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HODGE THEORY AND ARITHMETIC GROUPS

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I have been asked (with no objection on my part!) to give a unified account of the research program that began with [Z1], and which has continued with [Z2] and [Z3]. Here is a skeletal description. The objects of study in [Z1] are cohomology groups attached to a variation of Hodge structure; those of [Z3] are cohomology groups attached to a representation of a semi-simple Lie group (see [MM]). That there is a large common ground is the main point in [Z2]. A theme which has emerged throughout is the identification of L_2-cohomology groups on certain non-compact manifolds as some sheaf cohomology on suitable compactifications, where either of the two objects may be considered to be of primary interest. This sheaf cohomology is, in fact, intersection homology in our cases. Cheeger has obtained corresponding results for the L_2-cohomology of spaces with (iterated) conical singularities (see [Ch], [CGM]).

Without further ado, we begin the more detailed exposition.

1. Hodge theory with degenerating coefficients (HTDC). The results in [Z1] arose out of the need for a Hodge decomposition on the E_2 term of the Leray spectral sequence of certain morphisms of projective varieties. More precisely, we were interested, for application to a problem in algebraic geometry, in computing with the Hodge structure of the cohomology groups

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$H^1(\bar{S}, R^m\bar{f}_*\mathbb{C})$, where $\bar{f} : \bar{X} \to \bar{S}$ is a morphism of the non-singular projective variety $\bar{X}$ onto the smooth complete curve $\bar{S}$. Although $H^1(\bar{S}, R^m\bar{f}_*\mathbb{C})$ may be identified as a sub-quotient of $H^{m+1}(\bar{X}, \mathbb{C})$, and inherits a Hodge structure thereby, it became apparent that a construction of the Hodge structure <u>from a complex of sheaves on $\bar{S}$</u> was needed.

Let $f : X \to S$ denote the mapping obtained by deleting the (finitely many) singular fibers from $\bar{X}$ and their images from $\bar{S}$. With the help of the "local invariant cycle theorem" (see [Cℓ, (3.7)]), we can identify

(1) $$H^1(\bar{S}, R^m\bar{f}_*\mathbb{C}) \simeq H^1(\bar{S}, j_*\mathbb{W}),$$

where $j : S \to \bar{S}$ is the inclusion, and $\mathbb{W} = R^m f_*\mathbb{C}$. The essential property of $\mathbb{W}$ is that it is a <u>local system on</u> S <u>underlying a polarizable variation of Hodge structure of weight</u> m.

We should recall here what this last statement means. The sheaf $\mathbb{W}$ is the sheaf of germs of horizontal sections of a flat complex vector bundle $\mathcal{V}$ on S. The fiber V_s at the point $s \in S$ has a Hodge decomposition

(2) $$V_s = \bigoplus_{p+q=m} V_s^{p,q}.$$

These decompositions are subject to certain axioms, namely,

(3) a) The union of the $V_s^{p,q}$ forms a C^∞ sub-bundle $\mathcal{V}^{p,q}$ of $\mathcal{V}$.

b) One defines the <u>Hodge filtration</u> by

$$F_s^r = \bigoplus_{p \geq r} V_s^{p,q}.$$

The union of the F^r_s forms a holomorphic sub-bundle $\mathcal{F}^r$ of $\mathcal{V}$.

c) The flat differentiation of any local section of $\mathcal{F}^r$ with respect to to a holomorphic vector field gives a local section of $\mathcal{F}^{r-1}$.

d) There is a flat non-degenerate pairing on $\mathbb{W}$ which is alternately positive- and negative-definite on the successive terms of (2). (By correcting for these signs, one puts an Hermitian metric on $\mathcal{V}$.)

In the case where $S = \overline{S}$ (which, in the geometric situation from the beginning of this section, is the case where there are no singular fibers), but where S may be of arbitrary dimension, the desired construction had been carried out by Deligne, and it goes as follows. We have the usual de Rham quasi-isomorphism

$$\mathbb{W} \approx \Omega^{\cdot}_S \otimes_{\mathbb{C}} \mathbb{W} =: \Omega^{\cdot}_S(\mathbb{W}). \tag{4}$$

