Astérisque

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Astérisque, tome 87-88 (1981), p. 97-108

<http://www.numdam.org/item?id=AST_1981__87-88__97_0>

DESINGULARIZATIONS OF ORBITS OF CONCENTRATORS

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1. Introduction.

1.1 Let G be a connected reductive algebraic group over an algebraically closed field K. Let V be an affine G-variety and let X be a closed subvariety of V which is stable under the action of a parabolic subgroup P of G. We form the contracted product $Z := G \times^{P} X$ which is the quotient of $G \times X$ under the P-action given by $p(g,x) = (gp^{-1}, px)$. Since G/P is a projective variety, the morphism $Z \longrightarrow V$ which sends the class of (g,x) to the element gx, is proper. So the image $Y = GX$ is a closed subvariety of V. We have a proper surjective morphism $f: Z \longrightarrow Y$, which is called the collapsing morphism, cf. [5].

Definition. X is said to be P-resolute, if $f: Z \longrightarrow Y$ is birational; that is, if Y has an open dense subset U such that f induces an isomorphism between $f^{-1}U$ and U.

Remark. Usually Y has singularities, even if X is smooth. Then Z is also smooth. So, if X is P-resolute, $f: Z \longrightarrow Y$ is a desingularization of Y.

1.2 In this paper we give a method to determine some P-resolute subvarieties in V. This method is based on the theory of optimal concentration of Kempf [6] and Rousseau [8], see also [2]. In [3] we observed that equality of the optimal concentrator is an equivalence relation with nice geometric behaviour. In particular, the equivalence classes are locally closed and, if char(K) = 0, the closure of an equivalence class is P-resolute for some parabolic group P. In [4], after a quick introduction to the theory, we investigated the possibilities of scheme-theoretic generalizations.

The innovation in this paper is a more flexible objective of the concentration.

In [3] the points are concentrated on a fixed base point. In [6] and [4] the points are concentrated on a G-invariant subvariety. Here we consider concentration on a finite number of subvarieties with different lower bounds on the speed of concentration. These data are collected in the so-called objective. In this way we get more P-resolute subvarieties. In [3] and [4] we only considered points of the concentrated cone N(V). This set clearly depends on the objective. Here we admit a formally concentrating co-weight ∞, so that the non-concentrated points are no longer exceptional. In terms of 1.1 this concerns the trivial isomorphism $G \times^G V \longrightarrow V$, but there is no reason to exclude this case.

Since the innovation penetrates into the technical details of the two main proofs, we have given the proofs again. Some of the minor arguments have not been repeated. For these we refer to [3] and [4]. Nevertheless we hope that the present text can be read independently. The theory is exposed in the sections 2 and 3. In section 4 we introduce some concepts which may be indispensable for a systematic analysis of the possible stratifications. We give an example in a module over a group of type B_2.

I am greatly indebted to Tadeusz Józefiak and the Polish Academy of Sciences for the invitation to visit the Toruń Conference and for the stimulating request to prepare a paper for the proceedings.

2. Optimal concentration.

2.1 In this section G is a linear afgebraic group over an algebraically closed field K, acting on an affine variety V. The multiplicative group K^* of the field K is considered as an algebraic group and as an open subvariety of the affine line K.

Let $Y(G)$ be the set of the one-parameter subgroups $\mu: K^* \longrightarrow G$. The set $M(G)$ of the "finite co-weights" is defined as the quotient $(Y(G) \times \mathbb{N})/\sim$ where $(\mu,n)\sim(\nu,m)$ means that $\mu(t^n) = \nu(t^m)$ for every $t \in K^*$. If T is a torus, $Y(T)$ is a free $\mathbb{Z}$-module and $M(T) = Y(T) \otimes_{\mathbb{Z}} \mathbb{Q}$. The interior action $\mathrm{int}(g)h = ghg^{-1}$ of G induces an action of G on $M(G)$. An application $q: M(G) \longrightarrow \mathbb{Q}$ is called a norm if we have

a) $q(\mathrm{int}(g)\lambda) = q(\lambda)$ for every $g \in G$, $\lambda \in M(G)$.

b) If T is a torus the restriction of q is a positive definite quadratic form on the vector space $M(T)$.

