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SINGULAR PERTURBATION OF SYMBOLIC FLOWS AND THE MODIFIED LAX-PHILLIPS CONJECTURE

MITSURU IKAWA

1. Introduction. In the study of scattering by an obstacle consisting of several convex bodies, it is known that the distribution of poles of the scattering matrix has a close relationship to the zeta functions associated with a dynamical system in the exterior of the obstacle. When we want to consider the validity of the modified Lax-Phillips conjecture, we can derive it from the existence of poles of the zeta functions. That is, roughly speaking, if the zeta function has a pole in a certain region, the scattering matrix for the obstacle has an infinite number of poles in a strip $\{z \in \mathbf{C}; 0 < \operatorname{Im} z < \alpha\}$ for some $\alpha > 0$. The modified Lax-Phillips conjectrue will be explained in the next section.

Therefore, in order to consider distributions of poles of scattering matrices for an obstacle consisting of several convex bodies, the zeta functions play a crucial role. But unfortunately, it is not so easy to show the existence of a pole of the zeta functions in general.

In this talk, we shall develop a theory of singular perturbations of symbolic dynamics, with which we shall show the existence of a pole of the zeta function when the obstacle is consisted of several small balls.

In Section 2, we explain the modified Lax-Phillips conjecture and consider the scattering by obstacles consisting of several convex bodies. In Section 3, we shall discuss singular perturbations of symbolic dynamics. In Section 4, we shall show how to apply the theorem on singular perturbations of symbolic dynamics to considerations of the matrices for obstacles consisting of several small balls.

2. Scattering by several convex bodies.

Let $\mathcal{O}$ be a bounded open set in $\mathbf{R}^3$ with smooth boundary Γ. We set

$$\Omega = \mathbf{R}^3 - \overline{\mathcal{O}},$$

and assume that Ω is connected. Consider the following acoustic problem:

$$(2.1)\qquad \begin{cases} \Box u = \dfrac{\partial^2 u}{\partial t^2} - \Delta u = 0 & \text{in } \Omega \times (-\infty, \infty), \\ u = 0 & \text{on } \Gamma \times (-\infty, \infty), \\ u(x,0) = f_1(x), \dfrac{\partial u}{\partial t}(x,0) = f_2(x). \end{cases}$$

We denote by $\mathcal{S}(z)$ the scattering matrix for this problem. The scattering matrix $\mathcal{S}(z)$ is an $\mathcal{L}(L^2(S^2))$–valued function analytic in $\{z;\ \mathrm{Im}\, z \leq 0\}$ and meromorphic in the whole complex plane $\mathbf{C}$, and that the correspondance from obstacles to scattering matrices

$$\mathcal{O} \to \mathcal{S}(z)$$

is one to one(see for example [LP]).

Concerning the above correspondance, we are interested in the problem to know how the distribution of poles of scattering matrices relates to the geometry of obstacles. As to this problem, we would like to present the following conjecture:

Modified Lax-Phillips Conjecture. *When $\mathcal{O}$ is trapping, there is a positive constant α such that the scattering matrix $\mathcal{S}(z)$ has an infinite number of poles in $\{z; 0 < \mathrm{Im}\, z \leq \alpha\}$.*

Hereafter, we say that MLPC(abbreviation of the modified Lax-Phillips conjecture) is valid for obstacle $\mathcal{O}$, when there is $\alpha > 0$ such that the scattering matrix $\mathcal{S}(z)$ corresponding to $\mathcal{O}$ has an infinite number of poles in $\{z; \mathrm{Im}\, z \leq \alpha\}$.

About this conjecture, obstacles consisting of two convex bodies were studied first. By the works [BGR], [G], [Ik1] and [S], the distribution of poles are well studied, and it is shown that MLPC is valid for obstacles consisting of two convex bodies. It is very natural to proceed to obstacles consisting of three strictly convex bodies. But the problem for three bodies exposes an essential difference from that of two bodies. Namely, for an obstacle consisting of three bodies, there exist infinitely many primitive periodic rays in the exterior of the obstacle in general. Thus, we have to consider geometric property of the totality of the periodic rays in the exterior, and it seems that the asymptotic behavior of the periodic rays with very large period plays an essential role.

Here, we present a theorem in [Ik3,4], which allows us to connect the asymptotic behavior of the periodic rays and the distribution of poles of the scattering matrix.

Let $\mathcal{O}_j$, $j = 1, 2, \cdots, L$, be bounded open sets with smooth boundary Γ_j satisfying

(H.1) every $\mathcal{O}_j$ is strictly convex,

(H.2) for every $\{j_1, j_2, j_3\} \in \{1, 2, \cdots, L\}^3$ such that $j_l \neq j_{l'}$ if $l \neq l'$,

$$(\text{convex hull of } \overline{\mathcal{O}_{j_1}} \text{and } \overline{\mathcal{O}_{j_2}}) \cap \overline{\mathcal{O}_{j_3}} = \phi.$$

We set

$$\mathcal{O} = \cup_{j=1}^{L} \mathcal{O}_j, \quad \Omega = \mathbf{R}^3 - \overline{\mathcal{O}} \text{ and } \Gamma = \partial\Omega. \tag{2.2}$$

Denote by γ an oriented periodic ray in Ω, and we shall use the following notations:

d_γ : the length of γ,
T_γ : the primitive period of γ,
i_γ : the number of the reflecting points of γ,
P_γ : the Poincaré map of γ.

We define a function $F_D(s)$ ($s \in \mathbf{C}$) by

$$F_D(s) = \sum_{\gamma} (-1)^{i_\gamma} T_\gamma |I - P_\gamma|^{-1/2} e^{-s d_\gamma} \tag{2.3}$$

where the summation is taken over all the oriented periodic rays in Ω and $|I - P_\gamma|$ denotes the determinant of $I - P_\gamma$.