The right-hand side has a decreasing filtration $F^{\bullet}$ defined by

$$F^r(\Omega^p_S(\mathbb{W})) = \Omega^p_S \otimes_{\mathcal{O}_S} \mathcal{F}^{r-p}, \tag{5}$$

where we identify, as usual, the bundle $\mathcal{F}^{r-p}$ with its sheaf of holomorphic sections. By (3c), each $F^r(\Omega^{\cdot}_S(\mathbb{W}))$ is a sub-complex of $\Omega^{\cdot}_S(\mathbb{W})$. Then $H^i(S, \mathbb{W})$ has a natural Hodge structure of weight $m + i$, in which the Hodge filtration is induced by $F^{\bullet}$. In other words, the filtered complex $\Omega^{\cdot}_S(\mathbb{W})$ is a cohomological Hodge complex, in the sense of [D2, (8.1.2)]. Formula (5) is notable for its "mixing" of the Hodge filtration of $\Omega^{\cdot}_S$ with that of $\mathcal{V}$. This mixing is forced if one wants the resulting Hodge structure to be induced from $H^{\cdot}(\overline{X}, \mathbb{C})$ in the geometric situation.

In the general case, with $\dim S = 1$, we have extended (4) to provide a resolution of $j_*\mathbb{W}$ by locally free $\mathcal{O}_{\overline{S}}$-modules, such that (5) extends as well. The proof that $F^\cdot$ induces a Hodge structure of weight $m + i$ on $H^i(\overline{S}, j_*\mathbb{W})$ uses the L_2-cohomology* on S with respect to a Poincaré metric. The term "degenerating coefficients" was used because the dimension of the stalk of $j_*\mathbb{W}$ can drop at the points of $\overline{S} - S$. With a little hindsight, we note that

$$j_*\mathbb{W} \approx \tau_{\leq 0} Rj_*\mathbb{W} ,$$

the middle intersection homology complex of $\overline{S}$ with coefficients in $\mathbb{W}$. Thus, our construction provides a complete description of a Hodge structure on some intersection homology, beyond the classical Hodge theory of compact Kähler manifolds. Also, we deduce as a corollary the existence of a natural mixed Hodge structure on $H^\cdot(S, \mathbb{W})$, a result conjectured by Deligne [D2, p. 7].

The proof that the L_2-cohomology above coincides with intersection homology depends heavily on the asymptotics of the Hodge metric (3d) that follow from Schmid's SL_2-orbit theorem in one variable [Sc]. Since there is still no generalization of this result to several variables, we are unable at the present time to even attempt to generalize our theory to higher dimensional $\overline{S}$ (except in cases where $\overline{S} - S$ is smooth).

*For an exposition on L_2-cohomology, the reader is referred to [CGM], [Z3], or [Z4].

In the preceding paragraphs, there is a hint that there might be a relation between HTDC and automorphic forms. In [Z1, §12], we show that one can give an interpretation of the Eichler-Shimura isomorphism [Sh, p. 302], which relates the so-called parabolic cohomology and cusp forms for Fuchsian groups, as an example of the Hodge structure on some $H^1(\bar{S}, j_* \mathbb{W})$. In some cases, these groups come from a geometric situation (elliptic modular surfaces); we have generalized the Eichler-Shimura isomorphism to general elliptic surfaces in [CZ, § 3].

2. <u>The</u> L_2-<u>cohomology of arithmetic groups</u>. Several years ago, David DeGeorge encouraged me to read [MM] after I described HTDC to him. We begin with a description of the relevant part of [MM].

Let G be a semi-simple real algebraic group, K a maximal compact subgroup of G, and Γ a discrete subgroup of G that acts freely on the symmetric space $M = G/K$. We put $S = \Gamma\backslash M$. Let V be a finite dimensional representation of G. One defines a local system $\mathbb{W}$ on S by

$$\mathbb{W} = \Gamma\backslash(M \times V)$$

(where V is given the discrete topology). Since M is contractible, one obtains an isomorphism between cohomology groups:

$$H^\bullet(S, \mathbb{W}) \simeq H^\bullet(\Gamma, V) . \tag{6}$$

For certain G, there is on M a G-invariant (hence Kähler) complex structure, which therefore descends to S (the <u>Hermitian</u> cases). For such S, we have the holomorphic de Rham complex, $\Omega^\bullet_S(\mathbb{W})$, as before. One defines a second filtration (la filtration bête):

$$B^r\Omega^p_S(\mathbb{W}) = \begin{cases} \Omega^p_S(\mathbb{W}) & \text{if } p \geq r \\ 0 & \text{if } p < r, \end{cases}$$

i.e.,

$$B^r\Omega^\bullet_S(\mathbb{W}) = (F^r\Omega^\bullet_S) \otimes_{\mathbb{C}} \mathbb{W}.$$

It is shown, in effect, in [MM] that if S is compact, $B^\bullet$ induces a Hodge structure on $H^i(S, \mathbb{W})$ for all i.

