Le 2]).

Let $D \subset\subset \mathbb{C}^n$ be a C^3 intersection, $(U_{\overline{D}}, \rho_1, \dots, \rho_N, \rho_*)$ a frame for D, and let S_K be the corresponding manifolds introduced in Sect. 1.10.

Further, let ψ be a Leray map for the frame $(U_{\overline{D}}, \rho_1, \dots, \rho_N, \rho_*)$. Then we set

$$\psi_{OK}(z,\zeta,\lambda) = \overset{\circ}{\chi}(\lambda_0)\frac{\overline{\zeta} - \overline{z}}{|\zeta - z|^2} + \left(1 - \overset{\circ}{\chi}(\lambda_0)\right)\psi_K(z,\zeta,\overset{\circ}{\lambda}) \tag{4.1}$$

for $K \in P'(N,*)$ and $(z,\zeta,\lambda) \in D \times S_K \times \Delta_{OK}$. Note that $1 - \overset{\circ}{\chi}(\lambda_0) = 0$ for λ in the neighborhood $\Delta_{OK} \setminus \overset{\circ}{\Delta}_{OK}$ of Δ_0 and therefore ψ_{OK} is of class C^2. For $K \in P'(N,*)$ we introduce the differential form

$$\widehat{R}^{\psi}_K(z,\zeta,\lambda) = \frac{(-1)^{|K|}}{(2\pi i)^n} \det\Big(\overbrace{\psi_{OK}(z,\zeta,\lambda)}^{1}, \overbrace{d\psi_{OK}(z,\zeta,\lambda)}^{n-1}\Big) \wedge dz_1 \wedge \cdots \wedge dz_n$$

defined for $(z,\zeta,\lambda) \in D \times S_K \times \Delta_{OK}$, and the differential form

$$\widehat{L}^{\psi}_K(z,\zeta,\lambda) = \frac{1}{(2\pi i)^n} \det\Big(\overbrace{\psi_K(z,\zeta,\lambda)}^{1}, \overbrace{d\psi_K(z,\zeta,\lambda)}^{n-1}\Big) \wedge dz_1 \wedge \cdots \wedge dz_n$$

defined for $(z,\zeta,\lambda) \in D \times S_K \times \Delta_K$ (here d denotes the exterior differential operator with respect to all variables z,ζ,λ). If f is a continuous differential form on $\overline{D}$, then, for all $K \in P'(N,*)$, we set

$$R^{\psi}_K f(z) = \int_{(\zeta,\lambda)\in S_K \times \Delta_{OK}} f(\zeta) \wedge \widehat{R}^{\psi}_K(z,\zeta,\lambda), \quad z \in D,$$

and

$$L^{\psi}_K f(z) = \int_{(\zeta,\lambda)\in S_K \times \Delta_K} f(\zeta) \wedge \widehat{L}^{\psi}_K(z,\zeta,\lambda), \quad z \in D.$$

Then, for each continuous (n,r)-form f on $\overline{D}$, $0 \leqslant r \leqslant n$, such that df is also continuous on $\overline{D}$, one has the representation

$$\begin{aligned}(-1)^{n+r} f = dB_D f - B_D df &+ \sum_{K\in P'(N)} \left(L^{\psi}_K f + dR^{\psi}_K f - R^{\psi}_K df\right) \\ &+ \sum_{K\in P'(N)\cup\emptyset} \left(L^{\psi}_{K*} f + dR^{\psi}_{K*} f - R^{\psi}_{K*} df\right) \quad \text{on } D.\end{aligned} \tag{4.2}$$

This formula is basic for the present paper. It has different names and a long history (see Proposition 1.3.1 in [Ai/He], Sect. 3.12 in [He/Le 2] and the notes at the end of ch. 4 in [He/Le 1], we call it *Cauchy-Fantappie formula.*

4.1. **Cauchy-Fantappie formula for a local q-concave wedge.** — Let (E,D) be a local q-concave wedge, $0 \leqslant q \leqslant n-1$, $(U_{\overline{D}}, \rho_1, \dots, \rho_N, \rho_*)$ the associated frame satisfying conditions *(i), (ii)* and *(iii)* in Definition 2.2 and ψ the Leray map constructed in Section 3 for the frame $(U_{\overline{D}}, \rho_1, \dots, \rho_N, \rho_*)$.

We set

$$T^\psi = B_D + \sum_{K\in P'(N)} R_K^\psi + \sum_{K\in P'(N)\cup\emptyset} R_{K*}^\psi$$

and

$$L^\psi = \sum_{K\in P'(N)} L_K^\psi + \sum_{K\in P'(N)\cup\emptyset} L_{K*}^\psi$$

$$L_*^\psi = \sum_{K\in P'(N)\cup\emptyset} L_{K*}^\psi .$$

With this notation, for each continuous (n,r)-form f on $\overline{D}$, $0 \leqslant r \leqslant n$, such that df is also continuous on $\overline{D}$, (4.2) can be written

$$(-1)^{n+r} f = dT^\psi f - T^\psi df + L^\psi f \quad \text{on } D . \tag{4.3}$$

4.1.1. Theorem. — *If $0 \leqslant r \leqslant q-N$, for each continuous (n,r)-form f on $\overline{D}$ such that df is also continuous on $\overline{D}$*

$$(-1)^{n+r} f = dT^\psi f - T^\psi df + L_*^\psi f \quad \text{on } D .$$

Proof. — In view of the Cauchy-Fantappie formula (4.3) it is sufficient to prove that for $0 \leqslant r \leqslant q-N$, $K \in P'(N)$, $L_K^\psi f = 0$.

Let us denote by $[\widehat{L}_K^\psi]_{\deg \overline{\zeta}=k}$ the part of the form $\widehat{L}_K^\psi$ which is of type $(0,k)$ in ζ. Then, by Lemma 3.3, $[\widehat{L}_K^\psi]_{\deg \overline{\zeta}=k} = 0$ for $K \in P'(N)$ and $k \geqslant n-q$.

Since f is of type (n,r), $\dim \Delta_K = |K|-1$, $\dim S_K = 2n-|K|$ and $|K| \leqslant N$ we obtain, by definition of $L_K^\psi f$, that $L_K^\psi f = 0$ for $0 \leqslant r \leqslant q-N$ and $K \in P'(N)$. ∎

4.1.2. *Remark.* — In fact we can prove that, for $K \in P'(N)$, $L_K^\psi f = 0$ if $r \leqslant q-|K|$.

4.2. **The manifolds Γ_K.** — As we want to obtain an integral formula for forms which are not necessarily defined on ∂D, we are going to replace the integrals over the manifolds S_K in (4.2) by integrals over certain submanifolds Γ_K of D.

For $K = (k_1, \ldots, k_\ell) \in P(N,*)$ we set

$$U_{\overline{D}}^K = \{ \zeta \in U_{\overline{D}} : \rho_{k_1}(\zeta) = \cdots = \rho_{k_\ell}(\zeta) \}$$

if $k_1, \ldots, k_\ell$ are different in pairs, and $U_{\overline{D}}^K = \emptyset$ otherwise. By conditions *(i)* and *(ii)* in Definition 2.2 each $U_{\overline{D}}^K$ is a closed C^3 submanifold of $U_{\overline{D}}$. We denote by ρ_K, $K \in P(N,*)$, the function on $U_{\overline{D}}^K$ which is defined by

$$\rho_K(\zeta) = \rho_{k_\nu}(\zeta) \quad (\zeta \in U_{\overline{D}}^K; \nu = 1, \ldots, \ell) .$$

Now, for all $K \in P(N,*)$, we define

$$\Gamma_K = \{ \zeta \in U_{\overline{D}}^K : \rho_j(\zeta) \leqslant \rho_K(\zeta) \leqslant 0 \ \text{ for } \ j = 1, \ldots, N, * \} .$$

Then it is easy to see that all Γ_K are C^3 submanifolds of $\overline{D}$ with piecewise C^3 boundary, and that

$$\overline{D} = \Gamma_1 \cup \cdots \cup \Gamma_N \cup \Gamma_*$$

and

$$\partial\Gamma_K = S_K \cup \Gamma_{K1} \cup \cdots \cup \Gamma_{KN} \cup \Gamma_{K*}, \quad K \in P(N) .$$

We choose the orientation on Γ_K such that the orientation is skew symmetric in the components of K, and the following conditions hold :

$$\left.\begin{array}{l} \Gamma_1, \ldots, \Gamma_N, \Gamma_* \text{ carry the orientation of } \mathbb{C}^n, \text{ and if} \\ K \in P(N,*) \text{ and } 1 \leqslant j \leqslant N \text{ with } * \notin K, \text{ resp. } j \notin K, \text{ then} \\ \Gamma_{K*}, \text{ resp. } \Gamma_{Kj} \text{ are oriented just as } -\partial\Gamma_K \end{array}\right\}$$

As in [L-T/Le], we obtain the following lemmas :

4.2.1. LEMMA. — *If Γ_K are the above manifolds, then*

$$\partial\Gamma_K = S_K - \sum_{j=1}^{N} \Gamma_{Kj} - \Gamma_{K*}$$

for all $K \in P(N,*)$.

4.2.2. LEMMA. — *If Γ_K are the above manifolds and Δ_K, Δ_{OK} are oriented simplices introduced in Sect. 1.6, then*

$$\sum_{K \in P'(N,*)} (-1)^{|K|} \partial(\Gamma_K \times \Delta_{OK}) =$$

$$\overline{D} \times \Delta_O + \sum_{K \in P'(N,*)} (-1)^{|K|} S_K \times \Delta_{OK} - \sum_{K \in P'(N,*)} \Gamma_K \times \Delta_K . \qquad (4.4)$$

$$\sum_{K \in P'(N,*)} \partial(\Gamma_K \times \Delta_K) = \sum_{K \in P'(N,*)} S_K \times \Delta_K \qquad (4.5)$$

and

$$\sum_{K \in P'(N) \cup \emptyset} \partial(\Gamma_{K*} \times \Delta_{K*}) = \sum_{K \in P'(N) \cup \emptyset} S_{K*} \times \Delta_{K*} + \sum_{K \in P'(N)} \Gamma_{K*} \times \Delta_K . \qquad (4.6)$$

4.3. **The operators** L **and** M. — Let $w^*(z,\zeta)$, $\psi^*(z,\zeta)$, $w(z,\zeta,\lambda)$ and $\psi(z,\zeta,\lambda)$ be the maps defined in paragraph 3. We set

$$\Phi^*(z,\zeta) = \psi^*(z,\zeta) - 2\rho_*(\zeta) \quad \text{for } (z,\zeta) \in \mathbb{C}^n \times U_{\overline{D}}$$

and

$$\Phi(z,\zeta,\lambda) = \psi(z,\zeta,\lambda) + 2\rho_\lambda(\zeta) \quad \text{for } (z,\zeta,\lambda) \in \mathbb{C}^n \times U_{\overline{D}} \times \Delta_{1\cdots N}.$$

Then it follows from (3.1), (3.6) and (3.7) that $\Phi^*(z,\zeta) \neq 0$ for $(z,\zeta) \in D \times \overline{D}$ and $\Phi(z,\zeta,\lambda) \neq 0$ for $(z,\zeta,\lambda) \in D \times \overline{D} \times \Delta_{1\cdots N}$.