Concerning the periodic rays in Ω we have

$$\#\{\gamma; \text{periodic ray in } \Omega \text{ such that } d_\gamma < r\} < e^{a_0 r} \tag{2.4}$$

and

$$|I - P_\gamma| \geq e^{2a_1 d_\gamma}, \tag{2.5}$$

where a_0 and a_1 are positive constants depending on $\mathcal{O}$. The estimates (2.4) and (2.5) imply that the right hand side of (2.3) converges absolutely in $\{s \in \mathbf{C}; \operatorname{Re} s > a_0 - a_1\}$. Thus $F_D(s)$ is well defined in $\{s \in \mathbf{C}; \operatorname{Re} s > a_0 - a_1\}$, and holomorphic in this domain.

Now we have

Theorem 2.1. *Let $\mathcal{O}$ be an obstacle given by (2.2) satisfying (H.1) and (H.2). If $F_D(s)$ cannot be prolonged analytically to an entire function, then MLPC is valid for $\mathcal{O}$.*

We cannot give here the proof of the above theorem. We would like to refer that the trace formula due to [BGR] is the starting point of the proof. This trace formula is written as follows:

$$\begin{aligned} \text{(2.6)} \qquad & \mathrm{Trace}_{L^2(\mathbf{R}^3)} \int \rho(t) \left(\cos t\sqrt{-A} \oplus 0 - \cos t\sqrt{-A_0}\right) dt \\ & = \frac{1}{2}\sum_{j=1}^{\infty} \hat{\rho}(z_j), \qquad \text{for all } \rho \in C_0^\infty(0,\infty) \end{aligned}$$

where

$$\hat{\rho}(z) = \int e^{izt}\rho(t)dt,$$

$\{z_j\}_{j=1}^{\infty}$ is a numbering of all the poles of $\mathcal{S}(z)$, A is the selfajoint realization in $L^2(\Omega)$ of the Laplacian with the Dirichlet boundary condition and A_0 the one in $L^2(\mathbf{R}^3)$, and $\oplus 0$ indicates the extension into $\mathcal{O}$ by 0. It gives us an relationship between the distribution of poles of the scattering matrix and the singularities of the trace of the evolution operator of (2.1). We shall use (2.6) in the following way: Suppose that $F_D(s)$ has a singularity. This enable us to choose a sequence of ρ of the form

$$\rho_q(t) = \rho(m_q(t - l_q))$$

in such way that

$$l_q \to \infty, \quad m_q \to \infty \quad \text{as } q \to \infty,$$

and that the left hand side does not decay so fast as q tends to the infinity. But if MLPC is not valid, the right hand side of (2.6) for ρ_q decreases very rapidely. The difference in decreasing speeds brings a contradiction. Thus MLPC is valid. The detailed proof is given in [Ik3].

By virtue of Theorem 2.1, the proof of the validity of MLPC is transfered to the consideration of singularities of $F_D(s)$. But it is not easy to show the existence of singularities of $F_D(s)$ in general. At present we can show it only for obstacles consisting of small balls.

Theorem 2.2. *Let $P_j, j = 1, 2, \cdots, L$, be points in $\mathbf{R}^3$, and set for $\varepsilon > 0$*

$$\mathcal{O}_\varepsilon = \cup_{j=1}^{L}\mathcal{O}_{j,\varepsilon}, \quad \mathcal{O}_{j,\varepsilon} = \{x; |x - P_j| < \varepsilon\}.$$

Suppose that

(A) *any triple of P_j's does not lie on a straight line.*

Then, there is a positive constant ε_0 such that the modified Lax-Phillips conjecture holds for $\mathcal{O}_\varepsilon$ for all $0 < \varepsilon \le \varepsilon_0$.

To prove the above theorem we have to show the existence of singularities of $F_D(s)$ associated with $\mathcal{O}_\varepsilon$. To do this, we need a theory of singular perturbation of symbolic dynamics, which will be developed in the next section.

3. Singular perturbations of symbolic dynamics

In this section we consider singular perturbations. First we shall give some notations concerning the symbolic dynamics.

3.1. Notations and statement of a theorem.

Let $L \ge 2$ be an integer, and let $A = (A(i,j))_{i,j=1,2,\cdots,L}$ be a zero-one $L \times L$ matrix. We set

$$\Sigma_A^+ = \{\xi = (\xi_1, \xi_2, \cdots); 1 \le \xi_j \le L \text{ and } A(\xi_j, \xi_{j+1}) = 1 \text{ for } j = 1, 2, \cdots\},$$

and denote by σ_A the shift operator defined by

$$(\sigma_A(\xi))_j = \xi_{j+1} \quad \text{for all } j.$$

We regard Σ_A^+ as a compact metric space by introducing the usual discrete metric. Define $\mathrm{var}_n\, r$ and $\|r\|_\infty$ for $r \in C(\Sigma_A^+)$ by

$$\mathrm{var}_n\, r = \sup\{|r(\xi) - r(\psi)|; \xi, \psi \in \Sigma_A^+ \text{ and } \xi_j = \psi_i \text{ for } j \le n\},$$
$$\|r\|_\infty = \sup\{|r(\xi)|; \xi \in \Sigma_A^+\}.$$

We set for $0 < \theta < 1$

$$\|r\|_\theta = \sup_{n \ge 1} \frac{\mathrm{var}_n\, r}{\theta^n}, \quad |||r|||_\theta, = \max\{\|r\|_\theta, \|r\|_\infty\}$$
$$\mathcal{F}_\theta(\Sigma_A^+) = \{r \in C(\Sigma_A^+); |||r|||_\theta < \infty\}.$$

Assume that A satisfies

(3.1) $$A^N > 0 \quad \text{for some positive integer } N,$$

that is, all the entries of the matrix A^N are positive. Let $B = [B(i,j)]_{i,j=1,2,\cdots,L}$ be another zero-one $L \times L$ matrix.

Definition. Let $i, j \in \{1, 2, \cdots, L\}$. The notation

$$i \underset{B}{\longrightarrow} j$$

indicates the existence of a sequence $i_1, i_2. \cdots, i_p$ such that $B(i_1, i) = 1$, $B(i_{q+1}, i_q) = 1$ for $q = 1, 2, \cdots, p-1$ and $B(j, i_p) = 1$.