The preceding does not hold in the context of HTDC, and [Z2] was the result of an attempt to understand how such a result could be true for Hermitian locally symmetric spaces. What we discovered was that $\mathbb{W}$ underlies a (locally) homogeneous variation of Hodge structure, and thus $H^i(S, \mathbb{W})$ has <u>two</u> Hodge structures: one induced by $F^\bullet$, the other by $B^\bullet$. Moreover, the two are mutually compatible, so we get a double Hodge decomposition. The use of $F^\bullet$ enabled us to begin a generalization of the Eichler-Shimura isomorphism to higher dimensions, but we postpone the discussion of this until later.

The cohomology groups (6) are of especial interest when G is defined over $\mathbb{Q}$ and Γ is an arithmetic subgroup, because of applications in topology and number theory. The space S is then non-compact in general, and one gets Hodge decompositions only for the L_2-cohomology,* $H^\bullet_{(2)}(S, \mathbb{W})$, which we take with respect to natural metrics. The spaces of L_2 harmonic forms are finite

*Hodge decompositions are induced from decompositions of spaces of L_2 harmonic forms. A priori, one must take <u>reduced</u> L_2-cohomology, in case the range of d is not closed. It seems that this is never an issue for the class of spaces S that is being considered here.

dimensional, and are "computable" in terms of the representation theory of $L_2(\Gamma\backslash G)$ (see [BW], [B2]). For this to have any bearing on the cohomology groups (6), one must understand the mapping

$$H^{\cdot}_{(2)}(S, \mathbb{W}) \to H^{\cdot}(S, \mathbb{W}) \tag{7}$$

(see [B1] for an earlier treatment of this question), or better, know exactly what it is that L_2-cohomology computes from, say, a topological viewpoint.

When Γ is an arithmetic group, there is a normal projective variety S^*, the Baily-Borel-Satake compactification [BB], which contains S as a Zariski-open subset. Let $\mathcal{L}^{\cdot}_{(2)}(S^*, \mathbb{W})$ denote the L_2 complex of sheaves on S^* with coefficients in $\mathbb{W}$ (see [Z3, (1.16)] or [Z4]). We have shown in [Z3, § 3], for examples where S^* has only isolated singularities, that

$$H^{\cdot}_{(2)}(S, \mathbb{W}) \simeq IH^{\cdot}(S^*, \mathbb{W}), \tag{8}$$

where $IH^{\cdot}$ denotes intersection homology with middle perversity [GM]. This is achieved by proving that

$$\mathcal{L}^{\cdot}_{(2)}(S^*, \mathbb{W}) \approx \tau_{\leq n-1} Rj_* \mathbb{W}, \tag{9}$$

where n is the complex dimension of S, and $j : S \to S^*$ is the inclusion. We have conjectured that (8) is true in general (and will be proved by showing the appropriate generalization of (9); see [GM]).

According to Barry Mazur, the following consequence of (8) was not previously known. Let S be a Hilbert modular variety of dimension n.

Here, M is the product of n copies of the upper half-plane. Then above dimension n, the homology $H_\cdot(S^*, \mathbb{C})$ is in "Poincaré duality" with the algebra generated by the Poincaré metric volume forms of the factors of M [Z4].

It is time for a discussion of the (locally) homogeneous variations of Hodge structures of [Z2]. Because the construction is homogeneous, we may ignore Γ. For simplicity, we assume that M is irreducible as a symmetric space. We decompose V into weight spaces with respect to the (one-dimensional) center Z of K:

$$V = \oplus V_\chi .$$

Without specifying the indexing, which is a bit artificial anyway, we define the bundles $\mathcal{V}^{p,q}$ of (3a) to be the equivariant sub-bundles of $\mathcal{V}$ determined by the V_χ's.

In [Z1, § 12], the Eichler-Shimura isomorphism is deduced from the determination of the cohomology sheaves of the successive quotients of the $F^\cdot$ filtration:

$$\mathcal{H}^k \mathrm{Gr}_F^r \Omega_S^\cdot(\mathbb{W}) . \tag{10}$$

It is this much that we have been able to generalize. We observe that the terms of the complex $\mathrm{Gr}_F^r \Omega_S^\cdot(\mathbb{W})$ are equivariant bundles determined by representations of K, and that Z acts on all of them with a single weight, which we denote by χ_r. Moreover, if $r \neq r'$, then $\chi_r \neq \chi_{r'}$. From this, it is not hard to deduce that (10) is the equivariant bundle determined by the

χ_r weight space of the Lie algebra cohomology group $H^k(\underline{p}^+, V)$.

