Astérisque

ACTES DU COLLOQUE

DE

THÉORIE DE HODGE

Luminy, 1-6 Juin 1987

Comité d'organisation :
D. BARLET, H. ESNAULT, F. ELZEIN,
[†]J.L. VERDIER, E. VIEHWEG.

Jean-Louis VERDIER était coorganisateur du colloque de Théorie de Hodge et coéditeur de ce volume qui en rassemble quelques contributions.
Sa disparition accidentelle nous affecte profondément.
Nous dédions ces Proceedings à sa mémoire.

INTRODUCTION

Dans les années précédant la tenue du Colloque, diverses directions en "théorie de Hodge" avaient connu un développement spectaculaire : Cattani-Kaplan-Schmid et Kashiwara-Kawaï avaient démontré par des méthodes L^2 que la cohomologie d'intersection à valeur dans une variation de structure de Hodge polarisée sur une variété lisse kählérienne possède une structure de Hodge. A la demande des organisateurs, Cattani-Kaplan ont rédigé pour ces Proceedings une introduction au sujet. Looijenga et Saper-Stern avaient étendu ce résultat au cas des variétés localement symétriques pour la cohomologie d'intersection à coefficients constants. Une présentation en est donnée dans la contribution de Zucker.

Ces travaux ne peuvent être compris que dans le cadre général de la théorie de la cohomologie d'intersection et des faisceaux pervers, théorie dont un des fondateurs est O. Gabber. Sa présence à la conférence fut déterminante : ses remarques et critiques très constructives ont animé et clarifié beaucoup d'exposés.

Un des développements récents en théorie de Hodge, dont on mesure encore mal toutes les conséquences, est la théorie des modules de Hodge mixtes de M. Saito. Celle-ci devrait correspondre "philosophiquement" à celle des faisceaux pervers mixtes de Beilinson-Bernstein - Deligne-Gabber.

Dans un autre ordre d'idées, Deligne-Illusie avaient décomposé le complexe de De Rham d'une variété lisse en caractéristique $p > 0$ sous l'hypothèse de relèvement modulo p^2 de la variété, comblant ainsi un "trou" de la Géométrie Algébrique (exposé non rédigé). Diverses contributions aux "bords" de la théorie de Hodge furent présentées à la conférence : extensions de variations de structure de Hodge, cycles algébriques, théorème de comparaison analytique/algébrique, ainsi que des applications de la théorie de Hodge aux théorèmes d'annulation et aux singularités.

Different directions in "Hodge Theory" were developed in a spectacular way during the years before the conference : Cattani-Kaplan-Schmid and Kashiwara-Kawaï had shown via L^2-methods that the intersection cohomology with values in a polarized variation of Hodge structure on a kählerian manifold carries a Hodge structure. At the request of the organizers, Cattani-Kaplan contributed an introduction to the subject for these Proceedings. Looijenga and Saper-Stern had extended these results to the case of locally symmetric spaces for the intersection cohomology with constant coefficients. A presentation is given in Zucker's contribution.

These works can only be understood in the general framework of intersection cohomology and perverse sheaves, a theory one of whose founder was 0. Gabber. His participation to the conference was determinant, his very constructive remarks and criticisms enlived and clarified many talks.

One of the recent development in Hodge theory all of whose consequences are not yet understood is M. Saito's theory of mixed Hodge modules. This should correspond "philosophically" to the theory of mixed perverse sheaves of Beilinson-Bernstein-Deligne-Gabber.

In a different direction, Deligne-Illusie had decomposed the De Rham complex of a manifold in characteristic $p > 0$ under the assumption that the variety lifts modulo p^2, filling in a "gap" in Algebraic Geometry (contribution not written down). Several subjects at the border line of Hodge theory were presented at the conference : extensions of variations of Hodge structure, algebraic cycles, comparison theorems analytic versus algebraic, as well as applications of Hodge theory to vanishing theorems and to singularities.

