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ABSTRACTS OF TALKS

Exposé n° I

C. MARGERIN, Définitions, généralités, surfaces minimales dans $\mathbb{R}^n$ et S^n.

In this talk are rewied the most classical definitions results and examples in the theory of minimal submanifolds. Concerning minimal surfaces in Euclidean Spaces, the Weierstrass representation is given together with a classification of isometric minimal surfaces. Paragraph III is devoted to minimal surfaces in round spheres. A method to immerse isometrically and minimally any homogeneous space in a round sphere is described. The case of S^3 is presented with some more details allowing to prove that any compact Riemann surface can be minimally immersed into S^3 but that this is not true for $\mathbb{R}P^2$. Finally, criteria for minimality of orbits of isometry groups and of complex submanifolds of Kähler manifolds are given.

Exposé n° II

A.J. TROMBA, A proof of Douglas' theorem on the existence of disclike minimal surfaces spanning Jordan contours on $\mathbb{R}^n$.

An historical account of the Plateau problem is given together with a complete proof by a direct and elementary method of the fact that any minimum of the Dirichlet functional is conformal.

Exposé n° III

H.B. LAWSON, J.P. BOURGUIGNON, Formules de variations de l'aire et applications.

Where you can find, as a corollary of the First Variation Formula for the area functional, a monotonicity theorem and, in connection with the Second Variation Formula, a detailed study of the geometry of second fundamental forms, e.g. J. Simon's elliptic system, the Simon-Schoen-Yau a priori estimates, and the proof of the Bernstein conjecture in $\mathbb{R}^{n+1}$ ($n \geqslant 5$) which follows.

Exposé n° IV

W.K. ALLARD, Notes on the theory of varifolds.

Here is introduced the notion of "varifold", a natural measure-theoretic generalization of submanifolds in $\mathbb{R}^N$. The First Variation of such objects is defined, and a quasi-monotonicity formula for the mass is proved. This can be translated into a monotonicity formula for extremal varifolds which coincides with the classical one when the varifold is a smooth submanifold. From this and the Besicovitch lemma can be deduced an isoperimetric inequality.

The author goes on by presenting a sketch of proof for two fundamental results in the regularity theory for varifolds :

- the rectifiability theorem (cf.VIII) according to which, under very weak assumptions, a manifold can be described as a limit (in measure) of positive linear combination of C^1-manifolds ;
- the regularity theorem (cf IX) which gives for example a Hölder estimate of the variation of the tangent plane to a "nice" varifold.

Exposé n° V

D.B. O'SHEA, The Bernstein-Osserman-Xavier theorems.

This talk presents a review of the Nirenberg conjecture, a generalization of the Bernstein theorem according to which the set of oriented normals to a simply connected non planar minimal surface in $\mathbb{R}^3$ is dense in S^2. It also contains a more general result due to R. Osserman et S.S. Chern together with the striking result of F. Xavier thanks to which one knows that the image of the Gauss map of a complete non planar minimal surface in $\mathbb{R}^3$ "misses" at most 6 points in S^2.

Exposé n° VI

M.L. MICHELSOHN, Surfaces minimales dans les sphères.

It is a classical and elementary fact that any holomorphic map φ from a Riemann surface Σ in a Kählerian manifold X is minimal and that, π being a Riemannian submersion from X into a manifold Y, $\pi\circ\varphi$ is minimal as soon as φ is horizontal with respect to π. A converse due to E. Calabi is proven here in the case where $\Sigma = S^2$, $Y = S^{n-1} \subset \mathbb{R}^n$. One constructs a Riemannian submersion from a submanifold X of $\mathbb{C}P^N$ onto S^{N-1} and a holomorphic horizontal lift of φ. From this one can deduce that the image of a minimal immersion from S^2 into S^{n-1} is either contained in an equator, or has a "quantified" area $4\pi k$ for $k \geqslant \binom{m+1}{2}$ (Calabi-Barbosa).

Exposé n° VII

H.B. LAWSON, La classification des 2-sphères minimales dans l'espace projectif complexe.

A construction in holomorphic terms of minimal maps from the 2-sphere into

Euclidean spheres and in complex projective spaces is given. It is based on the consideration of isotropic subspaces which osculate the given map, hence generalizes the classical Weierstrass representation.

Exposé n° VIII

P. GAUDUCHON, Les immersions super-minimales d'une surface compacte dans une variété riemannienne orientée de dimension 4.