In the cases where it is known (and, of course, conjecturally in general), the isomorphism (8) imparts a Hodge structure to $IH^i(S^*, \mathbb{W})$. For applications, it would be useful to have a cohomological Hodge complex that restricts to $(\Omega^{\cdot}_S(\mathbb{W}), F^{\cdot})$ on S, preferably one that admits a description that does not mention L_2 (cf. [Z1, (4.1), (9.1)]). This has not yet been constructed beyond the case $G = SL(2, \mathbb{R})$, though it is clearly the next order of business.

3. Some calculations. I have found the following calculations instructive, and I hope the reader can benefit from them. (The number "1" or "2" before each paragraph selects the context and notation as those of one of the preceding sections.)

1A. It is an interesting and elementary exercise to see why HTDC works for $\mathbb{W} = \mathbb{C}$ when one takes $S \nsubseteq \overline{S}$, i.e., how one recovers the classical Hodge theory of $\overline{S}$ from L_2-cohomology on S with respect to a Poincaré metric. This is left to the reader.

2A. The non-fineness of $\mathcal{L}^{\cdot}_{(2)}(\overline{S}, \mathbb{W})$ on the Borel-Serre compactification $\overline{S}$ of S. In the case that M is the upper half-plane $\{(x, y) \in \mathbb{C} : y > 0\}$, the corner [BS, § 5] associated to the standard parabolic subgroup P of upper-triangular matrices is obtained by adjoining a line $y = \infty$, which we denote by L. Distinguished neighborhoods of L are just sets of the form

$$U_c = \{(x, y) \in \mathbb{C} : y > c\} .$$

There is a corresponding circle $(\Gamma \cap P)\backslash L$, which forms a component of the boundary of $\bar{S}$, with collars $(\Gamma \cap P)\backslash U_c$ for c sufficiently large. Let $z \in L$ represent a point of the boundary. We will show that the differential of any cut-off function f for a small neighborhood of z in M is of unbounded Riemannian norm, and therefore $\mathcal{L}^{\bullet}_{(2)}(\bar{S}, \mathbb{W})$ is not fine* (see [Z4, (6)]).

The Poincaré metric of the upper half-plane has the formula

$$(dy/y)^2 + y^{-2}(dx)^2 . \tag{11}$$

Suppose that f is equal to zero outside the set

$$\{(x, y) : a \leq x \leq b, \ y \geq c\} ,$$

and suppose that f extends smoothly to L. Then

$$|df|^2 \geq |(\frac{\partial f}{\partial x})dx|^2 .$$

We see from (11) that $|dx| = y$, so $|df|$ blows up along part of L like y. That any f must have unbounded differential follows by a standard smoothing argument, which we omit here.

2B. <u>The L_2-cohomology of a warped product</u>. Let M and N be Riemannian manifolds, with metric tensors g_M and g_N respectively. By definition, a warped product metric on $M \times N$ is a metric of the form

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*The point is that the definition of $\mathcal{L}^{\bullet}_{(2)}$ imposes an L_2 condition on both a form <u>and its exterior derivative</u>.

(12) $$g_M + w^2 g_N ,$$

where w is a positive function on M, e.g. (11). We let $M \times_w N$ denote the resulting Riemannian manifold. Under suitable hypotheses, one can compute the L_2-cohomology of $M \times_w N$ in terms of the L_2-cohomology of N and weighted L_2-cohomology on M by a generalized Künneth formula. The proof of this is a simplified version of the argument used to arrive at (8). See [Z3, § 2] for details.

2C/1B. L_2-cohomology with respect to different metrics. Let M be the ball in $\mathbb{C}^n$, an example of an Hermitian symmetric space, with $G = SU(n,1)$. Let S be a quotient of M by an arithmetic subgroup, and S^* its Baily-Borel-Satake compactification. Let $\mathbb{W}$ be a local system on S underlying a locally homogeneous variation of Hodge structure, equipped with its Hodge metric.