We assume on B the following:
There is $1 < K \leq L$ such that

$$B(i,j) = 0 \quad \text{for all } \ j \text{ if } \ i \geq K+1, \tag{3.2}$$

$$i \underset{B}{\longrightarrow} i \quad \text{for all } 1 \leq i \leq K, \tag{3.3}$$

$$i \underset{B}{\longrightarrow} j \quad \text{implies} \quad j \underset{B}{\longrightarrow} i \quad \text{if } i, j \leq K \tag{3.4}$$

and

$$B(i,j) = 1 \quad \text{implies} \quad A(i,j) = 1. \tag{3.5}$$

Let f_ε, h_ε are functions with parameter $\varepsilon \geq 0$ satisfying

$$f_\varepsilon,\ h_\varepsilon \in \mathcal{F}_\theta(\Sigma_A^+) \qquad \text{for all } 0 \leq \varepsilon \leq \varepsilon_1, \tag{3.5}$$

where ε_1 is a positive constant, and let $k \in \mathcal{F}_\theta(\Sigma_A^+)$ satisfy

$$k(\xi) = \begin{cases} k(\xi) = 0 & \text{if } \ B(\xi_1, \xi_2) = 1 \\ k(\xi) > 0 & \text{if } \ B(\xi_1, \xi_2) = 0. \end{cases} \tag{3.6}$$

Suppose that

$$|||f_\varepsilon - f_0|||_\theta,\ |||h_\varepsilon - h_0|||_\theta \to \infty \quad \text{as} \quad \varepsilon \to 0. \tag{3.7}$$

For $0 < \varepsilon \leq \varepsilon_1$, we define zeta function $Z_\varepsilon(s)$ by

$$Z(s;\varepsilon) = \exp\left(\sum_n^\infty \frac{1}{n} \sum_{\sigma_A^n \xi = \xi} \exp S_n r(\xi, s; \varepsilon)\right) \tag{3.8}$$

where

$$r(\xi, s; \varepsilon) = -s f_\varepsilon(\xi) + h_\varepsilon(\xi) + k(\xi) \log \varepsilon$$

and

$$S_n r(\xi, s; \varepsilon) = \sum_{j=0}^{n-1} r({\sigma_A}^j \xi, s; \varepsilon).$$

The following theorem is on the existence of singularities of $Z(s;\varepsilon)$, which is the main result of [Ik5].

Theorem 3.1. *Suppose that (3.1)∼(3.7) are satisfied, and that*

$$(3.9) \qquad f_0(\xi) > 0 \qquad \text{for all } \xi \in \Sigma_A^+,$$

$$(3.10) \qquad h_0(\xi) \text{ is real for all } \xi \in \Sigma_A^+ \text{ such that } B(\xi_1, \xi_2) = 1.$$

Then there exist $s_0 \in \mathbf{R}$, D a neighborhood of s_0 in $\mathbf{C}$ and $\varepsilon_0 > 0$ such that, for every $0 < \varepsilon \leq \varepsilon_0$, $Z(s;\varepsilon)$ is meromorphic in D and it has a pole s_ε in D with

$$s_\varepsilon \longrightarrow s_0 \qquad \text{as} \quad \varepsilon \longrightarrow 0.$$

Here we would like to mention about the reason why we call the above result as singular perturbation.

Let us set

$$C = (B(i,j))_{i,j=1,2,\cdots,K}$$

and

$$\Sigma_C^+ = \{\xi = (\xi_1, \xi_2, \cdots);\ 1 \leq \xi_j \leq K \text{ and } B(\xi_j, \xi_{j+1}) = 1 \text{ for all } j\}.$$

Consider a term in (3.8)

$$\sum_{\sigma_A^n \xi = \xi} \exp\, S_n\, r(\xi, s; \varepsilon).$$

If we make ε tend to zero, because of the effect of $k(\xi) \log \varepsilon$, for all $\xi \in \Sigma_A^+$ such that $k({\sigma_A}^m \xi) > 0$ for at least one m, $\exp\, S_n\, r(\xi, s; \varepsilon)$ tends to zero. Therefore, the above summation tends to

$$\sum_{\sigma_C^n \xi = \xi} \exp\, S_n\, r(\xi, s; \varepsilon).$$

If we set

$$\tilde{Z}_0(s) = \exp\{\sum_{n=1}^{\infty} \frac{1}{n} \sum_{\sigma_A^n \xi = \xi} \exp\, S_n(-s f_0(\xi) + h_0(\xi))\},$$

$\tilde{Z}_0(s)$ is a zeta function of the symbolic flow on (Σ_C^+, σ_C). Thus the above fact suggests us that $Z_\varepsilon(s)$ should be regarded as a perturbation of $\tilde{Z}_0(s)$. But when we compare these, not only the function $-f_0(\xi) + h_0(\xi)$ but also the structure matrix C are perturebed. Thus we should call it singular perturbation.

In the rest of this section we shall give only a sketch of the proof of Theorem 3.1. For the detailed proof, see [Ik5].

3.2. The Perron-Frobenius operators.

In order to find a pole of $Z_\varepsilon(s)$ it is important to examine the spectrum of the Perron-Frobenius operator associated with $Z(s;\varepsilon)$ defined by

$$\mathcal{L}_{\varepsilon,s} = \sum_{\sigma_A\eta=\xi} \exp(r_\varepsilon(\eta,s))\,u(\eta) \quad \text{for} \quad u\in C(\Sigma_A^+).$$

For the proof of the existence of poles of $Z(s;\varepsilon)$, if we use the results of [Po] or [H], it suffices to show the existence s for which $\mathcal{L}_{\varepsilon,s}$ has 1 as an eigenvalue.