By [7] p.58, norms exist. A different construction is given in [4] 2.3. For convenience we define the completion $M(G)_c$ to be the union $M(G) \cup \{\infty\}$. The elements of $M(G)_c$ are called co-weights. The action of G is extended by $\mathrm{int}(g)\infty = \infty$ for every $g \in G$. Every norm q is extended by $q(\infty) = \infty$.

2.2 Let $\mu \in Y(G)$ and $v \in V$. We say that $\lim \mu(t)v = w$ if there is a morphism of varieties $h: K \longrightarrow V$ with $h(0) = w$ and $h(t) = \mu(t)v$ whenever $t \neq 0$. Let C be a G-invariant closed subvariety of V. Following [7] we define the speed of concentration $s_\mu(v,C)$ to be the dimension of the co-ordinate ring of the affine scheme $h^{-1}C$ as a vector space over K. So we have $s_\mu(v,C) = +\infty$ if and only if $v \in C$. We have $s_\mu(v,C) = 0$ if and only if $w \notin C$. If the limit does not exist, we define $s_\mu(v,C) := -\infty$. If $\lambda \in M(G)$, we can define $s_\lambda(v,C) := m^{-1} s_\mu(v,C)$ where (μ,m) is an arbitrary element of the equivalence class λ. If $\lambda = \infty$ in $M(G)_c$, we define $s_\lambda(v,C) := +\infty$.

2.3 An objective γ in V of rank r is defined to be a triple (C_*, c_*, q) where C_* is a sequence of G-invariant closed subvarieties $C_1, \ldots C_r$, and c_* is a sequence of rational numbers $c_1,\dots c_r \geq 0$, and q is a norm on $M(G)$, see 2.1. We fix an objective γ. If $\lambda \in M(G)_C$, the concentrator $V(\lambda)_\gamma$ is defined to consist of the points $v \in V$ such that $s_\lambda(v,C) \geq c_i$ for every index i. Let X be a subset of V. We define the optimal value

$$|X|_\gamma := \inf\{q(\lambda) \mid \lambda \in M(G)_C,\ X \subset V(\lambda)_\gamma\}$$

and the optimal class

$$\Lambda(X)_\gamma := \{\lambda \in M(G)_C \mid X \subset V(\lambda)_\gamma,\ q(\lambda) = |X|_\gamma\}.$$

2.4 Proposition. $\Lambda(X)_\gamma$ is non-empty. If T is a torus in G then $M(T)_C$ contains at most one point of $\Lambda(X)_\gamma$.

The proof is given in 2.7. It uses two standard lemmas.

2.5 Lemma (quadratic programming) Fix rational numbers $c_0, c_1, \dots c_r$. Let E be a finite dimensional rational vector space with dual vector space E^*. Let R_* be a sequence of finite subsets $R_0, R_1, \dots R_r$ of E^*. Let $q: E \longrightarrow \mathbb{Q}$ be a positive definite quadratic form. Assume that

$$D(R_*) := \{x \in E \mid \forall i : u \in R_i \Rightarrow \langle u,x\rangle \geq c_i\}$$

is a non-empty set. Then there is a unique $d \in D(R_*)$ with $q(d) \leq q(x)$ for every $x \in D(R_*)$.

2.6 Lemma (linearization). There is a G-module V' with a sequence of submodules $C'_1,\dots C'_r$, and a G-equivariant closed immersion $j: V \to V'$ such that C_i is the schematic inverse image $j^{-1}(C'_i)$ for every index i.

2.7 Proof of 2.4. If $|X|_\gamma = \infty$, then clearly $\Lambda(X)_\gamma = \{\infty\}$. So we assume that $|X|_\gamma < \infty$. By lemma 2.6 we may assume that V is a G-module and that the subvarities $C_1,\dots C_r$ are submodules of V.

Choose a maximal torus T of G. The character group of T is the dual $Y(T)^*$ of the free $\mathbb{Z}$-module $Y(T)$. So it is a lattice in the dual vector space $M(T)^*$ of $M(T)$. The torus T induces on a G-module W a weight space decomposition $W = \Sigma W_\pi$ with $\pi \in Y(T)^*$. If F is a subset of W, let R(F) be the smallest subset R of $Y(T)^*$

such that F is contained in $\Sigma_{\pi \in R} W_\pi$.