So we can define the C^2 maps

$$\tilde{\psi}_K(z,\zeta,\lambda) = \overset{\circ}{\chi}(\lambda_*) \frac{w^*(\zeta)}{\Phi^*(z,\zeta)} + \left(1 - \overset{\circ}{\chi}(\lambda_*)\right) \frac{w(z,\zeta,\overset{*}{\lambda})}{\Phi(z,\zeta,\overset{*}{\lambda})}$$

for all $(z,\zeta,\lambda) \in D \times \overline{D} \times \Delta_K, K \in P'(N,*)$. Notice that $\tilde{\psi}_K(z,\zeta,\lambda) = \psi_K(z,\zeta,\lambda)$ when $(z,\zeta,\lambda) \in D \times S_K \times \Delta_K$.

We set for $(z,\zeta,\lambda) \in D \times \overline{D} \times \Delta_K$

$$\widehat{L}_K^{\tilde{\psi}}(z,\zeta,\lambda) = \frac{1}{(2i\pi)^n} \det \Big(\overbrace{\tilde{\psi}_K(z,\zeta,\lambda)}^{1}, \overbrace{d\tilde{\psi}_K(z,\zeta,\lambda)}^{n-1} \Big) \wedge dz_1 \wedge \cdots \wedge dz_n$$

and one has $\widehat{L}_K^{\tilde{\psi}} = \widehat{L}_K^{\psi}$ on $D \times S_K \times \Delta_K$.

We set also for $(z,\zeta,\lambda) \in D \times \overline{D} \times \Delta_K$

$$\widehat{M}_K^{\tilde{\psi}}(z,\zeta,\lambda) = \frac{1}{(2i\pi)^n} \det \Big(\overbrace{d\tilde{\psi}_K(z,\zeta,\lambda)}^{n} \Big) \wedge dz_1 \wedge \cdots \wedge dz_n \ .$$

4.3.1. *Remark.* — It comes from the properties of determinants that if $K \in P'(N)$

$$\widehat{L}_K^{\tilde{\psi}}(z,\zeta,\lambda) = \frac{1}{(2i\pi)^n \Phi^n(z,\zeta,\lambda)} \det \Big(\overbrace{w(z,\zeta,\lambda)}^{1}, \overbrace{dw(z,\zeta,\lambda)}^{n-1} \Big)$$

for $(z,\zeta,\lambda) \in D \times \overline{D} \times \Delta_K$, where $w(z,\zeta,\lambda)$ is $(q+1)$-holomorphic in ζ.

Now let us define the operators L, L^*, M and M^* on $C^0_{n,r}(D), 0 \leqslant r \leqslant n$, by

$$Lf(z) = \sum_{K \in P'(N,*)} \int_{\zeta \in \Gamma_K \times \Delta_K} f(\zeta) \wedge \widehat{L}_K^{\tilde{\psi}}(z,\zeta,\lambda), \quad z \in D$$

$$L^*f(z) = \sum_{K \in P'(N) \cup \emptyset} \int_{\zeta \in \Gamma_{K*} \times \Delta_{K*}} f(\zeta) \wedge \widehat{L}_{K*}^{\tilde{\psi}}(z,\zeta,\lambda), \quad z \in D$$

$$Mf(z) = \sum_{K \in P'(N,*)} \int_{\zeta \in \Gamma_K \times \Delta_K} f(\zeta) \wedge \widehat{M}_K^{\tilde{\psi}}(z,\zeta,\lambda), \quad z \in D$$

$$M^*f(z) = \sum_{K \in P'(N) \cup \emptyset} \int_{\zeta \in \Gamma_{K*} \times \Delta_{K*}} f(\zeta) \wedge \widehat{M}_{K*}^{\tilde{\psi}}(z,\zeta,\lambda), \quad z \in D$$

for $f \in C^0_{n,r}(D)$.

For $f \in C^0_{n,r}(D)$, the forms Lf, L^*f, Mf and M^*f are continuous on D.

4.3.2. LEMMA. — *Let f be a continuous (n,r)-form on $\overline{D}$. If we set*

$$\Lambda f(z) = \sum_{K \in P'(N) \cup \emptyset} \int_{\Gamma_{K*} \times \Delta_K} f(\zeta) \wedge \widehat{L}_K^{\tilde{\psi}}(z,\zeta,\lambda), z \in D \ ,$$

then $\Lambda f \equiv 0$ when $0 \leqslant r \leqslant q-N$.

Proof. — By remark 4.3.1, $[\widehat{L}_K^{\tilde{\psi}}]_{\deg \overline{\zeta}=k} = 0$ for $K \in P'(N)$ and $k \geqslant n-q$. Using that $\dim \Gamma_{K*} = 2n - |K|$ and $|K| \leqslant N$, the result follows easily from the definition of Λ. ■

4.3.3. PROPOSITION. — *Let f be a continuous (n,r)-form on $\overline{D}$ such that df is also continuous on $\overline{D}$, then*

$$L^{\psi} f = \sum_{K \in P'(N,*)} L_K^{\psi} f = Ldf - dLf + (-1)^{r+n} Mf$$

and, if $0 \leqslant r \leqslant q-N$

$$L_*^{\psi} f = \sum_{K \in P'(N) \cup \emptyset} L_{K*}^{\psi} f = L^* df - dL^* f + (-1)^{r+n} M^* f .$$

Proof. — As $\widehat{L}_K^{\tilde{\psi}} = \widehat{L}_K^{\psi}$ on $D \times S_K \times \Delta_K$, we have for $z \in D$

$$\sum_{K \in P'(N,*)} L_K^{\psi} f(z) = \sum_{K \in P'(N,*)} \int_{\zeta \in S_K \times \Delta_K} f(\zeta) \wedge \widehat{L}_K^{\tilde{\psi}}(z,\zeta,\lambda) .$$

Then using (4.5) in Lemma 4.2.2, we get

$$\begin{aligned}\sum_{K \in P'(N,*)} L_K^{\psi} f(z) &= \sum_{K \in P'(N,*)} \int_{(\zeta,\lambda) \in \partial(\Gamma_K \times \Delta_K)} f(\zeta) \wedge \widehat{L}_K^{\tilde{\psi}}(z,\zeta,\lambda) \\ &= \sum_{K \in P'(N,*)} \Big[\int_{(\zeta,\lambda) \in \Gamma_K \times \Delta_K} df(\zeta) \wedge \widehat{L}_K^{\tilde{\psi}}(z,\zeta,\lambda) \\ &\quad + (-1)^{n+r} \int_{(\zeta,\lambda) \in \Gamma_K \times \Delta_K} f(\zeta) \wedge d_{\zeta,\lambda} \widehat{L}_K^{\tilde{\psi}}(z,\zeta,\lambda) \Big]\end{aligned}$$

by Stokes'theorem.

As $d_{\zeta,\lambda} \widehat{L}_K^{\tilde{\psi}}(z,\zeta,\lambda) = -d_z \widehat{L}_K^{\tilde{\psi}}(z,\zeta,\lambda) + \widehat{M}_K^{\tilde{\psi}}(z,\zeta,\lambda)$, then we get

$$\sum_{K \in P'(N,*)} L_K^{\psi} f(z) = Ldf - dLf + (-1)^{r+n} Mf .$$

In the same way, using (4.6) in Lemma 4.2.2 and Lemma 4.3.2, we obtain the second relation in Proposition 4.3.3. ■

4.4. **The operator** H. — Using Φ^* and Φ (see Sect. 4.3), we can define the C^1 map

$$\eta(z,\zeta,\lambda) = \overset{\circ}{\chi}(\lambda_0) \frac{\overline{\zeta} - \overline{z}}{|\zeta - z|^2} + \big(1 - \overset{\circ}{\chi}(\lambda_0)\big) \Big[\overset{\circ}{\chi}(\overset{\circ}{\lambda}_*) \frac{w^*(\zeta)}{\Phi^*(z,\zeta)} + \big(1 - \overset{\circ}{\chi}(\overset{\circ}{\lambda}_*)\big) \frac{w(z,\zeta,\overset{\circ *}{\lambda})}{\Phi(z,\zeta,\overset{\circ *}{\lambda})} \Big]$$

for all $(z,\zeta,\lambda) \in D \times \overline{D} \times \Delta_{01 \cdots N*}$, with $z \neq \zeta$ (for the definitions of $\overset{\circ}{\chi}, \overset{\circ}{\lambda}_*$ and $\overset{\circ *}{\lambda}$ see Sect. 1.7 and 1.8). Note that

$$\eta(z,\zeta,\lambda) = \frac{\overline{\zeta} - \overline{z}}{|\zeta - z|^2} \quad \text{if} \quad 1/2 \leqslant \lambda_0 \leqslant 1 \tag{4.7}$$

$$\eta(z,\zeta,\lambda) = \overset{\circ}{\chi}(\overset{\circ}{\lambda}_*) \frac{w^*(\zeta)}{\Phi^*(z,\zeta)} + \big(1 - \overset{\circ}{\chi}(\overset{\circ}{\lambda}_*)\big) \frac{w(z,\zeta,\overset{\circ *}{\lambda})}{\Phi(z,\zeta,\overset{\circ *}{\lambda})} \quad \text{if} \quad 0 \leqslant \lambda_0 \leqslant 1/4$$

$$\eta(z,\zeta,\lambda) = \overset{\circ}{\chi}(\lambda_*) \frac{w^*(\zeta)}{\Phi^*(z,\zeta)} + \big(1 - \overset{\circ}{\chi}(\lambda_*)\big) \frac{w(z,\zeta,\overset{*}{\lambda})}{\Phi(z,\zeta,\overset{*}{\lambda})} \quad \text{if} \quad \lambda_0 = 0 .$$

In particular, for all $K \in P'(N,*)$ we have the relations

$$\eta(z,\zeta,\lambda) = \psi_{OK}(z,\zeta,\lambda) \quad \text{if} \quad (\zeta,\lambda) \in S_K \times \Delta_{OK} \tag{4.8}$$

(see (4.1) for the definition of ψ_{OK}) and

$$\eta(z,\zeta,\lambda) = \tilde{\psi}_K(z,\zeta,\lambda) \quad \text{if} \quad (\zeta,\lambda) \in \Gamma_K \times \Delta_K \ . \tag{4.9}$$

Now for $(z,\zeta,\lambda) \in D \times \overline{D} \times \Delta_{01\cdots N*}$ with $z \neq \zeta$ we introduce the continuous differential forms

$$\widehat{G}(z,\zeta,\lambda) = \frac{1}{(2i\pi)^n} \det \Big(\overbrace{\eta(z,\zeta,\lambda)}^{1}, \overbrace{d\eta(z,\zeta,\lambda)}^{n-1} \Big) \wedge dz_1 \wedge \cdots \wedge dz_n$$

and

$$\widehat{H}(z,\zeta,\lambda) = \frac{1}{(2i\pi)^n} \det \Big(\overbrace{d\eta(z,\zeta,\lambda)}^{n} \Big) \wedge dz_1 \wedge \cdots \wedge dz_n$$

where d is the exterior differential with respect to all variables z, ζ, λ.