Remark that it is difficult to consider directly the spectrum of $\mathcal{L}_{\varepsilon,s}$ since $r_\varepsilon(\xi,s)$ is of the form rather complex for $\varepsilon>0$. Thus, it is important to find its nice approximations. As the first approximation, we introduce an operator $\mathcal{L}'_s$ in Σ_A^+ by

$$\mathcal{L}'_s = \begin{cases} \sum_{B(\eta_1,\xi_1)=1} \exp(r_\varepsilon(\eta,s))\,u(\eta) & \text{for} \quad \xi\in\Sigma(1), \\ 0 & \text{for} \quad \xi\in\Sigma(2), \end{cases} \tag{3.11}$$

where

$$r_0(\xi;s) = -sf_0(\xi)+h_0(\xi),$$

$\sum_{B(\eta_1,\xi_1)=1}$ indicates the summation taken over all $\eta\in\Sigma_A^+$ such that $\sigma_A\eta=\xi$ and $B(\eta_1,\xi_1)=1$, and

$$\Sigma(1) = \{\xi\in\Sigma_A^+;\, B(l,\xi_1)=1 \quad \text{for some } 1\le l\le K\},$$
$$\Sigma(2) = \{\xi\in\Sigma_A^+;\, B(l,\xi_1)=0 \quad \text{for all } 1\le l\le K\}.$$

Since $r_0(\xi,s)$ is not necessarily real even for $\varepsilon=0$ and real s, we have to introduce an approximation $\tilde{\mathcal{L}}_s$ of $\mathcal{L}'_s$ defined by

$$\tilde{\mathcal{L}}_s\, v(\xi) = \sum_{\sigma_C\eta=\xi} \exp(r_\varepsilon(\eta,s))\,u(\eta) \quad \text{for} \quad v\in C(\Sigma_C^+). \tag{3.12}$$

Now $r_0(\xi,s)$ is real valued for all $\xi\in\Sigma_C^+$ and real s and we can apply the generalized Perron-Frobenius theorem and find $s_0\in\mathbf{R}$ such that $\tilde{\mathcal{L}}_{s_0}$ has 1 as an eigenvalue.

Of course, in using these approximations of $\mathcal{L}_{\varepsilon,s}$, we have to compare the spectra of these operators. In our reasoning, the most crucial step is to give a relationships between spectra of $\tilde{\mathcal{L}}_s$ and $\mathcal{L}'_s$.

3.3. On the decomposition of $\tilde{\mathcal{L}}_s$.

First recall the generalized Perron-Frobenious Theorem of [AS] for symbolic dynamics which is not necessarily mixing. We introduce the following definition of indecomposability of a matrix.

Definition. We say that an $L \times L$ zero-one matrix C is indecomposable when $i \underset{C}{\longrightarrow} j$ for any $i, j \in \{1, 2, \cdots, L\}$.

Then the following theorem holds:

Theorem 3.2. *Suppose that a zero-one matrix C is indecomposable, and r is a real valued function belonging to $\mathcal{F}_\theta(\Sigma_C^+)$. The the operator in $\mathcal{F}_\theta(\Sigma_C^+)$ defined by*

$$\mathcal{L}\, u(\xi) = \sum_{\sigma_C \eta = \xi} \exp\big(r(\eta)\big)\, u(\eta)$$

has the following decomposition:

$$\mathcal{L} = \sum_{k=1}^{k_0} \lambda_k\, E_k + S$$

where

$$\begin{gathered} \lambda_1 > 0, \quad \textit{and} \quad \lambda_k = \lambda_1 \exp(i(k-1)2\pi/k_0), \quad \textit{for } k = 2, \ldots, k_0, \\ E_k\, E_l = \delta_{kl} E_k, \quad E_k S = S E_k = 0, \\ \textit{dimension of the range } E_k = 1, \\ \textit{the spectral radius of } S < \lambda_1(1-\delta) \textit{ for some } \delta > 0. \end{gathered}$$

The constant k_0 is the greatest common divisor of all the periods of periodic elements in Σ_C^+

Let us say that i and j are equivalent when $i \underset{B}{\longrightarrow} j$. Then the conditions (3.2) and (3.3) on B imply that this gives an equivalent relation in $\{1, 2, \ldots, K\}$. Therefore, by changing the numbering of the elements of $\{1, 2, \ldots, K\}$, we may assume that the set $\{1, 2, \ldots, K\}$ is decomposed into equivalents classes

$$M_j = \{i_j, i_j + 1, \cdots, i_{j+1} - 1\} \quad (j = 1, 2, \cdots, l).$$

We shall denote by C_j the $(i_{j+1} - i_j) \times (i_{j+1} - i_j)$ matrix $[B(i,j)]_{i,j \in M_j}$. Note that each C_j is indecomposable. We set

$$\Sigma_{C_j}^+ = \{\xi = (\xi_1, \xi_2, \cdots);\ \xi_i \in M_j \text{ and } \ B(\xi_i, \xi_{i+1}) = 1 \ \text{ for all } i\}$$

and

$$\Sigma_C^+ = \{\xi = (\xi_1, \xi_2, \cdots);\ 1 \le \xi_i \le K \text{ and } \ B(\xi_i, \xi_{i+1}) = 1 \ \text{ for all } i\}.$$

Regarding $\Sigma_{C_j}^+$ and Σ_C^+ as subsets of Σ_A^+, we have a decomposition

$$C(\Sigma_C^+) = C(\Sigma_{C_1}^+) \oplus C(\Sigma_{C_2}^+) \oplus \cdots \oplus C(\Sigma_{C_l}^+).$$

For $u \in C(\Sigma_A^+)$ we denote by $[u]$ and $[u]_j$ the restrictions of u to Σ_C^+ and $\Sigma_{C_j}^+$ respectively. Conversely, for functions in Σ_C^+ or in $\Sigma_{C_j}^+$ we shall often treat them as functions defined in Σ_A^+ by extending them by zero in the outside of Σ_C^+ or of $\Sigma_{C_j}^+$.

Let $\tilde{\mathcal{L}}_s$ be the operator in $C(\Sigma_C^+)$ defined by

$$\tilde{\mathcal{L}}_s v(\xi) = \sum_{\sigma_C \eta = \xi} \exp\big(r_0(\eta; s)\big)\, v(\eta) \qquad \text{for } v \in C(\Sigma_C^+),$$

and let $\tilde{\mathcal{L}}_{j,s}$ be the operators in $C(\Sigma_{C_j}^+)$ defined by

$$\tilde{\mathcal{L}}_{j,s}\, v(\xi) = \sum_{\sigma_{C_j} \eta = \xi} \exp\big(r_0(\eta; s)\big)\, v(\eta) \quad \text{for} \quad v \in C(\Sigma_{C_j}^+)$$

where σ_C and σ_{C_j} denote the restrictions of σ_A to Σ_C^+ and $\Sigma_{C_j}^+$ respectively. Then $\tilde{\mathcal{L}}_s$ has a decomposition

$$\tilde{\mathcal{L}}_s = \tilde{\mathcal{L}}_{1,s} \oplus \tilde{\mathcal{L}}_{2,s} \oplus \cdots \oplus \tilde{\mathcal{L}}_{l,s}.$$