There is a non-singular model $\overline{S}$ of S^*, in which each point of $S^* - S$ is replaced by an Abelian variety (see [He]; this is a trivial instance of the compactifications introduced in [AMRT]). We have considered two metrics on S:

i) The G-invariant Bergman metric. We have in this case by (8)

$$H^{\bullet}_{(2)}(S, \mathbb{W}) \simeq IH^{\bullet}(S^*, \mathbb{W}) = \mathbb{H}^{\bullet}(S^*, \tau_{\leq n-1} Rj_* \mathbb{W}) .$$

ii) A Poincaré metric relative to $\overline{S}$ (see [Z1, § 3]). We then have, by a mild generalization of HTDC,

$$H^{\bullet}_{(2)}(S, \mathbb{W}) \simeq IH^{\bullet}(\overline{S}, \mathbb{W}) = H^{\bullet}(\overline{S}, i_*\mathbb{W}) ,$$

where $i : S \to \overline{S}$ is the inclusion.

In local coordinates on $\overline{S}$, S is given as $\Delta^* \times \Delta^{n-1}$, and the two metrics can be compared as follows: (ii) presents S as being locally quasi-isometric to the Riemannian product of Δ^* (with the Poincaré metric of a larger punctured disc, so that the only singularity is at the origin) and Δ^{n-1} (with the Euclidean metric), whereas (i) presents S locally as a warped product of these two spaces, with $w = (-\log |u|)^{-1/2}$ in (12) for $u \in \Delta^*$.

2D. <u>Examples of Hodge norm asymptotics</u>. The locally homogeneous variations of Hodge structure are so explicit that it should be possible to use them to generate, without much difficulty, examples of Hodge norm asymptotics. These examples should shed some light on the nature of the general situation.

We carry out this calculation here in a case that is perhaps too simple, but which illustrates the method clearly. Let S be a Hilbert modular surface associated to the real quadratic number field $\mathbb{F}$. In [Hi], a resolution of singularities $\overline{S}$ of S^* is constructed such that $\overline{S} - S$ is a union of smooth curves crossing transversally. (This also fits into the general framework of [AMRT].)

Explicitly, let $\mathbb{C}^+$ denote the upper half-plane. The symmetric space M is, in this case, just $\mathbb{C}^+ \times \mathbb{C}^+$; let $\varphi : \mathbb{C}^+ \times \mathbb{C}^+ \to S$ be the canonical mapping. One embeds the product of punctured discs, $\Delta^* \times \Delta^*$, in S by projecting via φ multi-valued mappings of the form

$$(13)\qquad \begin{cases} z_1 = A\lambda(t_1) + B\lambda(t_2) \\ z_2 = A'\lambda(t_1) + B'\lambda(t_2) \end{cases}$$

(for sufficiently small t_1 and t_2), where $\lambda(t) = (2\pi i)^{-1}\log t$, A and B are suitable totally positive elements of $\mathbb{F}$, and "prime" denotes the Galois conjugation of $\mathbb{F}$ (see [Hi, p. 207]). Put $x_j = \mathrm{Re}\, z_j$ and $y_j = \mathrm{Im}\, z_j$ for $j = 1, 2$, and let $a = (y_1 y_2)^{1/4}$ and $b = (y_1 y_2^{-1})^{1/4}$. We have from (13)

$$(14)\qquad \begin{cases} y_1 = A|\lambda(|t_1|)| + B|\lambda(|t_2|)| \\ y_2 = A'|\lambda(|t_1|)| + B'|\lambda(|t_2|)| \end{cases}$$

The group G is the product of two copies of $SL(2, \mathbb{R})$. Let

$$V = \mathrm{Symm}^{\ell}(\mathbb{C}^2) \otimes \mathrm{Symm}^{m}(\mathbb{C}^2)\,,$$

an irreducible representation of G in the usual way. The local system $\mathbb{W}$ is trivial on sets represented by subsets of M defined by inequalities $a > c >> 0$ and b, x_1, x_2 are restricted to lie in sufficiently small intervals. From (14), one sees that products of sectors in $\Delta^* \times \Delta^*$ are contained in such sets. The flat section determined by the element

$$e_{j,k} = \begin{bmatrix}1\\0\end{bmatrix}^{j}\begin{bmatrix}0\\1\end{bmatrix}^{\ell-j} \otimes \begin{bmatrix}1\\0\end{bmatrix}^{k}\begin{bmatrix}0\\1\end{bmatrix}^{m-k}$$

of V has length asymptotic to $a^{2j-\ell+2k-m}$ (cf. [Z1, p. 460], [Z3, (3.7)]). Using (14), we see that this is equivalent to

$$|e_{j,k}| \sim (|\lambda(t_1)| + |\lambda(t_2)|)^{j+k-(\ell+m)/2} .$$

Similar calculations can be carried out for bigger symmetric spaces, by using the compactifications constructed in [AMRT].

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