Then it is easy to see that

$$d\widehat{G} = \widehat{H} \tag{4.10}$$

It follows from the definitions of the kernels $\widehat{B}, \widehat{R}_K^{\psi}, \widehat{L}_K^{\tilde{\psi}}$ and from the relations (4.7), (4.8) and (4.9) that

$$\widehat{G}\,\big|_{D\times\overline{D}\times\Delta_0} = \widehat{B} \tag{4.11}$$

$$\widehat{G}\,\big|_{D\times S_K\times\Delta_{0K}} = (-1)^{|K|}\widehat{R}_K^{\psi} \quad \text{for all} \quad K \in P'(N,*) \tag{4.12}$$

$$\widehat{G}\,\big|_{D\times\Gamma_K\times\Delta_K} = \widehat{L}_K^{\tilde{\psi}} \quad \text{for all} \quad K \in P'(N,*)\ . \tag{4.13}$$

Like in [L-T/Le] we can describe the singularity of $\widehat{G}$ and $\widehat{H}$ at $z = \zeta$.

4.4.1. LEMMA. — *Denote by $[\widehat{G}(z,\zeta,\lambda)]_{\deg\lambda=k}$ and $[\widehat{H}(z,\zeta,\lambda)]_{\deg\lambda=k}$ the parts of the forms $\widehat{G}(z,\zeta,\lambda)$ and $\widehat{H}(z,\zeta,\lambda)$, respectively, which are of degree k in λ. Then the following statements hold :*

(i) The singularity at $z = \zeta$ of the form $[\widehat{G}(z,\zeta,\lambda)]_{\deg\lambda=k}$ is of order $\leqslant 2n-2k-1$;

(ii) The singularities at $z = \zeta$ of the first-order derivatives with respect to z of the coefficients of $[\widehat{G}(z,\zeta,\lambda)]_{\deg\lambda=k}$ are of order $\leqslant 2n-2k$;

(iii) The singularity at $z = \zeta$ of the form $[\widehat{H}(z,\zeta,\lambda)]_{\deg\lambda=k}$ is of order $\leqslant 2n-2k+1$.

As (E,D) is a local q-concave wedge, the map w is $(q+1)$-holomorphic in ζ (Lemma 3.3) and therefore

4.4.2. LEMMA. — *If $f \in C^0_{n,r}(\overline{D})$ with $r \leqslant q-N+1$, then*

$$\int_{(\zeta,\lambda)\in\Gamma_K\times\Delta_K} f(\zeta) \wedge \widehat{G}(z,\zeta,\lambda) = 0$$

for all $K \in P'(N)$ and $z \in D$.

Proof. — Let us remark that for $K \in P'(N)$

$$\widehat{G}\,|_{D\times\Gamma_K\times\Delta_K} = \frac{1}{(2i\pi)^n}\frac{1}{\Phi^n}\det\Big(w(z,\zeta,\lambda),\overbrace{dw(z,\zeta,\lambda)}^{n-1}\Big)\wedge dz_1\wedge\cdots\wedge dz_n$$

where w is $(q+1)$-holomorphic in ζ. Therefore $[\widehat{G}(z,\zeta,\lambda)]_{\deg\overline{\zeta}=k} = 0$ for $K \in P'(N)$, $(z,\zeta,\lambda) \in D \times \Gamma_K \times \Delta_K$, $k \geqslant n-q$.

Since f is of type (n,r), $\dim \Delta_K = |K|-1$, $\dim \Gamma_K = 2n-|K|+1$ and $|K| \leqslant N$, we get

$$\int_{(\zeta,\lambda)\in\Gamma_K\times\Delta_K} f(\zeta)\wedge\widehat{G}(z,\zeta,\lambda) = 0$$

when $r \leqslant q-N+1$ and $K \in P'(N)$. ∎

Let $f \in B^{\beta}_{n,*}(D), 0 \leqslant \beta < 1$ (see Sect. 1.13). Then, for all $K \in P'(N,*)$, we define

$$H_K f(z) = \int_{(\zeta,\lambda)\in\Gamma_K\times\Delta_{OK}} f(\zeta)\wedge\widehat{H}(z,\zeta,\lambda), \quad z\in D\,. \tag{4.14}$$

It follows from Lemma 4.4.1 *(iii)* that these integrals converge and the so defined differential forms $H_K f$ are continuous on D. We set

$$Hf = \sum_{K\in P'(N,*)} (-1)^{|K|} H_K f$$

for $f \in B^{\beta}_{n,*}(D)$, $0 \leqslant \beta < 1$.

Now let $f \in B^{\beta}_{n,r}(D)$, $0 \leqslant \beta < 1$, $0 \leqslant r \leqslant n$. Since $\widehat{H}(z,\zeta,\lambda)$ is of degree $2n$ and contain the factor $dz_1\wedge\cdots\wedge dz_n$ and since $\dim_{\mathbb{R}} \Gamma_K \times \Delta_{OK} = 2n+1$, then only such monomials of $\widehat{H}(z,\zeta,\lambda)$ contribute to the integral in (4.14) which are of degree $(n+1-r)$ in (ζ,λ) and hence of bidegree $(n,r-1)$ in z. This implies that $H_K f = 0$ if $r = 0$ or $n+1-r < |K| = \dim_{\mathbb{R}} \Delta_{OK}$.

Hence, for $f \in B^{\beta}_{n,r}(D)$, $0 \leqslant \beta < 1$, $0 \leqslant r \leqslant n$, we have

$$\left.\begin{aligned} &Hf = \sum_{\substack{K\in P'(N,*)\\ |K|\leqslant n+1-r}} (-1)^{|K|} H_K f,\\ &Hf = 0 \text{ if } r = 0, \text{ and } Hf \in C^0_{n,r-1}(D) \text{ if } 1 \leqslant r \leqslant n. \end{aligned}\right\} \tag{4.15}$$

4.4.3. THEOREM. — *Let (E,D) be a local q-concave wedge, $0 \leqslant q \leqslant n-1$ and $f \in B^{\beta}_{n,r}(D)$ an (n,r)-form, $0 \leqslant r \leqslant n$, $0 \leqslant \beta < 1$ such that $df \in B^{\beta}_{*}(D)$. Then*

$$f = dHf + Hdf + Mf \quad on\ D\,.$$

Let $(U_{\overline{D}},\rho_1,\dots,\rho_N,\rho_*)$ the frame associated to (E,D) in Definition 2.2, then, if $0 \leqslant r \leqslant q-N$,

$$f = dHf + Hdf + M^*f \quad on\ D\,.$$

In particular, if $r = 0, f = Hdf + M^*f$ on D.

Proof. — The proof of this theorem is analogous to that of Theorem 4.11 in [L-T/Le]. For the convenience of the lecturer we will repeat it here

First consider a form $g \in C^0_{n,j}(\overline{D})$. Then by (4.10)

$$d_{\zeta,\lambda}(g \wedge \widehat{G}) = dg \wedge \widehat{G} - d_z(g \wedge \widehat{G}) + (-1)^{n+j} g \wedge \widehat{H}$$

and it follows from Stokes'formula (which can be applied in view of Lemma 4.4.1) that

$$\int_{\partial(\Gamma_K \times \Delta_{OK})} g \wedge \widehat{G} = \int_{\Gamma_K \times \Delta_{OK}} dg \wedge \widehat{G} + d \int_{\Gamma_K \times \Delta_{OK}} g \wedge \widehat{G} + (-1)^{n+j} H_K g$$

for all $K \in P'(N,*)$. In view of (4.4) this implies that

$$\int_{D \times \Delta_0} g \wedge \widehat{G} + \sum_{K \in P'(N,*)} (-1)^{|K|} \int_{S_K \times \Delta_{OK}} g \wedge \widehat{G} - \sum_{K \in P'(N,*)} \int_{\Gamma_K \times \Delta_K} g \wedge \widehat{G}$$

$$= \sum_{K \in P'(N,*)} (-1)^{|K|} \left(\int_{\Gamma_K \times \Delta_{OK}} dg \wedge \widehat{G} + d \int_{\Gamma_K \times \Delta_{OK}} g \wedge \widehat{G} + (-1)^{n+j} H_K g \right).$$

Taking into account (4.11) and (4.12) as well as the definitions of T^ψ and H, this can be written

$$T^\psi g - \sum_{K \in P'(N,*)} \int_{\Gamma_K \times \Delta_K} g \wedge \widehat{G}$$

$$= \sum_{K \in P'(N,*)} (-1)^{|K|} \left(\int_{\Gamma_K \times \Delta_{OK}} dg \wedge \widehat{G} + d \int_{\Gamma_K \times \Delta_{OK}} g \wedge \widehat{G} \right) + (-1)^{j+n} H g. \quad (4.16)$$

Now we consider a form $f \in C^0_{n,r}(\overline{D})$ with $0 \leqslant r \leqslant n$ such that df is also continuous on $\overline{D}$. Setting $g = df$ in (4.16), we obtain that

$$T^\psi df = \sum_{K \in P'(N,*)} (-1)^{|K|} d \int_{\Gamma_K \times \Delta_{OK}} df \wedge \widehat{G} + (-1)^{r+1+n} H df + \sum_{K \in P'(N,*)} \int_{\Gamma_K \times \Delta_K} df \wedge \widehat{G}.$$

Setting $g = f$ in (4.16), applying d to the resulting relation, we obtain that

$$dT^\psi f = \sum_{K \in P'(N,*)} (-1)^{|K|} d \int_{\Gamma_K \times \Delta_{OK}} df \wedge \widehat{G} + (-1)^{r+n} dH f + \sum_{K \in P'(N,*)} d\Big(\int_{\Gamma_K \times \Delta_K} f \wedge \widehat{G} \Big).$$

Using (4.13) and Proposition 4.3.3, these two relations imply that

$$dT^\psi f - T^\psi df + L^\psi f = (-1)^{r+n}(dHf + Hdf + Mf)$$

and hence by (4.3)

$$f = dHf + Hdf + Mf. \quad (4.17)$$

If moreover $0 \leqslant r \leqslant q - N$, then by Lemma 4.4.2, we obtain

$$dT^\psi f - T^\psi df = (-1)^{r+n}(dHf + Hdf) + \sum_{K \in P'(N)} \left[d\Big(\int_{\Gamma_{K*} \times \Delta_{K*}} f \wedge \widehat{G} \Big) - \int_{\Gamma_{K*} \times \Delta_{K*}} df \wedge \widehat{G} \right].$$