By using the notaion introduced in the above, we have for all $u \in \Sigma_A^+$

$$\tilde{\mathcal{L}}_s\,[u] = \tilde{\mathcal{L}}_{1,s}[u]_1 \oplus \tilde{\mathcal{L}}_{2,s}[u]_2 \oplus \cdots \oplus \tilde{\mathcal{L}}_{l,s}[u]_l.$$

Note that the conditions (3.9) and (3.10) imply that r_0 is real valued in $\Sigma_{C_j}^+$ for $s \in \mathbf{R}$. Thus, taking acount of the indecomposability of C_j we can apply the above Theorem 3.2 to $\tilde{\mathcal{L}}_{j,s}$ and get the following

Lemma 3.3. *For $s \in \mathbf{R}$, $\tilde{\mathcal{L}}_{j,s}$ has a decomposition*

$$\tilde{\mathcal{L}}_{j,s} = \sum_{k=1}^{k_j} \tilde{\lambda}_{j,k,s}\, \tilde{E}_{j,k,s} + \tilde{S}_{j,s},$$

with the following properties:

(i) $$\tilde{\mathcal{L}}_{j,s}\, \tilde{E}_{j,k,s} = \tilde{\lambda}_{j,k,s} \tilde{E}_{j,k,s}.$$

(ii) $$\tilde{\lambda}_{j,1,s} > 0 \quad \textit{and} \quad -\frac{d\,\tilde{\lambda}_{j,1,s}}{ds} > 0.$$

(iii) $$|\tilde{\lambda}_{j,k,s}| = \tilde{\lambda}_{j,1,s} \quad \textit{and} \quad \tilde{\lambda}_{j,k,s} \neq \tilde{\lambda}_{j,k',s} \quad \textit{if}\ k \neq k'.$$

(iv) $$\tilde{E}_{j,k,s}\, u(\xi) = \nu_{j,k,s}(u)\, p_{j,k,s}(\xi),$$

where $\nu_{j,k,s} \in \cap_{\theta'>0} \mathcal{F}_{\theta'}(\Sigma_{C_j}^+)^*$ satisfying $\nu_{j,k,s}(p_{j,k,s}) = 1$,

(v) $$\tilde{E}_{j,k,s} \tilde{E}_{j,k',s} = \delta_{k,k'} \tilde{E}_{j,k,s}, \quad \tilde{E}_{j,k,s} \tilde{S}_{j,s} = \tilde{S}_{j,s} \tilde{E}_{j,k,s} = 0,$$

(vi) *the spectral radius of $\tilde{S}_{j,s} < \tilde{\lambda}_{j,k,s}$.*

Hereafter, we shall denote often $\tilde{\lambda}_{j,1,s}$ as $\tilde{\lambda}_{j,s}$. Note that we have for each j

$$\tilde{\lambda}_{j,s} \to \infty \qquad \text{as} \quad s \to -\infty,$$
$$\tilde{\lambda}_{j,s} \to 0 \qquad \text{as} \quad s \to \infty.$$

Thus, by changing the numbering of $\tilde{\lambda}_{j,s}$ if necessary, we may suppose that for some $s_0 \in \mathbf{R}$

$$1 = \tilde{\lambda}_{1,s_0} = \tilde{\lambda}_{2,s_0} = \cdots = \tilde{\lambda}_{h,s_0} > \tilde{\lambda}_{h+1,s_0} \geq \cdots \geq \tilde{\lambda}_{l,s_0}.$$

Then, by using the perturbation theory we have immediately the following

Lemma 3.4. *There are a neighborhood D of s_0 in $\mathbf{C}$ and a constant $\delta > 0$ such that for all $s \in D$ we have a decomposition*

$$\tilde{\mathcal{L}}_s = \sum_{j=1}^{h} \sum_{k=1}^{k_j} \tilde{\lambda}_{j,k,s} \, \tilde{E}_{j,k,s} + \tilde{S}_s$$

with the following properties:

(i) $$\tilde{E}_{j,k,s} u(\xi) = \nu_{j,k,s}([u]_j) \, p_{j,k,s}(\xi),$$

(ii) $$\tilde{E}_{j,k,s} \tilde{E}_{j',k',s} = \delta_{j,j'} \delta_{k,k'} \tilde{E}_{j,k,s}$$

(iii) $$\tilde{E}_{j,k,s} \, \tilde{S}_s = \tilde{S}_s \, \tilde{E}_{j,k,s} = 0,$$

(iv) $$|\tilde{\lambda}_{j,s} - 1| < \delta,$$

(v) $$|\tilde{\lambda}_{j,k,s} - 1| > 2\delta, \quad 1 - \delta < |\tilde{\lambda}_{j,k,s}| < 1 + \delta \quad \textit{for } k \geq 2,$$

(vi) *the spectral radius of* $\tilde{S}_s < 1 - 2\delta$.

3.4. On eigenvalues of $\mathcal{L}'_s$.

With the aid of the results of the previous subsection, we shall consider the decomposition of $\mathcal{L}'_s$. First remark that for any positive integer m and for $\xi \in \Sigma(1)$ we have an expression

$$\mathcal{L}'^{m}_s u(\xi) = \sum_{\eta_1, \cdots, \eta_m} \exp\big(S_m r_0(\eta_m, \eta_{m-1}, \cdots, \eta_1, \xi; s)\big) \cdot u(\eta_m, \eta_{m-1}, \cdots, \eta_1, \xi), \tag{3.13}$$

where the summation is taken over all $\eta_1, \eta_2, \cdots, \eta_m$ satisfying $B(\eta_1, \xi_1) = 1$, $B(\eta_2, \eta_2) = 1, \cdots, B(\eta_m, \eta_{m-1}) = 1$.