In this talk a twistorial construction of certain minimal immersions in a 4-dimensional manifold as holomorphic horizontal maps from the surface into one of the twistor spaces of the manifold is given. By specializing it to the Riemann sphere, one obtains a description of the minimal immersions of the 2-sphere respectively in S^4 and in $\mathbb{C}P^2$ as holomorphic horizontal curves respectively in $\mathbb{C}P^3$ and the flag manifold of $\mathbb{C}^3$.

Exposé n° IX

P. GAUDUCHON, La correspondance de Bryant.

As an extension of Exposé VIII, one describes a correspondance established by R. Bryant between twistor spaces of S^4 and $\mathbb{C}P^2$. Thanks to this geometric construction based on appropriate identifications, from minimal maps from any Riemann surface Σ into $\mathbb{C}P^2$ one can deduce minimal maps of Σ into S^4 , henceforth solving a classical problem of the theory of Riemann surfaces.

Exposé n° X

H.B. LAWSON Jr. Sous-variétés associatives et courbes holomorphes dans S^6 .

Here is studied the geometry of certain minimizing submanifolds of $\mathbb{R}^7$ and $\mathbb{R}^8$ connected with Cayley's octonions. These submanifolds give rise to minimal immersions of compact Riemann surfaces as pseudo-holomorphic curves of S^6 as shown by R. Bryant.

Exposé n° XI

B. CHARLET, The spherical Bernstein problem.

Any minimal embedding of S^2 into S^3 it known to be totally geodesic (cf. Exposé I). Here, one gets infinitely many minimal embeddings of S^{n-1} into S^n , $n = 4,5,6$ or 7, which are counter examples to the spherical Bernstein problem, since not totally geodesic. The reader will find a developped version of W.Y. Hsiang's ideas , looking for minimal embeddings of S^{n-1} into S^n which are invariant under a cohomogeneity 2 group action through the study of an ordinary differential equation on the orbit space.

Exposé n° XII

J.P. BOURGUIGNON, Sphères minimales d'après J. Sacks et K. Uhlenbeck

In this talk is given a survey of results due to J. Sacks, K. Uhlenbeck (and also to L. Lemaire, R. Schoen, S.T. Yau and M. Struwe) on the existence of minimal 2-spheres in a homotopy class of maps from S^2 to a general Riemannian manifold. In this context appears the phenomenon of loss of compactness in a conformally invariant variational problem by concentration of a solution at a point (the "bubbling off" phenomenon). Thanks to these results, minimal surfaces have been turned into powerful tools for geometric investigations.

Exposé n° XIII

A. EL SOUFI, Immersions minimales sphériques : restrictions sur la courbure et la codimension.

Here is studied the set of manifolds admitting a minimal immersion in a Euclidean sphere of radius 1. This study can be made precise if one controls Riemannian invariants as a lower bound on the scalar curvature and an upper bound for the diameter (in this way one can obtain an upper bound on the dimension of the sphere in which the immersion is full) or in the case of the standard metric on the sphere.

Exposé n° XIV

F. ALMGREN, Basic techniques of geometric measure theory.

The reader will find here many basic definitions in Geometric Measure Theory, all well illustrated and motivated. He will also find basic properties and a deformation theory which leads to isoperimetric inequalities and the existence of isomorphisms between homology (and homotopy) groups of some current spaces and the singular homology groups of the underlying spaces.

In a second paragraph, it is shown how lipschitz multiple valued functions can be used to approximate generalized surfaces (cf. theorem 2.11).

In the last paragraph the author sketches a proof of a compactness theorem for size bounded real currents.

Exposé n° XV

J.E. TAYLOR, Some crystalline variational techniques and results.

For an isotropic surface energy the equilibrium configuration is described by a minimal surface. When this energy is orientation-dependent (i.e. anisotropic), as for metals, further tools are here introduced to study the (local) structure of extrema. "Wulff shape" (the equilibrium configuration), "n-diagram" and "Labelled cycle" are defined in order to state some relations between the n-diagram of an energy functionnal and the structure of minimizing cones. Interaction with an external gravitational field is also considered. We end up with an a priori bound on the combinatorial complexity of any minimum.

Exposé n° XVI

R.L. BRYANT, Surfaces of mean curvature one in hyperbolic space.

In this talk are studied surfaces of mean curvature one in 3-dimensional hyperbolic space. They have a Weierstrass representation as minimal surfaces in Euclidean spaces do. By considering the Gauss maps of such surfaces, many of the "regularity" results for minimal surfaces can be carried over to those which are complete and have finite total curvature in 3-dimensional hyperbolic space.