It follows from Theorem 4.1.1, Proposition 4.3.3 and (4.13) that

$$f = dHf + Hdf + M^* f. \quad (4.18)$$

Now we consider the general case. Let $f \in B^{\beta}_{n,r}(D)$, $0 \leqslant \beta < 1$, $0 \leqslant r \leqslant n$, such that also $df \in B^{\beta}_{*}(D)$. Choose $\varepsilon > 0$ with $\beta + \varepsilon < 1$. Then, by local shifts of f and a partition of unity argument, we can find a sequence of forms $f_\nu \in C^0_{n,r}(\overline{D})$ such that also the forms df_ν are continuous on $\overline{D}$ and

$$f_\nu \longrightarrow f \text{ and } df_\nu \longrightarrow df$$

in the space $B^{\beta+\varepsilon}_{*}(D)$. By Lemma 4.4.1 *(iii)*, then

$$Hf_\nu \longrightarrow Hf \text{ and } Hdf_\nu \longrightarrow Hdf$$

uniformly on the compact subsets of D. Moreover the kernels $\widehat{M}^{\tilde{\psi}}_K$ are of class C^1 in $D \times \overline{D} \times \Delta_K$ and therefore

$$Mf_\nu \longrightarrow Mf \text{ and } M^*f_\nu \longrightarrow M^*f$$

uniformly on the compact subsets of D. Since, by (4.17) and (4.18),

$$f_\nu = dHf_\nu + Hdf_\nu + Mf_\nu$$

and

$$f_\nu = dHf_\nu + Hdf_\nu + M^*f_\nu, \text{ if } 0 \leqslant r \leqslant q - N,$$

this implies that

$$f = dHf + Hdf + Mf$$
$$f = dHf + Hdf + M^*f, \text{ if } 0 \leqslant r \leqslant q - N. \blacksquare$$

5. Homotopy formula and solution of the $\overline{\partial}$-equation

in local q-concave wedges

Let (E, D) be a local q-concave wedge, $0 \leqslant q \leqslant n-1$, $(U_{\overline{D}}, \rho_1, \dots, \rho_N, \rho_*)$ the associated frame satisfying conditions *(i)*, *(ii)* and *(iii)* in Definition 2.2.

5.1. LEMMA. — *Let ξ be a fixed point in E, then there exists a neighborhood W of ξ in $\mathbb{C}^n$ such that for each $f \in B^{\beta}_{n,r}(D)$, $0 \leqslant \beta < 1$, $0 \leqslant r \leqslant n$, the differential form $M^*f = \sum_{K \in P'(N)} \int_{\Gamma_{K*} \times \Delta_{K*}} f(\zeta) \wedge \widehat{M}_{K*}(\cdot, \zeta, \lambda)$ is of class C^1 in W and $D \subset W$. Moreover M^* is a bounded operator from $B^{\beta}_{n,*}(D)$ into $C^1_{n,*}(W)$.*

Proof. — Recall that $\widehat{M}_{K*}(z, \zeta, \lambda) = \frac{1}{(2i\pi)^n} \det\big(d\tilde{\psi}_{K*}(z, \zeta, \lambda)\big)$ where

$$\tilde{\psi}_{K*}(z, \zeta, \lambda) = \overset{\circ}{\chi}(\lambda_*) \frac{w^*(\zeta)}{\Phi^*(z, \zeta)} + \big(1 - \overset{\circ}{\chi}(\lambda_*)\big) \frac{w(z, \zeta, \overset{*}{\lambda})}{\Phi(z, \zeta, \overset{*}{\lambda})},$$

for $(z, \zeta, \lambda) \in D \times \overline{D} \times \Delta_{K*}$.

Moreover, we know from (3.1) and the definition of Φ^* in Section 4.3 that

$$\Phi^*(z, \zeta) \neq 0 \text{ for all } (z, \zeta) \in \{x \in U_{\overline{D}} / \rho_*(x) < 0\} \times \{y \in U_{\overline{D}} / \rho_*(y) \leqslant 0\}. \quad (5.1)$$

From (3.6), (3.7) and the definition of Φ in Section 4.3 we get

$$\operatorname{Re}\Phi(z,\zeta,\lambda) \leqslant \rho_\lambda(z)+\rho_\lambda(\zeta)-\frac{\alpha}{2}|\zeta-z|^2 \text{ for all } (z,\zeta,\lambda)\in U_{\overline{D}}\times U_{\overline{D}}\times\Delta_K\ . \tag{5.2}$$

Set $\delta=\operatorname{dist}(\xi,\Gamma_{1\cdots N*})$, if $z\in B(\xi,\tau\delta)$, $\tau<1$, and $\zeta\in\Gamma_{K*}$, then $|z-\zeta|>(1-\tau)\delta$.

Let $W_{\tau,\lambda}^{\ *}=\{z\in B(\xi,\tau\delta)\mid \rho_\lambda^*(z)<\frac{\delta\alpha(1-\tau)}{2}\}$, then $W_\tau=\bigcap_{\lambda\in\Delta_{K*}} W_{\tau,\lambda}^{\ *}$ is a neighborhood of ξ, which contains $D\cap B(\xi,\tau\delta)$.

We set $W=\left[(\bigcup_{\tau<1}W_\tau)\cup D\right]\cap\{z\in U_{\overline{D}}\mid\rho_*(x)<0\}$, W is a neighborhood of ξ in $\mathbb{C}^n$, which contains D. We deduce from (5.1) and (5.2) that $\Phi^*(z,\zeta)\neq 0$ and $\Phi(z,\zeta)\neq 0$ for $(z,\zeta,\lambda)\in W\times\Gamma_{K*}\times\Delta_{K*}$.

Consequently $\widehat{M}_{K*}$ is a C^1 differential form on $W\times\Gamma_{K*}\times\Delta_{K*}$, which defines a bounded operator M^* from $B^\beta_{n,*}(D)$ into $C^1_{n,*}(W)$. ■

5.2. Lemma. — *Let $f\in B^\beta_{n,r}(D)$ a (n,r)-differential form, $0\leqslant\beta<1$, such that $df\in B^\beta_*(D)$. Then if $0\leqslant r\leqslant q-N-1$, $dM^*f=M^*df$ on W.*

Proof. — We consider first the case, where $f\in C^0_{n,r}(\overline{D})$ and df is also continuous on $\overline{D}$. If $z\in W$

$$dM^*f(z)=(-1)^{r+1}\sum_{K\in P'(N)\cup\emptyset}\int_{(\zeta,\lambda)\in\Gamma_{K*}\times\Delta_{K*}} f(\zeta)\wedge d_{\zeta,\lambda}\widehat{M}_{K*}(z,\zeta,\lambda)$$

since $d\widehat{M}_{K*}=0$ by definition of $\widehat{M}_{K*}$.

Therefore, using Stokes'theorem and (4.6) we get

$$\begin{aligned} dM^*f(z)=M^*df(z)&-\sum_{K\in P'(N)\cup\emptyset}\int_{(\zeta,\lambda)\in S_{K*}\times\Delta_{K*}} f(\zeta)\wedge\widehat{M}_{K*}(z,\zeta,\lambda)\\ &-\sum_{K\in P'(N)}\int_{(\zeta,\lambda)\in\Gamma_{K*}\times\Delta_K} f(\zeta)\wedge\widehat{M}_{K*}(z,\zeta,\lambda).\end{aligned}$$

But we have $\widehat{M}_{K*}\ |_{S_{K*}\times\Delta_{K*}}=d\widehat{L}^{\tilde\psi}_{K*}=0$, then

$$dM^*f(z)=M^*df(z)-\sum_{K\in P'(N)}\int_{(\zeta,\lambda)\in\Gamma_{K*}\times\Delta_K} f(\zeta)\wedge\widehat{M}_{K*}(z,\zeta,\lambda)\ . \tag{5.3}$$

Since $\widehat{M}_{K*}\ |_{\Gamma_{K*}\times\Delta_K}=d\widehat{L}^{\tilde\psi}_K\ |_{\Gamma_{K*}\times\Delta_K}$, we have

$$\left.\begin{aligned}\int_{(\zeta,\lambda)\in\Gamma_{K*}\times\Delta_K} f(\zeta)\wedge\widehat{M}_{K*}(z,\zeta,\lambda)&=\int_{(\zeta,\lambda)\in\Gamma_{K*}\times\Delta_K} f(\zeta)\wedge d_{z,\zeta,\lambda}\widehat{L}^{\tilde\psi}_K(z,\zeta,\lambda)\\ &=(-1)^r d_z\Big(\int_{(\zeta,\lambda)\in\Gamma_{K*}\times\Delta_K} f(\zeta)\wedge\widehat{L}^{\tilde\psi}_K(z,\zeta,\lambda)\Big)\\ &\quad+(-1)^{r+1}\int_{(\zeta,\lambda)\in\Gamma_{K*}\times\Delta_K} df(\zeta)\wedge\widehat{L}^{\tilde\psi}_K(z,\zeta,\lambda)\\ &\quad+(-1)^r\int_{(\zeta,\lambda)\in\Gamma_{K*}\times\Delta_K} d_{\zeta,\lambda}\big(f(\zeta)\wedge\widehat{L}^{\tilde\psi}_K(z,\zeta,\lambda)\big)\end{aligned}\right\} \tag{5.4}$$

By Lemma 4.3.1 we get that, if $0 \leqslant r \leqslant q-N$,

$$\int_{(\zeta,\lambda)\in\Gamma_{K*}\times\Delta_K} f(\zeta)\wedge \widehat{L}_K^{\tilde{\psi}}(z,\zeta,\lambda)=0 \tag{5.5}$$

and if $0 \leqslant r \leqslant q-N-1$ or $df=0$

$$\int_{(\zeta,\lambda)\in\Gamma_{K*}\times\Delta_K} df(\zeta)\wedge \widehat{L}_K^{\tilde{\psi}}(z,\zeta,\lambda)=0\,. \tag{5.6}$$

One can easily prove that

$$\sum_{K\in P'(N)} \partial(\Gamma_{K*}\times\Delta_K)=\sum_{K\in P'(N)} S_{K*}\times\Delta_K\,. \tag{5.7}$$

Then, from Stokes'theorem and (5.7) we deduce

$$\sum_{K\in P'(N)}\int_{(\zeta,\lambda)\in\Gamma_{K*}\times\Delta_K} d_{\zeta,\lambda}\big(f(\zeta)\wedge \widehat{L}_K^{\tilde{\psi}}(z,\zeta,\lambda)\big)$$

$$=\sum_{K\in P'(N)}\int_{(\zeta,\lambda)\in S_{K*}\times\Delta_K} f(\zeta)\wedge \widehat{L}_K^{\tilde{\psi}}(z,\zeta,\lambda). \tag{5.8}$$

Using $[\widehat{L}_K^{\tilde{\psi}}]_{\deg\overline{\zeta}=k}=0$ for $K\in P'(N)$, $k\geqslant n-q$, and $\dim S_{K*}=2n-|K|-1$ for $K\in P'(N)$, we obtain that

$$\int_{(\zeta,\lambda)\in S_{K*}\times\Delta_K} f(\zeta)\wedge \widehat{L}_K^{\tilde{\psi}}(z,\zeta,\lambda)=0 \quad\text{if}\quad 0\leqslant r\leqslant q-N-1\,. \tag{5.9}$$

Therefore using (5.3), (5.4), (5.5), (5.6), (5.8) and (5.9) the lemma is proved for $f\in C^0_{n,r}(\overline{D})$ such that df is continuous on $\overline{D}$.