In the expression of (3.13), by using the fact the $r_0 \in \mathcal{F}_\theta(\Sigma_A^+)$ and the decomposition of $\tilde{\mathcal{L}}_s$ shown the the previous subsection, we have the following

Lemma 3.5. *For each pair j, k in Lemma 3.4, there is a function $w_{j,k,s}(\xi) \in \mathcal{F}_\theta(\Sigma_A^+)$ satisfying*

(3.14)

$$|(\tilde{\lambda}_{j,k,s})^{-m} \sum_{\eta_m,\cdots,\eta_2,l} \exp\left(S_q r_0(\eta_m,\cdots,\eta_2,l,\xi)\right) p_{j,k,s}(\eta_q,\cdots,l,\eta^{(l)}) - w_{j,k,s}(\xi)| \le C\gamma_1^m \quad \text{for } m = 1,2,\cdots,$$

and

$$\mathcal{L}'_s\, w_{j,k,s} = \tilde{\lambda}_{j,k,s}\, w_{j,k,s}.$$

Here γ_1 is a constant such that $0 \le \gamma_1 < 1$.

Remark that we have from (3.14) and (iv) of Lemma 3.3

$$w_{j,k,s}(\xi) = p_{j,k,s}(\xi) \quad \text{for all} \quad \xi \in \Sigma_C^+,$$

from which it follows that

$$\nu_{j,k,s}([w_{j',k',s}]_j) = \delta_{j,j'}\delta_{k,k'}.$$

Define $E'_{j,k,s}$ by

$$E'_{j,k,s}u(\xi) = \nu_{j,k,s}([u]_j)w_{j,k,s}(\xi).$$

Then, we have

$$E'_{j,k,s}\, E'_{j',k',s} = \delta_{j,j'}\delta_{k,k'}.E'_{j,k,s},$$

and

$$\mathcal{L}'_s\, E'_{j,k,s} = \tilde{\lambda}_{j,k,s}E'_{j,k,s}.$$

In the expression (3.13), by using the decomposition of $\tilde{\mathcal{L}}_s$ and Lemma 3.5, we have the following lemma which is crusial for the proof of Theorem 2.1.

Lemma 3.6. *There exist a neighborhood D_1 of s_0 in $\mathbf{C}$ and a positive constant δ_2 such that we have for all $s \in D_1$*

$$1 - \delta_2/2 \le |\tilde{\lambda}_{j,k,s}| \le 1 + \delta_2/2, \tag{3.15}$$

$$|||\mathcal{L}'^m_s u - \sum_{j=1}^{h}\sum_{k=1}^{k_j} (\tilde{\lambda}_{j,k,s})^m E'_{j,k,s}u(\xi)|||_\theta \le C|||u|||_\theta\,(1-2\delta_2)^m. \tag{3.16}$$

3.5. On the decomposition of $\mathcal{L}'_s$.

By using the same argument as in [Ik3], we have the following two estimates concerning $\mathcal{L}'_{s_0}$ for any $u \in \mathcal{F}_\theta(\Sigma_A^+)$

$$\|{\mathcal{L}'_{s_0}}^m u\|_\infty \leq C_1 \|u\|_\infty,$$
$$|||{\mathcal{L}'_{s_0}}^m u|||_\theta \leq C_2\, \theta^m\, |||u|||_\theta + C_3 \|u\|_\infty.$$

Thus, by applying the theorem of [IM] to the pair of the spaces $C(\Sigma_A^+)$ and $\mathcal{F}_\theta(\Sigma_A^+)$, we have from the above inequalities the following decomposition of $\mathcal{L}'_{s_0}$ in $\mathcal{F}_\theta(\Sigma_A^+)$

$$\mathcal{L}'_{s_0} = \sum_{j=1}^{J} c_j E'_j + S' = E' + S',$$

where

$$\begin{aligned} &\mathcal{L}'_{s_0} E'_j = c_j E'_j \quad \text{and} \quad |c_j| = 1 \quad \text{for all } \ j, \\ &E'_j\, E'_l = \delta_{jl}\, E'_j \quad \text{for all } \ j,\ l, \\ &E'_j\, S' = S'\, E'_j = 0 \quad \text{for all } \ j, \\ &\text{the spectral radius of } S' < 1. \end{aligned}$$

With the aid of Lemma 3.5 we can show easily that there is no eigenvalue of E' besides $\tilde{\lambda}_{j,k,s_0}$. Now we have the following proposition from the standard perturbation theory:

Proposition 3.7. *There are $s_0 \in \mathbf{R}$, a neighborhood D_2 of s_0 in $\mathbf{C}$ and a positive constant δ_3 such that, for all $s \in D_2$, $\mathcal{L}'_s$ has a decomposition*

$$\mathcal{L}'_s = \sum_{l=1}^{l_0} F'_{l,s} + S'_s$$

satisfying the following:
(1) $F'_{l,s}\, S'_s = S'_s\, F'_{l,s} = 0$, for all $l = 0, 1, \cdots, l_0$.
(2) $F'_{l,s}\, F'_{k,s} = F'_{k,s}\, F'_{l,s} = 0$ for all $l, k = 0, 1, \cdots, l_0$ such that $l \neq k$.
(3) For $0 \leq l \leq l_0$, the dimension of the range of $F'_{l,s} = i_l$ for all $s \in D_2$ and the eigenvalues of $F'_{l,s}$ are $\mu_{(l,i),s}$ $i = 1, 2, \cdots, i_l$, which satisfy

$$|\mu_{(l,i),s} - \mu_l^0| < \frac{1}{3}\delta_3 \quad |\mu_l^0 - \mu_{l'}^0| > \delta_3\ (l \neq l').$$

Especially, $\mu_0^0 = 1$, $i_0 = h$ and $\mu_{(0,j),s} = \tilde{\lambda}_{j,s}$ $(j = 1, 2, \cdots, h)$.

(4) the spectral radius of $S'_s < 1 - 3\delta_3$.