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LISTE DES EXPOSÉS

Lundi 1er juin :

9 h 30   A. Kaplan : Degeneration of Hodge structures.

11 h 15   S. Zucker : Hodge theory with degenerating coefficients.

16 h 00   L. Illusie : Relèvements mod.p^2 et dégénérescence de la suite spectrale de Hodge.

17 h 45   H. Esnault : Complexe de De Rham logarithmique et théorème d'annulation.

Mardi 2 juin :

9 h 30   E. Cattani : L^2 and intersection cohomology.

11 h 15   M. Kashiwara : Hodge modules with normal crossing singularities.

16 h 00   M. Saito : Mixed Hodge modules.

17 h 45   K. Timmerscheidt : Mixed Hodge theory for unitary systems.

Mercredi 3 juin :

9 h 30   J. Steenbrink : Courbes polaires et cycles évanescents.

11 h 15   J. Carlson : Extension of variations of Hodge structures.

16 h 00   R. Buchweitz : Algebraic theory of algebraic cycles.

17 h 45   A. Beauville : Théorème d'annulation générique de Green–Lazarsfeld.

Jeudi 4 juin :

9 h 30   F. Elzein : Théorie de Hodge à coefficients dans une variation de structure de Hodge mixte.

11 h 15   Z. Mebkhout : Théorème de comparaison entre cohomologies de De Rham.

Vendredi 5 juin :

9 h 30   H. Hamm : Cohomologie à valeurs dans $Z_{(p)}$ et cohomologie de De Rham.

11 h 15   Usui : Degeneration of Kunev varieties.

14 h   C. Sabbah : Proximité évanescente.

15 h 30   D. Barlet : Symétrie de Hodge pour les racines du polynôme de Bernstein-Sato.

ASYMPTOTIC FILTRATION and POLES of $\int_X |f|^{2\lambda} \square$

par Daniel BARLET

Abstract : We consider an "Asymptotic filtration" $F^\bullet$, following A.N. Varchenko's idea, on the cohomology of the Milnor fiber of a germ $f : (\mathbb{C}^{n+1}, 0) \to (\mathbb{C}, 0)$ without assumption on the singularity of $\{f = 0\}$.

Then we give a relation between the nilpotent logarithm N of the unipotent part of the monodromy, thefiltrations $F^\bullet$ and $\overline{F}^\bullet$ and the poles of the meromorphic continuation of the distribution $\int |f|^{2\lambda} \square$

This contains estimates of the integral shift of the roots of the Bernstein-Sato polynomial of f in term of the action of N on the filtration $F^\bullet$.

EXTENSIONS OF VARIATIONS OF MIXED HODGE STRUCTURE

by James A. CARLSON and Richard M. HAIN

Abstract : We compute the Yoneda extension groups of unipotent variations of mixed Hodge structure. When the variations are $\mathbb{Q}$ and $\mathbb{Q}(p)$, we show that the group of extensions is the continuous Deligne-Beilinson cohomology of the fundamental group of the base space.

DEGENERATING VARIATIONS OF HODGE STRUCTURE

by Eduardo CATTANI and Aroldo KAPLAN

Abstract : This is a survey on some techniques and results on degerations of polarized, pure Hodge structures. We review Schmid's orbit theorems and their generalization to variations of several variables emphasizing the relationships : nilpotent orbit $\leftrightarrow$ polarized mixed Hodge structure and SL$_2$-orbit $\leftrightarrow$ polarized mixed Hodge structure split over $\mathbf{R}$. These are then used to describe the asymptotic behavior of a period mapping and to give estimates on the degeneration of the Hodge norm. We conclude with a brief sketch of the proof of the isomorphism between the L_2 and intersection cohomologies of a compact Kähler manifold with values on a local system underlying a polarizable VHS.