Now, let $f\in B^\beta_{n,r}(D)$, $0\leqslant\beta<1$, $0\leqslant r\leqslant q-N-1$, such that also $df\in B^\beta_*(D)$. Choose $\varepsilon>0$ with $\beta+\varepsilon<1$. Then as in the proof of Theorem 4.4.3, we can find a sequence of forms $f_\nu\in C^0_{n,r}(\overline{D})$ such that the forms df_ν are also continuous on $\overline{D}$ and

$$f_\nu\longrightarrow f \quad\text{and}\quad df_\nu\longrightarrow df$$

in the space $B^{\beta+\varepsilon}_*(D)$.

As the kernels $\widehat{M}_{K*}$ are of class C^1 in $W\times\Gamma_{K*}\times\Delta_{K*}$, $K\in P'(N)\cup\emptyset$, $M^*f_\nu\to M^*f$ and $M^*df_\nu\to M^*df$ for the C^1 topology in the open set W. Since $dM^*f_\nu=M^*df_\nu$ by the first part of the proof we get that $dM^*f=M^*df$ for $0\leqslant r\leqslant q-N-1$. ∎

5.3. Theorem. — *Let (E,D) be a local q-concave wedge, $0\leqslant q\leqslant n-1$, $(U_{\overline{D}},\rho_1,\dots,\rho_N,\rho_*)$ the frame associated to (E,D) in Definition 2.2 and ξ a fixed point in E. Then there exists a real R, $R>0$, such that for each $f\in B^\beta_{n,r}(D)$, $0\leqslant\beta<1$, $1\leqslant r\leqslant q-N-1$, with $df\in B^\beta_*(D)$ we have*

$$f=Sdf+dSf \quad\text{on}\quad D\cap B(\xi,R)$$

where $S=H+TM^*$, T being the Henkin operator for solving the $\overline{\partial}$-equation in $B(\xi,R)$.

Proof. — In Theorem 4.4.3, we have proved that, if $1 \leqslant r \leqslant q-N$, we have

$$f = dHf + Hdf + M^*f \quad \text{on} \quad D . \tag{5.10}$$

Let W be the neighborhood of ξ defined in Lemma 5.1. Then, there exists, $R > 0$, such that $\overline{B}(\xi, R) \subset W$ and M^*f is a C^1 differential form on $\overline{B}(\xi, R)$.

Let T be the operator defined by Corollary 1.12.2 in [He/Le 1] with the Leray map associated to $B(\xi, R)$ (see Definition 2.1.2 and Corollary 2.1.4 in [He/Le 1]). Then we have

$$M^*f = dTM^*f + TdM^*f \quad \text{on} \quad B(\xi, R) . \tag{5.11}$$

Setting $S = H + TM^*$, (5.10), (5.11) and Lemma 5.2 imply

$$f = dSf + Sdf \quad \text{on} \quad D \cap B(\xi, R). \blacksquare$$

5.4. LEMMA. — *Let us suppose that (E, D) is a local q-concave wedge defined by a q-configuration, ξ a fixed point in E and W the neighborhood of ξ defined in Lemma 5.1 using a constant α satisfying the properties of Remark 2.6. Then for each $(z, \lambda) \in W \times \Delta_{1\cdots N}$ there exists a strictly q-convex domain G such that*

a) $S_{1\cdots N*} \subset\subset G$;

b) $U_{\overline{D}}$ *is a q-convex extension of G ;*

c) $[\widehat{L}^{\psi}_{1\cdots N}]_{\deg \overline{\zeta}=n-q-1}$ *is a $\overline{\partial}$-closed form on a neighborhood of $\overline{G}$.*

Proof. — Set $\tilde{\rho}_i(\zeta) = \rho_i(\zeta) - \rho_i(z) + \frac{\alpha}{2}|\zeta-z|^2$, $i = 1, \ldots, N$ and for $\varepsilon > 0$, sufficiently small

$$\tilde{\varphi} = \max(-\tilde{\rho}_1, \ldots, -\tilde{\rho}_N, \rho_*-\varepsilon) .$$

By definition of W, if $z \in W$, we have

$$S_{1\cdots N*} \subset\subset \{\zeta \in U_{\overline{D}} \mid \tilde{\varphi}(\zeta) < 0\} .$$

Consequently there exists $\beta > 0$ such that

$$S_{1\cdots N*} \subset\subset \{\zeta \in U_{\overline{D}} \mid \tilde{\varphi}^{\beta}(\zeta) < 0\}$$

where $\tilde{\varphi}^{\beta} = \max_{\beta}(-\tilde{\rho}_1, \ldots, -\tilde{\rho}_N, \rho_*-\varepsilon)$.

Since $\tilde{\rho}_\lambda$ is strictly $(q+1)$-convex for each $\lambda \in \Delta_{1\cdots N}$ and ρ_* is convex, the function $\tilde{\varphi}^{\beta}$ is strictly $(q+1)$-convex on $U_{\overline{D}}$. Without loss of generality, we can assume that ρ_* is an unbounded exhausting function for $U_{\overline{D}}$. Then also $\tilde{\varphi}^{\beta}$ is an unbounded exhausting function for $U_{\overline{D}}$.

Since $-\operatorname{Re}\psi(z, \zeta, \lambda) > \tilde{\rho}_\lambda(\zeta)$ for $(z, \zeta, \lambda) \in U_{\overline{D}} \times U_{\overline{D}} \times \Delta_{1\cdots N}$, for each $(z, \lambda) \in W \times \Delta_{1\cdots N}$, $\widehat{L}^{\psi}_{1\cdots N}(z, \cdot, \lambda)$ is defined on $\{\zeta \in U_{\overline{D}} \mid \tilde{\varphi}^{\beta}(\zeta) < 0\}$.

Using the $(q+1)$-holomorphy of ψ and the definition of $\widehat{L}^{\psi}_{1\cdots N}$ we get

$$[L^{\psi}_{1\cdots N}]_{\deg \overline{\zeta}=n-q} = 0 \quad \text{and} \quad d_{z,\zeta,\lambda}L^{\psi} = 0 ,$$

therefore

$$\overline{\partial}_\zeta [L^\psi_{1\cdots N}]_{\deg \overline{\zeta}=n-q-1} = -(\partial_\zeta + d_{z,\lambda})[L^\psi_{1\cdots N}]_{\deg \overline{\zeta}=n-q} = 0 \,.$$

For $(z,\lambda) \in W \times \Delta_{1\cdots N}$, $\widehat{L}^\psi_{1\cdots N}(z,\cdot,\lambda)$ is $\overline{\partial}$-closed on $\{\, \zeta \in U_{\overline{D}} \mid \tilde{\varphi}^\beta(\zeta) < 0 \,\}$ and for sufficiently small $c > 0$, $G = \{\, \zeta \in U_{\overline{D}} \mid \tilde{\varphi}^\beta(\zeta) < -c \,\}$ has the required properties. ■

5.5. LEMMA. — *Under the hypothesis of Lemma 5.4, let $f \in B^\beta_{n,q-N}(D)$ an $(n, q-N)$ differential form, $0 \leqslant \beta < 1$, such that $df = 0$ then*

$$dM^* f = 0 \quad \text{on} \quad W \,.$$

Proof. — First let us assume that f is continuous on $\overline{D}$. Using (5.3), (5.4), (5.5), (5.6) and (5.8) we get for $z \in W$

$$dM^* f(z) = \sum_{K \in P'(N)} \int_{(\zeta,\lambda) \in S_{K*} \times \Delta_K} f(\zeta) \wedge \widehat{L}^{\tilde{\psi}}_K(z,\zeta,\lambda) \,.$$

Since on $W \times S_{K*} \times \Delta_K$, $\widehat{L}^{\tilde{\psi}}_K = \widehat{L}^\psi_K$ and $[\widehat{L}^{\tilde{\psi}}_K]_{\deg \overline{\zeta}=k} = 0$ for $K \in P'(N)$, $k \geqslant n-q$, we obtain

$$\begin{aligned} dM^* f(z) &= \int_{(\zeta,\lambda) \in S_{1\cdots N*} \times \Delta_{1\cdots N}} f(\zeta) \wedge [\widehat{L}^\psi_{1\cdots N}]_{\deg \overline{\zeta}=n-q-1}(z,\zeta,\lambda) \\ &= \int_{\lambda \in \Delta_{1\cdots N}} \Big(\int_{\zeta \in S_{1\cdots N*}} f(\zeta) \wedge [\widehat{L}^\psi_{1\cdots N}]_{\deg \overline{\zeta}=n-q-1}(z,\zeta,\lambda) \Big). \end{aligned} \tag{5.12}$$

We fix $(z,\lambda) \in W \times \Delta_{1\cdots N}$, by Lemma 5.4 $[\widehat{L}^\psi_{1\cdots N}]_{\deg \overline{\zeta}=n-q-1}$ is a $\overline{\partial}$-closed form on a neighborhood of a strictly q-convex domain G containing $S_{1\cdots N*}$. Moreover U is a q-convex extension of G and by Corollary 12.12 *(ii)* in [He/Le 2] we can approach $[\widehat{L}^\psi_{1\cdots N}]_{\deg \overline{\zeta}=n-q-1}$ uniformly on $\overline{G}$ by a sequence $(F_j)_{j \in N}$ of $\overline{\partial}$-closed form on U. Therefore we have

$$\int_{\zeta \in S_{1\cdots N*}} f(\zeta) \wedge [\widehat{L}^\psi_{1\cdots N}]_{\deg \overline{\zeta}=n-q-1}(z,\zeta,\lambda) = \lim_{j \to \infty} \int_{\zeta \in S_{1\cdots N*}} f(\zeta) \wedge F_j(\zeta) \,.$$

Since $S_{1\cdots N*}$ is the boundary of $S_{1\cdots N}$ and $f(\zeta) \wedge F_j(\zeta)$ is closed on $S_{1\cdots N}$ we obtain

$$\int_{\zeta \in S_{1\cdots N*}} f(\zeta) \wedge [\widehat{L}^\psi_{1\cdots N}]_{\deg \overline{\zeta}=n-q-1}(z,\zeta,\lambda) = 0$$

and consequently using (5.12) $dM^* f = 0$ on W.