Let F be a subset of V. Let C_0 denote the zero submodule of V. If $0 \leq i \leq r$, put $R_i(F) := R(F_i)$ where F_i is the image of F in the quotient module V/C_i. By an analysis of the speed of concentration, one obtains

$$\{\mu \in M(T) \mid F \subset V(\mu)_\gamma\} = D(R_*(F))$$

Here we use the notation of 2.5 with $E = M(T)$, and $c_0 = 0$, and $c_1, \dots c_r$ and q as given in the objective γ. The index $i = 0$ with $R_0(F) = R(F)$ and $c_0 = 0$ had to be introduced in order to quarantee convergence.

In the definition of $|X|_\gamma$ we substitute $\lambda = \mathrm{int}(g)\mu$ with $\mu \in M(T)$ and $g \in G$. Then we have $q(\lambda) = q(\mu)$, and $X \subset V(\lambda)_\gamma$ if and only if $g^{-1}X \subset V(\mu)_\gamma$. It follows that

$$|X|_\gamma = \inf \{q(\mu) \mid \mu \in D(R_*(g^{-1}X)),\ g \in G\}$$

Since each $R_i(g^{-1}X)$ is contained in the finite set R(V), lemma 2.5 implies the existence of $h \in G$ and $\mu \in D(R_*(h^{-1}X))$ with $q(\mu) = |X|_\gamma$. Then we have $\mathrm{int}(h)\mu \in \Lambda(X)_\gamma$. The second part of 2.4 is now also clear.

3. Stratification and geometry.

3.1 From now we assume that the group G is connected and reductive. Recall that V is an affine G-variety and that γ is an objective in V. Let $\lambda \in M(G)_c$. Using the interior action of G on itself we define $G(\lambda)$ to consist of the points $g \in G$ with $s_\lambda(g,G) = \infty$. It is a parabolic subgroup of G and it stabilizes $V(\lambda)_\gamma$. The equivalence relation $\sim$ on $M(G)_c$ is defined by $\lambda \sim \mu$ if and only if $\mu = \mathrm{int}(g)\lambda$ with $g \in G(\lambda)$. If $\lambda \sim \mu$ then $V(\lambda)_\gamma = V(\mu)_\gamma$ and $G(\lambda) = G(\mu)$. Let X be a subset of V. The class $\Lambda(X)_\gamma$ is clearly a union of equivalence classes in $M(G)_c$. Now 2.4 and [3] 2.5 (b) together imply the

Theorem of Kempf [6] and Rousseau [8]. The class $\Lambda(X)_\gamma$ is one equivalence class in $M(G)_c$.

3.2 It follows that we may define the Kempf group $P(X)_\gamma := G(\lambda)$ and the γ-saturation $S(X)_\gamma := V(\lambda)_\gamma$ where λ is an arbitrary element of $\Lambda(X)_\gamma$. It is clear that $\Lambda(S(X)_\gamma)_\gamma = \Lambda(X)_\gamma$. As in [3] 2.8 one proves that

(*) $\quad P(X)_\gamma = \{g \in G \mid gX \subset S(X)_\gamma\}$

In particular $P(X)_\gamma$ is the normalizer of $S(X)_\gamma$. Since $S(X)_\gamma$ is closed in V, the set $GS(X)_\gamma$ is also closed by 1.1. If $x \in V$ we write $|x|_\gamma$ instead of $|\{x\}|_\gamma$, etc.. The set of values $|x|_\gamma$ is finite. If $s \in \mathbb{Q} \cup \{\infty\}$, the locus of the points $x \in V$ with $|x|_\gamma < s$ is closed. See [3] 2.9.

3.3 Points x and y of V are said to be γ-equivalent if they have the same optimal class (or equivalently: the same γ-saturation). The equivalence class of x is called the γ-blade $[x]_\gamma$ of x. It consists of the points $y \in S(x)_\gamma$ with $|y|_\gamma = |x|_\gamma$. It is an open subset of the closed set $S(x)_\gamma$. So if $S(x)_\gamma$ is irreducible, then it is the closure of $[x]_\gamma$.