VANISHING AND NON VANISHING THEOREMS
by Hélène ESNAULT and Eckart VIEHWEG

Abstract : We present several applications of our vanishing theorem for cohomology groups of sheaves of twisted logarithmic differentials. We improve A. Sommese's vanishing theorem for k-ample sheaves. Moreover we generalize A.N. Varchenko's and our own description by global logarithmic differential forms of certain non trivial cohomology classes with values in local constant systems.

COHOMOLOGY WITH COEFFICIENTS IN Z(p) AND DE RHAM COHOMOLOGY : EXAMPLES
by Helmut A. HAMM

Abstract : If p is a prime number and X a smooth complete intersection over $Z(p)$ one can use a calculation of P. Deligne to show that the groups $H^r(X^{an} ; Z_{(p)})$ and $H^r(X, \Omega^{\bullet}_{X/Z_{(p)}})$ have the same rank and torsion. This result is extended to similar classes of examples ; here the De Rham complex is modified in view of the fact that the spaces in question may not be smooth nor proper.

"INTRODUCTION TO MIXED MODULES"
by Morihiko SAITO

Abstract : We introduce the notion of Mixed Hodge Modules which corresponds philosophically to that of mixed perverse sheaf of Beilinson-Bernstein-Deligne-Gabber, and explain some consequences of the stability of its bounded derived categories $D^bMHM(X)$ by the standard functors f_* , $f_!$, f^*, $f^!$, ψ_g , $\phi_{g,1}$, D, $\boxtimes$, $\otimes$, <u>Hom</u>, and the relation with Hodge cycle, Griffiths normal function, Deligne cohomology, admissible variation of mixed Hodge structure in the sense of Steenbrink-Zucker and Kashiwara, etc.

THE SPECTRUM OF HYPERSURFACE SINGULARITIES
by Joseph H.M. STEENBRINK

Abstract : The notion of the spectrum is generalized from isolated to arbitrary hypersurface singularities. A formula for the behaviour of the spectrum in Yomdin series of isolated singularities is conjectured and proven in some special cases : curve singularities and certain types of surface singularities and homogeneous ones. Examples are given of curve singularities with different imbedded topological types and equal spectra, and of surface singularities with equal spectra but different absolute topological type. A formula is given for the spectrum of a homogeneous singularity with one-dimensional singular locus.

TYPE I DEGENERATION OF KUNEV SURFACES
by Sampei USUI

Abstract : We determine here completely the main components of the degenerations of Kunev surfaces with finite local monodromy on the second cohomology. There are two series of degenerations. One is $\mathscr{S}_0 \cap \mathscr{T}_0 := \{$Kunev surfaces$\} \to \mathscr{S}_1 := \{$numerical K3 surfaces with one double fiber$\} \to \mathscr{S}_2 := \{$K3 surfaces$\}$, and the other is $\mathscr{S}_0 \cap \mathscr{T}_0 \to \mathscr{T}_1 \cap \mathscr{S}_0 := \{$elliptique surfaces with $P_g = q = 1\} \to \mathscr{T}_2 := \{$splitting abelian surfaces$\}$. In particular, this explains uniformly the appearence of positive dimensional fibers of the period maps of the surfaces $\mathscr{S}_0 \cap \mathscr{T}_0$, $\mathscr{S}_1$ and $\mathscr{T}_1 \cap \mathscr{S}_0$ by degeneration.

L²-COHOMOLOGY AND INTERSECTION HOMOLOGY OF LOCALLY SYMMETRIC VARIETIES, III
by Steve ZUCKER

Abstract : In 1987, two proofs were announced for the so-called Zucker Conjecture, giving a topological interpretation of the L^2-cohomology of a locally symmetric variety (Shimura variety). One, by E. Looijenga, is algebro-geometric in character ; the other, by L. Saper and M. Stern, uses no algebraic geometry at all! In this article, we give an exposition of both proofs, and discuss their relation to previous developments. As the title suggests, this is the third article in a series, each of which was different in tone : the first was a brief account of the author's results that led to the conjecture ; the second was written as a "last testament" on the subject... .