This proves the lemma when f is continuous on $\overline{D}$. The same argument as in the proof of Lemma 5.2, implies this lemma when $f \in B^\beta_{n,q-N}(D)$.

5.6. THEOREM. — *Let (E, D) be a local q-concave wedge defined by a q-configuration (see Definition 2.4), $1 \leqslant q \leqslant n-1$, ξ a fixed point in E and N the real codimension of E in $\mathbb{C}^n$.*

Then there exists a real R, $R > 0$, such that for each $f \in B^\beta_{n,q-N}(D)$, $0 \leqslant \beta < 1$, $q-N \geqslant 1$, with $df = 0$ on D we have

$$f = dSf \quad \text{on} \quad D \cap B(\xi, R)$$

where $S = H + TM^*$, T being the Henkin operator for solving the $\overline{\partial}$-equation in $B(\xi, R)$.

Proof. — From Theorem 4.4.3, we know that

$$f = dHf + M^*f \quad \text{on} \quad D\,. \tag{5.13}$$

Let W be the neighborhood of ξ defined in Lemma 5.1. Then there exists $R > 0$ such that $\overline{B}(\xi, R) \subset W$ and M^*f is a C^1 differential form on $\overline{B}(\xi, R)$. Moreover by Lemma 5.5, M^*f is $\overline{\partial}$-closed on $B(\xi, R)$.

Let T be the operator defined by Corollary 1.12.2 in [He/Le 1] with the Leray map associated to $B(\xi, R)$ (see Definition 2.1.2 and Corollary 2.1.4 in [He/Le 1]).

Then we have

$$M^*f = dTM^*f \quad \text{on} \quad B(\xi, R)\,. \tag{5.14}$$

Setting $S = H + TM^*$, (5.13) and (5.14) imply

$$f = dSf \quad \text{on} \quad D \cap B(\xi, R). \blacksquare$$

5.7. THEOREM. — *Let (E, D) be a local q-concave wedge, defined by a q-configuration, $1 \leqslant q \leqslant n-1$, N the real codimension of E and ξ a fixed point in E. Let us suppose that $q - N \geqslant 0$, then there exists a neighborhood W of ξ in $\mathbb{C}^n$, $D \subset W$, such that each holomorphic function in D has an holomorphic extension to W.*

Proof. — Let f be a holomorphic function in D and $\varepsilon > 0$ a real number. We set $\rho_j^\varepsilon = \rho_j + \varepsilon$, $j = 1 \cdots N, *$. For ε sufficiently small, the frame $(U_{\overline{D}}, \rho_1^\varepsilon, \ldots, \rho_N^\varepsilon, \rho_*^\varepsilon)$ defines a new local q-concave wedge, denoted by $(E_\varepsilon, D_\varepsilon)$, which has the same properties than (E, D). Let $d_\varepsilon = \mathrm{dist}(\xi, E_\varepsilon)$ and $\xi_\varepsilon \in E_\varepsilon$ a point such that $|\xi - \xi_\varepsilon| = d_\varepsilon$.

Set $\tilde{f}(\zeta) = f(\zeta) d\zeta_1 \wedge \cdots \wedge d\zeta_n$, $\tilde{f}$ is a d-closed $(n,0)$-form which is continuous in $\overline{D}_\varepsilon$. Since $q \geqslant N$, Theorem 4.4.3, applied to $\tilde{f}$ and D_ε, implies that

$$\tilde{f} = M_\varepsilon^* f \quad \text{in} \quad D_\varepsilon\,.$$

As in the proof of Lemma 5.1 we have to consider the functions Φ^* and Φ_ε associated to $(E_\varepsilon, D_\varepsilon)$.

If $\zeta \in \Gamma_{K*}^\varepsilon$, then $\Phi^*(z, \zeta) \neq 0$ for all $z \in U_{\overline{D}}$ such that $\rho_*^\varepsilon(z) < 0$, *i.e.* $\rho^*(z) < -\varepsilon$.

On the other hand, for all $(z, \zeta, \lambda) \in U_{\overline{D}} \times U_{\overline{D}} \times \Delta_K$

$$\mathrm{Re}\, \Phi_\varepsilon(z, \zeta, \lambda) \leqslant \rho_\lambda^\varepsilon(z) + \rho_\lambda^\varepsilon(\zeta) - \frac{\alpha}{2}|\zeta - z|^2$$

where the constant α depends only on the second derivatives of ρ_λ^ε and consequently is independent of ε.

Following the proof of Lemma 5.1, if $\delta_\varepsilon = \mathrm{dist}(\xi_\varepsilon, \Gamma_{1\cdots N*}^\varepsilon)$ set $W^\varepsilon_{\tau,\lambda\,*} = \{z \in B(\xi_\varepsilon, \tau\delta_\varepsilon) \mid \rho_{*\lambda}^\varepsilon < \frac{\delta_\varepsilon \alpha(1-\tau)}{2}\}$, then $W_\tau^\varepsilon = \bigcap_{\lambda \in \Delta_{K*}} W^\varepsilon_{\tau,\lambda\,*}$ is a neighborhood of ξ_ε.

We shall prove that for some τ and for sufficiently small ε, then W_τ^ε is a neighborhood of ξ.

Since $\Gamma^\varepsilon_{1\cdots N*} = \Gamma_{1\cdots N*} \cap D_\varepsilon$, we have $\delta_\varepsilon \geqslant \delta - d_\varepsilon$. Choose $\varepsilon_0 > 0$ such that for all $\varepsilon < \varepsilon_0$, $\delta - d_\varepsilon > \frac{\delta}{2}$ and τ such that $d_{\varepsilon_0} < \frac{\tau\delta}{2}$.

Then if $\varepsilon < \inf(\frac{\alpha}{4}(1-\tau)\delta, \varepsilon_0)$, the point ξ belongs to $\{ z \in B(\xi_\varepsilon, \tau\frac{\delta}{2}) \mid \rho^\varepsilon_{\underset{\lambda}{*}} < \frac{\delta_\varepsilon \alpha(1-\tau)}{2} \}$ and therefore $\xi \in W_\tau^\varepsilon$ and $\Phi_\varepsilon(z,\zeta,\lambda) \neq 0$ on $W_\tau^\varepsilon \times \Gamma_{K*} \times \Delta_{K*}$.

Choose such an ε, it follows from the definition of M_ε^* that $M_\varepsilon^* \tilde{f}$ is a C^1, $(n,0)$-form in W_τ^ε, moreover by Lemma 5.5 $dM_\varepsilon^* \tilde{f} = 0$.

Finally the $(n,0)$-form $\tilde{h}$ defined by $\tilde{h} = \tilde{f}$ on D and $\tilde{h} = M_\varepsilon^* \tilde{f}$ on W_τ^ε defined a holomorphic function h on $W = W_\tau^\varepsilon \cup D$ such that $h = f$ on D. ∎

6. Estimates

In this section we denote by (E, D) a local q-concave wedge, $0 \leqslant q \leqslant n-1$, and by $(U_{\overline{D}}, \rho_1, \dots, \rho_N, \rho_*)$ the associated frame satisfying *(i)*, *(ii)* and *(iii)* in Definition 2.2. Let $\Gamma_K, K \in P(N,*)$ be the submanifolds of $\overline{D}$ defined in Section 4.2 and $\Phi(z,\zeta,\lambda)$ the function defined in Section 4.3.

In Section 4.3, we have defined an operator H from $B^\beta_{n,*}(D)$ into $C^0_{n,*}(D)$ by

$$Hf = \sum_{K \in P'(N,*)} (-1)^{|K|} H_K f \quad \text{for} \quad f \in B^\beta_{n,*}(D)$$

where the H_K's are given by (4.14).

Let us set $H'f = \sum_{K \in P'(N)} (-1)^{|K|} H_K f$ and $H^* f = \sum_{K \in P'(N) \cup \emptyset} (-1)^{|K|+1} H_{K*} f$.

Let us recall some definitions and propositions given in [L-T/Le].

6.1. Definition. — Let $K \in P'(N,*)$ and let s be an integer.

A *form of type* O_s (or *of type* $O_s(z,\zeta,\lambda)$) *on* $D \times \Gamma_K \times \Delta_{OK}$ is, by definition, a continuous differential form $f(z,\zeta,\lambda)$ defined for all $(z,\zeta,\lambda) \in D \times \Gamma_K \times \Delta_{OK}$ with $z \neq \zeta$ such that the following conditions are fulfilled :

(i) All derivatives of the coefficients of $f(z,\zeta,\lambda)$ which are of order 0 in ζ, of order $\leqslant 1$ in z, and of arbitrary order in λ are continuous for all $(z,\zeta,\lambda) \in D \times \Gamma_K \times \Delta_{OK}$ with $z \neq \zeta$.

(ii) Let $\nabla_z^\kappa, \kappa = 0, 1$, be a differential operator with constant coefficients which is of order 0 in ζ, of order κ in z, and of arbitrary order in λ. Then there is a constant $C > 0$ such that, for each coefficient $\varphi(z,\zeta,\lambda)$ of the form $f(z,\zeta,\lambda)$,

$$|\nabla_z^\kappa \varphi(z,\zeta,\lambda)| \leqslant C|\zeta - z|^{s-\kappa}$$

for all $(z,\zeta,\lambda) \in D \times \Gamma_K \times \Delta_{OK}$ with $z \neq \zeta$.

(iii) There exist neighborhood $U_0, U_K \subseteq \Delta_{OK}$ of Δ_0 and Δ_K, respectively, such that $f(z,\zeta,\lambda) = 0$ for all $(z,\zeta,\lambda) \in D \times \Gamma_K \times (U_0 \cup U_K)$.

The symbols $O_s(z,\zeta,\lambda)$ and O_s will be used also to denote forms of this type, also in formulas. For example :

$$f = O_s \text{ means } : f \text{ is a form of type } O.$$

$O_s \wedge f = O_k \wedge g + O_m$ means : for each form h of type O_s there exist a form u of type O_k and a form v of type O_m such that $h \wedge f = u \wedge g + v$.

The equation

$$Ef(z) = \int_{(\zeta,\lambda)\in S_K \times \Delta_{OK}} O_s(z,\zeta,\lambda) \wedge f(z,\zeta,\lambda)$$

means : there exists a form $\widehat{E}$ of type O_s such that

$$Ef(z) = \int_{(\zeta,\lambda)\in S_K \times \Delta_{OK}} \widehat{E}(z,\zeta,\lambda) \wedge f(z,\zeta,\lambda)$$

for all f.