The set $G[x]_\gamma$ is called the γ-stratum of x. It consists of the points $y \in GS(x)_\gamma$ with $|y|_\gamma = |x|_\gamma$. It is an open subset of the closed set $GS(x)_\gamma$. The variety V is a finite disjoint union of the γ-strata.

3.4 Theorem. Let X_0 be a γ-blade in V with γ-stratum $U = GX_0$. Let X be the closure of X_o. Let P be the normalizer of X in G.

a) If $x \in X_o$ then $P = P(x)_\gamma$; so P is a parabolic subgroup.

b) Let $f\colon G \times^P X \longrightarrow GX$ be the collapsing morphism. The set U is open and dense in the closed set GX, and $f^{-1}U = G \times^P X_o$.

c) The restriction $f\colon f^{-1}U \longrightarrow U$ is a universal homeomorphism.

d) Assume char(K) = 0. Then $f\colon f^{-1}U \longrightarrow U$ is an isomorphism. The set X is a P-resolute subset of V.

Proof. a) Put $X_1 := S(x)_\gamma$ and $P_1 := P(x)_\gamma$. We have $X_o \subset X \subset X_1$. By 3.2(*) it follows that $P \subset P_1$. Since the function $|?|_\gamma$ is G-invariant and P_1 stabilizes X_1, it stabilizes X_o by 3.3. Therefore P_1 stabilizes X. This proves that $P_1 = P$.

b) Since U is open in GX_1 it is also open in GX. It is clearly dense in GX. The set GX is closed by 1.1. If $g \in G$ and $x \in X$ and $gx \in U$, then we have $x \in X_o$ by 3.3. This proves that $f^{-1}U = Gx^PX_o$.

c) The morphism $f\colon f^{-1}U \longrightarrow U$ is clearly proper and surjective. Let (g,x) and (h,y) in $G \times X_o$ satisfy $gx = hy$. As $h^{-1}gx \in X_o$, it follows with (a) and 3.2(*) that $h^{-1}g \in P$. So (g,x) and (h,y) represent the same point of $G x^P X$. This proves that the restriction of f is injective. So by [1] III 4.4.2 and IV 2.4.5, it is a universal homeomorphism.

d) As in the proof of [3] 4.7 and [4] 3.6 it suffices to prove that the restriction of f is unramified. By lemma 2.6 we may assume that V is a G-module and that the subvarieties C_i are submodules. Now it suffices to prove the Lie algebra analogue of 3.2(*):

3.5 <u>Lemma</u>. Assume char(K) = 0. Let V be a G-module and $C_1, \dots C_r$ be submodules. Let $v \in V$. The Lie algebra $\underline{\underline{p}}$ of $P(v)_\gamma$ consists of the elements X of the Lie algebra $\underline{\underline{g}}$ of G with $Xv \in S(v)_\gamma$.

Proof. If $X \in \underline{\underline{p}}$ then clearly $Xv \in S(v)_\gamma$. Suppose $X \in \underline{\underline{g}}$ satisfies $X \notin \underline{\underline{p}}$ and $Xv \in S(v)_\gamma$. Choose $\lambda \in \Lambda(v)_\gamma$. Then $\lambda \neq \infty$, so it induces $\mathbb{Q}$-gradings $V = \Sigma V_s$ and $\underline{\underline{g}} = \Sigma \underline{\underline{g}}_s$ with $s \in \mathbb{Q}$. Write $v = \Sigma v_s$ and $X = \Sigma X_s$ with $v_s \in V_s$ and $X_s \in \underline{\underline{g}}_s$. Let m be minimal with $X_m \neq 0$. Since $X \notin \underline{\underline{p}}$ we have $m < 0$. An element $x = \Sigma x_s$ with $x_s \in V_s$ belongs to $S(v)_\gamma$

if and only if $x_s \in C_i$ whenever $s < c_i$. Here $0 \leq i \leq r$ and $C_0 = \{0\}$ and $c_0 = 0$, as in 2.7. Since both v and Xv belong to $S(v)_\gamma$ and since C_i is a submodule of V, we obtain

Rule 1: If $s = c_i$ then $X_m v_s \in C_i$.