3.6. Spectrum of $\mathcal{L}_{\varepsilon,s}$.

Suppose that Lemmas 3.6 and Proposition 3.7 hold for the open disk $D_2 = \{s; |s - s_0| < \alpha_0\}$ ($\alpha_0 > 0$). Recall that $\tilde{\lambda}_{j,s}$, $j = 1, 2, \cdots, h$ are analytic in D_2, and satisfies

$$\tilde{\lambda}_{j,s_0} = 1, \qquad -\frac{d}{ds}\tilde{\lambda}_{j,s}\Big|_{s=s_0} > 0.$$

Thus, by exchanging α_0 by a smaller one if necessary, we may assume the following:

$$|\tilde{\lambda}_{s,j} - 1| \le \delta_3/3 \qquad \text{for all } s \in D_2,$$

$$|\tilde{\lambda}_{s,j} - 1| \ge c_1|s - s_0| \quad \text{for all } s \in \{s; |s - s_0| \le \alpha_0\} \quad (c_1 > 0).$$

By the same argument as in [Ik3, Section 3] we have

$$|||\mathcal{L}'_{0,s} - \mathcal{L}_{\varepsilon,s}|||_\theta \to 0 \quad \text{uniformly in } s \in D_2 \text{ as } \varepsilon \to 0.$$

Therefore by applying the standard perturbation theory we have

Lemma 3.8. *There are positive constants ε_0 and δ_4 such that for all $0 < \varepsilon \le \varepsilon_0$ and $s \in D_2$ we have the following decomposition of $\mathcal{L}_{\varepsilon,s}$:*

(i) $$\mathcal{L}_{\varepsilon,s} = \sum_{l=0}^{l_0} \mathcal{E}_{(l),\varepsilon,s} + \mathcal{S}_{\varepsilon,s}$$

where

(ii) $$\mathcal{E}_{(l),\varepsilon,s}\,\mathcal{E}_{(k),\varepsilon,s} = \mathcal{E}_{(k),\varepsilon,s}\,\mathcal{E}_{(l),\varepsilon,s} = 0 \quad \text{if } l \ne k,$$

(iii) $$\mathcal{E}_{(l),\varepsilon,s}\,\mathcal{S}_{\varepsilon,s} = \mathcal{S}_{\varepsilon,s}\,\mathcal{E}_{(l),\varepsilon,s} = 0,$$

(iv) $$\text{the spectral radius of } \mathcal{S}_{\varepsilon,s} < 1 - 2\delta_3,$$

$$\dim \operatorname{Range} \mathcal{E}_{(l),\varepsilon,s} = i_l \quad \text{for all } 0 < \varepsilon < \varepsilon_0,$$

(v) $$\sum_{\sigma_A^n \xi = \xi} \exp\big(\operatorname{Re} r_\varepsilon(\xi, s)\big) \le C(1 + \delta_3)^n \quad \text{for all } n.$$

Moreover, denoting the eigenvalues of $\mathcal{E}_{(l),\varepsilon,s}$ by $\lambda_{l,i}(\varepsilon, s), i = 0, 1, \cdots, i_l$, $l = 1, 2, \cdots, h$, we have for all $0 < \varepsilon \le \varepsilon_0$

(vi) $$|\lambda_{l,j}(\varepsilon, s) - \mu_l^0| \le \frac{2}{3}\delta_3 \quad \text{for all } s \in D_2, \quad l = 0, 1, \cdots, l_0,$$

(vii) $$|\lambda_{0,j}(\varepsilon, s) - 1| > \delta_4 \quad \text{for all } s \in \{s; |s - s_0| = \alpha_0\}.$$

3.7. Proof of Theorem.

Set

$$f_l(\lambda, s; \varepsilon) = \prod_{i=0}^{i_l} (\lambda - \lambda_{l,i}(\varepsilon, s)).$$

It is easy to check that, for each l, $f_l(\lambda, s; \varepsilon)$ is holomorphic in s. With the aid of Rouché's theorem, we can show easily from (vii) of Lemma 3.8 that for each $0 < \varepsilon \leq \varepsilon_0$, $f_0(1, s; \varepsilon) = 0$ has exactly h zeros in $\{s; |s - s_0| < \alpha_0\}$ $(\alpha_0 > 0)$.

Now we apply Theorem 2 of [Po] or Theorem 4 of [H] to $\mathcal{L}_{\varepsilon,s}$. By exchanging ε_0 by a smaller one if necessary we may assume that

$$\theta(1 + \delta_3) < 1.$$

Then, the application of the theorems of [Po, H] to $\mathcal{L}_{\varepsilon,s}$ assures that

$$Z_\varepsilon(s) \text{ is meromorphic in } \operatorname{Re} s > s_0 + \alpha_0$$

and is of the form

$$Z_\varepsilon(s) = \exp(\phi(s, \varepsilon)) \prod_{l=0}^{l_0} f_l(1, s; \varepsilon)^{-1},$$

where $\phi(\cdot, \varepsilon)$ is holomorphic in $\operatorname{Re} s > s_0 + \alpha_0$. Recall that f_0 has h zeros near s_0. Thus Theorem 3.1 is proved.

4. Application to small balls

Let $\mathcal{O}$ be the obstacle defined by (2.2) satisfying (H.1) and (H.2). Now we explain briefly the relationship between symbolic dynamics and bounded rays in the exterior of $\mathcal{O}$.

Let $A = (A(i,j))_{i,j=1,\cdots,L}$ be the $L \times L$ matrix defined by

$$A(i,j) = \begin{cases} 1, & \text{if } i \neq j \\ 0, & \text{if } i = j, \end{cases}$$

and set

$$\Sigma_A = \{\xi = (\cdots, \xi_{-1}, \xi_0, \xi_1, \cdots) \in \prod_{i=-\infty}^{\infty} \{1, 2, \cdots, L\}; \\ A(\xi_j, \xi_{j+1}) = 1 \text{ for all } j\}.$$

Let $X(s)$ $(s \in \mathbf{R})$ be a representation of an orientated broken ray by the arc length such that $X(0) \in \Gamma$ and $X(s)$ moves in the orientation as s increases. When $\{|X(s)|; s \in \mathbf{R}\}$ is bounded, $X(s)$ repeats reflections on the boundary Γ infinitely many times as s tends to $\pm\infty$. Let the j-th reflection point X_j be on Γ_{l_j}. Then a bounded broken ray defines an infinite sequence $\xi = \{\cdots, l_{-1}, l_0, l_1, \cdots\}$, which is called the reflection order of $X(s)$. Remark that, for a bounded broken ray with direction, there is freedom of such representation, that is, the freedom of the choice of $X(0)$. Therefore the correspondance between bounded broken rays and Σ_A is not one to one. We set

$$f(\xi) = |X_0 X_1|$$

where X_j denote the j-th reflection point of the broken ray corresponding to ξ.