6.2. Definition. — Let $m \geqslant 0$ be an integer. An *operator of type* m is, by definition, a map

$$E : \cup_{0\leqslant\beta<1} B^{\beta}_{n,*}(D) \longrightarrow C^{0}_{n,*}(D)$$

such that there exist

- an integer $k \geqslant 0$,
- $K \in P'(N)$,
- a form $\widehat{E}(z,\zeta,\lambda)$ of type $O_{|K|-2n+2k+m}$ on $D \times \Gamma_K \times \Delta_{OK}$ such that, for all $f \in B^{\beta}_{n,*}(D), 0 \leqslant \beta < 1$,

$$Ef(z) = \int_{(\zeta,\lambda)\in \Gamma_K \times \Delta_{OK}} \widetilde{f}(\zeta) \wedge \frac{\widehat{E}(z,\zeta,\lambda) \wedge \Theta(\zeta)}{\Phi^{k+m}(z,\zeta,\mathring{\lambda})}$$

where $\widetilde{f} \in B^{\beta}_{0,*}(D)$ is the form with

$$f(\zeta) = \widetilde{f}(\zeta) \wedge d\zeta_1 \wedge \cdots \wedge d\zeta_n \ ,$$

and for Θ holds the following :

if $m = 0$, then $\Theta = 1$;

if $m \geqslant 1$, then there exist indices $i_1, \ldots, i_m \in K$ such that either

$$\Theta = \partial\rho_{i_1} \wedge \cdots \wedge \partial\rho_{i_m} \quad \text{or} \quad \Theta = \overline{\partial}\rho_{i_1} \wedge \partial\rho_{i_2} \wedge \cdots \wedge \partial\rho_{i_m}$$

(for the definition of $\mathring{\lambda}$, see Sect. 1.8).

6.3. Proposition. — *Let us consider an operator E of type $m, m \geqslant 0$.*

(i) *Let $0 \leqslant \beta < 1/2$, $0 < \varepsilon \leqslant 1/2-\beta$, and $1 \leqslant r \leqslant n$. Then*

$$E\big(B^{\beta}_{n,r}(D)\big) \subset C^{1/2-\beta-\varepsilon}_{n,r-1}(\overline{D})$$

and the operator E is compact as operator between the Banach spaces $B^{\beta}_{n,r}(D)$ and $C^{1/2-\beta-\varepsilon}_{n,r-1}(\overline{D})$

(ii) *Let $1/2 \leqslant \beta < 1$, $0 < \varepsilon \leqslant 1-\beta$, and $1 \leqslant r \leqslant n$. Then*

$$E\big(B^{\beta}_{n,r}(D)\big) \subset B^{\beta+\varepsilon-1/2}_{n,r-1}(\overline{D})$$

and the operator E is compact as operator between the Banach spaces $B^{\beta}_{n,r}(D)$ and $B^{\beta+\varepsilon-1/2}_{n,r-1}(D)$.

For the proof of this proposition see the proof of Theorem 4.12 in Section 8 of [L-T/Le].

6.4. THEOREM. — *The operator H' is a finite sum of operators of type $m, m \geqslant 0$.*

Proof. — It comes from the definition of H' that the calculations are exactly the same than in the proof of Theorem 5.4 in [L-T/Le]. The only change is that we have exchange the roles of z and ζ in the definition of w. But using that, for all $k = 1, \dots, N$, ρ_k is of class C^3, we get that

$$\begin{aligned} O_0 \wedge W &= O_0 \wedge \sum_{j=1}^{n} w^j(z,\zeta,\lambda)d\zeta_j = O_0 \wedge \sum_{k\in K} \frac{\partial \rho_k}{\partial z_j}(z)d\zeta_j + O_1 \\ &= O_0 \wedge \partial\rho_j(\zeta) + \sum_{k\in K}\Big(\frac{\partial \rho_k}{\partial z_j}(z) - \frac{\partial \rho_k}{\partial z_j}(\zeta)\Big)d\zeta_j + O_1 \\ &= O_0 \wedge \partial\rho_j(\zeta) + O_1 \end{aligned}$$

and in the same way $O_0 \wedge d_\lambda W = \sum_{j\in K} O_0 \wedge \partial\rho_j(\zeta) + O_1$ and $O_0 \wedge \overline{\partial}_{z,\zeta}\Phi = \sum_{j\in K} O_0 \wedge \overline{\partial}\rho_j(\zeta) + O_1$ on $D \times \Gamma_K \times \Delta_{OK}$, $K \in P'(N)$, which are exactly the same estimates than in [L-T/Le]. ■

6.5. PROPOSITION. — *Let ξ be a fixed point in E and W the neighborhood of ξ defined in Lemma 5.1. Then for each $f \in B^{\beta}_{n,r}(D)$, $0 \leqslant \beta < 1$, $0 \leqslant r \leqslant n$ the differential form H^*f is of class C^1 in W and the operator H^* is a bounded linear operator from $B^{\beta}_{n,*}(D)$ into $C^1_{n,*}(W)$.*

Proof. — By definition of W, $\Phi^*(z,\zeta) \neq 0$, $\Phi(z,\zeta) \neq 0$ and $|z-\zeta| \neq 0$ for $(z,\zeta,\lambda) \in W \times \Gamma_{K*} \times \Delta_{OK*}$.

Therefore the kernels, which are used to define the operator H^*, are C^1 differential forms on $W \times \Gamma_{K*} \times \Delta_{OK*}$. Then it follows easily from the definition of H^* that H^* is a bounded linear operator from $B^{\beta}_{n,*}(D)$, $0 \leqslant \beta < 1$, into $C^1_{n,*}(W)$.

6.6. THEOREM. — *Let ξ be a fixed point in E and R be a positive real number such that $\overline{B}(\xi, R) \subset W$, where W is the neighborhood of ξ defined in Lemma 5.1. Then* *the operator $S = H + TM^*, T$ being the Henkin operator for solving the $\overline{\partial}$-equation in $B(\xi, R)$ has the following properties :*

i) For $0 \leqslant \beta < 1/2$, $0 < \varepsilon \leqslant 1/2-\beta$ and $1 \leqslant r \leqslant n$, S is a compact operator between the Banach spaces $B^\beta_{n,r}(D)$ and $C^{1/2-\beta-\varepsilon}_{n,r-1}(\overline{D} \cap \overline{B}(\xi, R))$.

ii) For $1/2 \leqslant \beta < 1$, $0 < \varepsilon \leqslant 1-\beta$ and $1 \leqslant r \leqslant n$, S is a compact operator, between the Banach spaces $B^\beta_{n,r}(D)$ and $B^{\beta+\varepsilon-1/2}_{n,r-1}(D \cap B(\xi, R))$.

Proof. — Recall that $S = H' + H^* + TM^*$. It follows from Proposition 6.3 and Theorem 6.4 that H' satisfies the conclusions *i)* and *ii)* of the theorem.

By Lemma 5.1 and Theorem 2.2.2 in [He/Le 1], TM^* is a bounded operator from $B^\beta_{n,*}(D)$, $0 \leqslant \beta < 1$, into $C^{1/2}_{n,*}(\overline{D} \cap \overline{B}(\xi, R))$ and, by Proposition 6.5, H^* is a bounded operator from $B^\beta_{n,*}(D)$, $0 \leqslant \beta < 1$, into $C^1_{n,*}(\overline{D} \cap \overline{B}(\xi, R))$.

Now let $0 \leqslant \beta < 1/2$. It follows from Ascoli's theorem that the injection maps from $C^{1/2}_{n,*}(\overline{D} \cap \overline{B}(\xi, R))$ and $C^1_{n,*}(\overline{D} \cap \overline{B}(\xi, R))$ into $C^{1/2-\beta-\varepsilon}(\overline{D} \cap \overline{B}(\xi, R))$ are compact. This ends the proof of the theorem in the first case.

Finally, suppose that $1/2 \leqslant \beta < 1$. By Ascoli's theorem, $H^* + TM^*$ is a compact operator from $B^\beta_{n,*}(D)$ into $C^0_{n,*}(\overline{D} \cap \overline{B}(\xi, R))$. Moreover the injection map from $C^0_{n,*}(\overline{D} \cap \overline{B}(\xi, R))$ into $B^{\beta+\varepsilon-1/2}_{n,*}(D \cap B(\xi, R))$ is bounded and the second assertion of the theorem is proved. ■

Combining Theorem 5.3, Theorem 5.6 and Theorem 6.6, we obtain the main result of this paper :

6.7. THEOREM. — *Let (E, D) be a local q-concave wedge, $0 \leqslant q \leqslant n-1$, and ξ be a fixed point in E. Then there exists a real R, $R > 0$, and a linear operator S from $B^\beta_{n,r}(D)$ into $C^0_{n,r-1}(D \cap B(\xi, R))$, $1 \leqslant r \leqslant n$, such that :*

i) If $0 \leqslant \beta < 1/2$ and $0 < \varepsilon \leqslant 1/2-\beta$, S is compact from $B^\beta_{n,*}(D)$ into $C^{1/2-\beta-\varepsilon}_{n,*}(\overline{D} \cap \overline{B}(\xi, R))$.

ii) If $1/2 \leqslant \beta < 1$ and $0 < \varepsilon \leqslant 1-\beta$, S is compact from $B^\beta_{n,*}(D)$ into $B^{\beta+\varepsilon-1/2}_{n,*}(D \cap B(\xi, R))$.

iii) For each $f \in B^\beta_{n,r}(D)$, $0 \leqslant \beta < 1$, $1 \leqslant r \leqslant q-\operatorname{codim}_{\mathbb{R}} E-1$ with $df \in B^\beta_*(D)$ we have

$$f = Sdf + dSf \quad \text{on} \quad D \cap B(\xi, R) .$$

iv) If moreover the local q-concave wedge (E, D) is defined by a q-configuration and $1 \leqslant r = q-\operatorname{codim}_{\mathbb{R}} E$, then for each d-closed form $f \in B^\beta_{n,r}(D)$, $0 \leqslant \beta < 1$ we have

$$f = dSf \quad \text{on} \quad D \cap B(\xi, R) .$$

7. Globalization

Let us denote by E a holomorphic vector bundle over an n-dimensional complex manifold X, by Ω and Δ two domains in X such that $\Omega \subset\subset \Delta \subset\subset X$ and by D the domain $\Delta \setminus \Omega$. Further, let $C^{\alpha}_{n,r}(\overline{D}, E)$, $B^{\beta}_{n,r}(D, E)$ *etc...* the Banach spaces of E-valued differential forms on D, which are obtained canonically extending the definitions of Section 1.13.

7.1. DEFINITION. — Let q and q' be two integers, $0 \leqslant q, q' \leqslant n-1$. A domain $D \subset\subset X$ will be called *a q-concave, q'-convex domain of order* N, $1 \leqslant N \leqslant 2n$, if there exist two domains $\Omega \subset\subset \Delta \subset\subset X$ such that $D = \Delta \setminus \Omega$ and satisfying the following properties :

(i) For each point $\xi \in \partial\Omega$, there exists a neighborhood U_ξ of ξ in X contained in a coordinate domain, such that, after identification with its image in $\mathbb{C}^n$, U_ξ contains a local q-concave wedge (E_ξ, D_ξ) with

(a) $\xi \in E_\xi$;

(b) $\operatorname{codim}_{\mathbb{R}} E_\xi \leqslant N$;

(c) (E_ξ, D_ξ) is defined by a q-configuration ;

(d) If $(U_{\overline{D}_\xi}, \rho_1, \dots, \rho_{N_\xi}, \rho_*)$ is a frame for (E_ξ, D_ξ) then $D \cap U_\xi \cap \{ z \in U_{\overline{D}_\xi} \mid \rho_*(z) < 0 \} = D_\xi$.