For a real valued function $g(\xi) \in \mathcal{F}_\theta(\Sigma_A)$, we define $\zeta(s)$ by

$$\zeta(s) = \exp\left(\sum_{n=1}^{\infty} \frac{1}{n} \sum_{\sigma_A^n \xi = \xi} \exp S_n(-sf(\xi) + g(\xi) + \pi i)\right).$$

Denote by ν_0 the abscissa of convergence of $F_D(s)$, that is,

$$\nu_0 = \inf\{\nu;\ F_D(s) \text{ converges absolutely for } \mathrm{Re} s > \nu\}$$

If we choose $g(\xi)$ in a suitable way, there is $a_2 > 0$, which is a constant determined by $\mathcal{O}$, such that the singularities of $F_D(s)$ and $-\dfrac{d}{ds}\log \zeta(s)$ are coincide in $\{s; \mathrm{Re}\, s \geq \nu_0 - a_2\}$. The function $g(\xi)$ with the above property is determined uniquely by the geometry of $\mathcal{O}$.

Thus, if we can show the existence of poles of $-\frac{d}{ds}\log\zeta(s)$ in $\{s; \mathrm{Re}\, s \geq \nu_0 - a_2\}$, we get the existence of poles of $F_D(s)$.

Now we turn to considerations on the singularities of $\zeta(s)$ corresponding to $\mathcal{O}_\varepsilon$ of in Theorem 3.2. Remark that (A) in Theorem 2.2 implies (H.2) for $\mathcal{O}_\varepsilon$ when ε is small.

We denote $f(\xi), g(\xi)$ and $\zeta(s)$ attached to $\mathcal{O}_\varepsilon$ by $f_\varepsilon(\xi), g_\varepsilon(\xi)$ and $\zeta_\varepsilon(s)$ respectively. It is easy to see that, by setting $f_0(\xi) = |P_{\xi_0}P_{\xi_1}|$,

$$|\log\varepsilon|\ |||f_\varepsilon - f_0|||_\theta \to 0 \qquad \text{as} \quad \varepsilon \to 0. \tag{4.1}$$

By using the relationship between the curvatures of the wave fronts of incident and reflected waves we have

$$|||g_\varepsilon(\xi) - \left(\log\varepsilon + \frac{1}{2}\log\frac{1}{4}(\cos\frac{\Theta(\xi)}{2})\right)|||_\theta \to 0 \qquad \text{as} \quad \varepsilon \to 0,$$

where $\Theta(\xi) = \angle P_{\xi_{-1}}P_{\xi_0}P_{\xi_1}$. Then, by setting $\tilde{g}_\varepsilon(\xi) = g_\varepsilon(\xi) - \log\varepsilon$ and $\tilde{g}_0(\xi) = \frac{1}{2}\log\frac{1}{4}(\cos\frac{\Theta(\xi)}{2})$ we have

$$|||\tilde{g}_\varepsilon - \tilde{g}_0|||_\theta \to 0 \qquad \text{as} \quad \varepsilon \to 0. \tag{4.2}$$

Set

$$d_{\max} = \max_{i\neq j}|P_iP_j|$$

and

$$B(i,j) = \begin{cases} 1 & \text{if} \ \ |P_iP_j| = d_{\max}, \\ 0 & \text{if} \ \ |P_iP_j| < d_{\max}. \end{cases}$$

By changing the numbering of the points if necessary, we may suppose that

$$\begin{aligned} B(i,j) &= 0 \qquad \text{for all } j \quad \text{if} \ \ i \geq K+1, \\ B(i,j) &= 1 \qquad \text{for some } j \quad \text{if} \ \ i \leq K, \end{aligned}$$

holds for some $2 \leq K \leq L$.

Define $k(\xi)$ by

$$k(\xi) = 1 - f_0(\xi)/d_{\max}.$$

By putting $s' = s - (\log\varepsilon + \sqrt{-1}\,\pi)/d_{\max}$ we have

$$-sf_\varepsilon + g_\varepsilon + \sqrt{-1}\,\pi = -s'f_\varepsilon + h_\varepsilon + k\log\varepsilon,$$

where

$$h_\varepsilon = \tilde{g}_\varepsilon + \sqrt{-1}\,\pi\, k + (\log\varepsilon + \sqrt{-1}\,\pi)\frac{(f_0 - f_\varepsilon)}{d_{\max}}.$$

By tending ε to the zero, it follows that

$$h_0 = \tilde{g}_0 + \sqrt{-1}\,\pi\, k,$$

hence we have

$$h_0(\xi) = \tilde{g}_0(\xi) \quad \text{for } \xi \text{ satisfying } B(\xi_0, \xi_1) = 1.$$

Thus $f_\varepsilon, h_\varepsilon, k$ satisfy the conditions required in Theorem 2.1.

Let $Z_\varepsilon(s)$ be the zeta function defined by using these $f_\varepsilon, h_\varepsilon, k$. Note that we have the relation

$$\zeta_\varepsilon(s) = Z_\varepsilon(s - (\log\,\varepsilon + \sqrt{-1}\,\pi)/d_{\max}).$$

On the other hand, Theorem 3.1 says that there exists $\varepsilon_0 > 0, s_0 \in \mathbf{R}$ and D_0 such that $Z_\varepsilon(s)$ has a pole in D_0, which implies that $\zeta_\varepsilon(s)$ is meromorphic in $D_\varepsilon = \{s = z + (\log\,\varepsilon + \sqrt{-1}\,\pi)/d_{\max}; z \in D_0\}$ and has a pole near $s_0 + (\log\,\varepsilon + \sqrt{-1}\,\pi)/d_{\max}$. It is evident that this pole of $\zeta_\varepsilon(s)$ stays in the domain where the singularities of $\zeta_\varepsilon(s)$ and $F_{D,\varepsilon}(s)$ coincide. Thus the existence of singularities of $F_{D,\varepsilon}(s)$ is proved.

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