(ii) Δ is a local q'-convex domain.

7.2. Examples. — The simplest example of such domains is given by $D = B(0,R') \setminus B(0,R)$, $0 < R < R'$ in $\mathbb{C}^n$, this is a $(n-1)$-concave, $(n-1)$-convex domain of order 1. Another simple example is $D = \Delta \setminus \Omega$ with Δ a C^2 smooth q'-convex domain and Ω a C^3 smooth q-convex domain.

A more interesting example is given by $D = \Delta \setminus \Omega$ where Δ is a strictly pseudoconvex domain with C^2-smooth boundary and Ω is the union of N strictly pseudoconvex domains with C^3-smooth boundary, whose boundaries are intersecting transversally. Such a domain is a $(n-1)$-concave, $(n-1)$-convex domain of order N.

The case where Δ is a strictly pseudoconvex domain with C^2-smooth boundary and $\Omega = \Omega_1 \cup \Omega_2$ with Ω_i, $i = 1,2$, two strictly q-convex domains with C^3-smooth boundary intersecting themselves transversally defined by $\Omega_i = \{ z \in U_{\partial\Omega_i} \mid \varphi_i(z) < 0 \}$ and such that for each $\lambda \in [0,1]$ and $\xi \in \partial\Omega_1 \cap \partial\Omega_2$ the Levi form $L_{\lambda\varphi_1+(1-\lambda)\varphi_2}(\xi)$ restricted to $T^{\mathbb{C}}_\xi(\partial\Omega_1 \cap \partial\Omega_2)$ has at least $\dim_{\mathbb{C}} T^{\mathbb{C}}_\xi(\partial\Omega_1 \cap \partial\Omega_2) - n + q + 1$ positive eigenvalues, defines a q-concave, $(n-1)$-convex domain of order 2 (*cf.* remark 2.5).

7.3. THEOREM. — *Let D be a q-concave, q'-convex domain of order N in X. We suppose that $q+q'-N \geqslant n$. Then there exist linear operators*

$$\widetilde{T}_r : \bigcup_{0 \leqslant \beta < 1} B^{\beta}_{n,r}(D, E) \longrightarrow C^{0}_{n,r-1}(D, E)$$

and

$$K_r : \bigcup_{0 \leqslant \beta < 1} B^{\beta}_{n,r}(D, E) \longrightarrow C^{0}_{n,r}(D, E)$$

for $n-q' \leqslant r \leqslant q-N$ such that the following holds :

(i) *if $n-q' \leqslant r \leqslant q-N-1$, then*

$$f = d\widetilde{T}_r f + \widetilde{T}_{r+1} df + K_r f$$

for all $f \in B^{\beta}_{n,r}(D,E)$, $0 \leqslant \beta < 1$, such that df also belongs to $B^{\beta}_{*}(D,E)$;

(ii) *if $r = q-N$, then for all d-closed $f \in B^{\beta}_{n,r}(D,E)$, $0 \leqslant \beta < 1$,*

$$f = d\widetilde{T}_r f + K_r f \ ;$$

(iii) *if $0 \leqslant \beta < 1/2$ and $0 < \varepsilon \leqslant 1/2-\beta$, then $\widetilde{T}_r$ and $K_r, n-q' \leqslant r \leqslant q-N$, are compact operators from $B^{\beta}_{n,r}(D,E)$ into $C^{1/2-\beta-\varepsilon}_{n,r-1}(\overline{D},E)$, resp. $C^{1/2-\beta-\varepsilon}_{n,r}(\overline{D},E)$;*

(iv) *if $1/2 \leqslant \beta < 1$ and $\varepsilon > 0$, then $\widetilde{T}_r$ and $K_r, n-q' \leqslant r \leqslant q-N$, are compact operators from $B^{\beta}_{n,r}(D,E)$ into $B^{\beta+\varepsilon-1/2}_{n,r-1}(D,E)$, resp. $B^{\beta+\varepsilon-1/2}_{n,r}(D,E)$*

Proof. — By Definition 7.1 and Lemma 2.4 in [L-T/Le] there exists a finite number of open sets $U_1, \dots, U_m \subset X$ such that $\overline{D} \subset U_1 \cup \cdots \cup U_m$ and each $U_j \cap D$, $1 \leqslant j \leqslant m$ is either a local q'-convex domain or a local q-concave wedge defined by a q-configuration. The second case occurs, when $U_j \cap \Omega \neq \emptyset$. Moreover, we may assume that E is trivial over some neighborhood of each $\overline{U_j \cap D}$, $1 \leqslant j \leqslant m$.

Let A_j be the operators which are induced in

$$\bigcup_{0 \leqslant \beta < 1} B^{\beta}_{n,*}(D,E)$$

by the local operators in the following way : if $U_j \cap D$ is a local q-concave wedge $A_j f = S(f|_{U_j \cap D})$ where S is defined in Theorems 5.3 and 5.6 and if $U_j \cap D$ is a local q'-convex domain $A_j f = H(f|_{U_j \cap D})$ where H is defined in Section 4 of [L-T/Le].

We choose non negative C^{∞} functions χ_j with compact support in U_j such that $\chi_1 + \cdots + \chi_m = 1$ in a neighborhood of $\overline{D}$ and we set

$$\widetilde{T}_r f = \sum_{j=1}^{m} \chi_j A_j f$$

and

$$K_r f = \sum_{j=1}^{m} d\chi_j \wedge A_j f$$

for $n-q' \leqslant r \leqslant q-N, f \in B^{\beta}_{n,r}(D)$, $0 \leqslant \beta < 1$. ■

Up to the end of this part we will suppose that $X = \mathbb{C}^n$.

7.4. Definition. — A q-concave, q'-convex domain of order N, $1 \leqslant N \leqslant 2n$, D contained in $\mathbb{C}^n$ will be *of special type* if $D = \Delta \setminus \Omega$ where Δ is a local q'-convex domain and Ω is the union of N strictly q-convex domains Ω_i, $1 \leqslant i \leqslant N$, with C^3 smooth boundary intersecting themselves transversally, defined by $\Omega_i = \{z \in U_{\partial\Omega_i} \mid \varphi_i(z) < 0\}$ and such that for each multi-index $K \in \mathcal{P}(N)$, each $\lambda \in \Delta_K$ and each $\xi \in \bigcap_{k_\nu \in K} \partial\Omega_{K_\nu}$ the Levi form $L_{\lambda_1\varphi_{k_1}+\cdots+\lambda_\ell\varphi_{k_\ell}}(\xi)$ restricted to $T^{\mathbb{C}}_{\xi}(\partial\Omega_{k_1} \cap \cdots \cap \partial\Omega_{k_\ell})$ has at least $\dim_{\mathbb{C}} T^{\mathbb{C}}_{\xi}(\partial\Omega_{k_1} \cap \cdots \cap \partial\Omega_{k_\ell})-n+q+1$ positive eigenvalues.

7.5. PROPOSITION. — *Let $D \subset\subset \mathbb{C}^n$ be a q-concave, q'-convex domain of order N and of special type and suppose that $q+q'-N \geqslant n$. If f is a continuous (n,r)-form in some neighborhood $U_{\overline{D}}$ of $\overline{D}$, $n-q' \leqslant r \leqslant q-N$, such that $\overline{\partial} f = 0$ in $U_{\overline{D}}$, then there exists a form $u \in \bigcap_{\varepsilon>0} C^{1/2-\varepsilon}_{n,r-1}(\overline{D})$ such that $\overline{\partial} u = f$ in D.*

Proof. — This proposition is the analogous in the case of q-concave, q'-convex domains of Lemma 2.3.4 in [He/Le 1]. Using Theorem 7.3 at the place of Lemma 2.3.1 ([He/Le 1]) we can repeat the proof of Lemma 2.3.4 in [He/Le 1]. We have only to remark that there exists a q-concave, q'-convex domain of order N and of special type G such that $D \subset\subset G \subset\subset U_{\overline{D}}$.

Let us consider $\Omega_{i,\alpha} = \{ z \in U_{\partial\Omega_i} \mid \varphi_i(z) > \alpha \}$. For $\alpha > 0$, sufficiently small it is easy to verify that $\Omega_\alpha = \bigcup_{i=1}^{N} \Omega_{i,\alpha}$ has the same properties than Ω. Moreover if $\Delta = \{ z \in U_{\overline{\Delta}} \mid \rho_j < 0,\ j = 1, \dots, N \}$ then $\Delta_\beta = \{ z \in U_{\overline{\Delta}} \mid \rho_j < -\beta,\ j = 1, \dots, N \}$ is also a local q-convex domain for sufficiently small $\beta > 0$. Then it suffices to take $G = \Delta_\beta \setminus \Omega_\alpha$ for some small α and β. ∎

Following the same methods than in part 2.3 of [He/Le 1], we get the following theorem on the resolution of the $\overline{\partial}$-equation in q-concave, q'-convex domains with estimates up to the boundary.

7.6. THEOREM. — *Let $D \subset\subset \mathbb{C}^n$ be a q-concave, q'-convex domain of order N and of special type such that $q+q'-N \geqslant n$ and for $0 \leqslant \beta < 1$, let $f \in B^{\beta}_{n,r}(D)$ be a $\overline{\partial}$-closed form on D, $n-q' \leqslant r \leqslant q-N$.*

(i) if $0 \leqslant \beta < 1/2$, there exists $u \in \bigcap_{\varepsilon>0} C^{1/2-\beta-\varepsilon}_{n,r-1}(\overline{D})$ such that $\overline{\partial} u = f$ and for each $\varepsilon > 0$ there exists also a constant C_ε such that

$$\|u\|_{1/2-\beta-\varepsilon} \leqslant C_\varepsilon \|f\|_{-\beta} ;$$

(ii) if $1/2 \leqslant \beta < 1$, there exists $u \in \bigcap_{\varepsilon>0} B^{\beta+\varepsilon-1/2}_{n,r-1}(D)$ such that $\overline{\partial} u = f$ and for each $\varepsilon > 0$ there exists also a constant C_ε such that

$$\|u\|_{1/2-\beta-\varepsilon} \leqslant C_\varepsilon \|f\|_{-\beta} .$$

Proof. — As in the proof of Theorem 2.3.5 in [He/Le 1], we deduce the existence of the solution u from Proposition 7.5 by the bumping method. The estimates are a consequence of the Banach's open mapping theorem and of Theorem 7.3 (*cf.* [He/Le 1] appendix 2). ∎

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