Since $m < 0$ the element X_m is nilpotent in $\underline{\underline{g}}$. Using the Jacobson-Morozov theory we construct a reductive subgroup L of G of semisimple rank one with a maximal torus T and root system $\{\alpha,-\alpha\}$ such that $\lambda \in M(T)$ and that X_m is a root vector of α in the Lie algebra of L. Let (,) be the inner product on M(T) with $(\mu,\mu) = q(\mu)$ for every $\mu \in M(T)$. This inner product is used to identify $M(T)^*$ with M(T). Since $X_m \in \underline{\underline{g}}_m$ we have $(\alpha,\lambda) = m$. The representation theory of $\underline{\underline{sl}}(2)$ says the following. If an element w is annihilated by X_m in an L-module W, then every weight $\pi \in R(w)$ satisfies $(\pi,\alpha) \geq 0$. Here we use the notation of 2.7. Now rule 1 implies

Rule 2: If $\pi \in R_i(v)$ satisfies $(\pi,\lambda) = c_i$, then $(\pi,\alpha) \geq 0$.

Recall that $(\pi,\lambda) \geq c_i$ whenever $\pi \in R_i(v)$. Now it is clear that there exists a positive number ε such that for every rational number $t \in [0,\varepsilon]$ and every index $i \in [0,r]$ and every weight $\pi \in R_i(v)$ we have $(\pi,\lambda + t\alpha) \geq c_i$. The optimality of λ implies that $q(\lambda) \leq q(\lambda + t\alpha)$ and hence that $(\alpha,\lambda) \geq 0$. This contradicts $m < 0$.

4. Additional remarks and an example.

4.1 A co-weight λ is said to be γ-balanced if $\lambda \in \Lambda(x)_\gamma$ with $x \in V$. The map which associates to a γ-stratum U the union of the classes $\Lambda(x)_\gamma$ with $x \in U$, is a bijection between the finite set of the γ-strata in V and the set of the conjugacy classes of γ-balanced co-weights. A co-weight λ is said to be weakly γ-balanced if $\lambda \in \Lambda(V(\lambda)_\gamma)_\gamma$. Every γ-balanced co-weight is weakly γ-balanced, but not vice versa.

Remark. The (weakly) balanced co-weights were introduced in [3] 6.8, for a special case and with a slightly different terminology.

We fix a Borel group B of G. A co-weight λ is said to be dominant if $B \subset G(\lambda)$. We fix a maximal torus T of B. If C is a conjugacy class in $M(G)_C$, there is a unique dominant co-weight in $C \cap M(T)_C$. So the γ-stratification of V is determined by the set of the dominant γ-balanced co-weights in $M(T)_C$. Determination of the weakly γ-balanced co-weights is usually a first step, since the verification of 4.2(b) is much easier than of 4.2(a):

4.2 Lemma. a) A co-weight $\lambda \in M(G)_C$ is γ-balanced if and only if $V(\lambda)_\gamma$ is not contained in the union of the sets $GV(\mu)_\gamma$ where μ runs through the set of the γ-balanced (or weakly γ-balanced) dominant co-weights $\mu \in M(T)$ with $q(\mu) < q(\lambda)$.
b) A co-weight $\lambda \in M(T)_C$ is weakly γ-balanced if and only if every co-weight $\mu \in M(T)$ with $V(\lambda)_\gamma \subset V(\mu)_\gamma$ satisfies $q(\lambda) \leq q(\mu)$.

Proof. a) is trivial. b) Assume that $\lambda \in M(T)_C$ satisfies $q(\lambda) \leq q(\mu)$ for every $\mu \in M(T)$ with $V(\lambda)_\gamma \subset V(\mu)_\gamma$. Put $X = V(\lambda)_\gamma$. The intersection of the parabolic subgroups $G(\lambda)$ and $P(X)_\gamma$ contains a maximal torus S of G. Since S and T are maximal tori in $G(\lambda)$, we have $T = \mathrm{int}(g)S$ with $g \in G(\lambda)$. Then $gX = X$ and therefore

$$T \subset \mathrm{int}(g)P(X)_\gamma = P(gX)_\gamma = P(X)_\gamma.$$

It follows that $M(T)_C$ contains an element of $\Lambda(X)_\gamma$. The assumption on λ implies that $\lambda \in \Lambda(X)_\gamma$. This proves that λ is weakly γ-balanced. The other implication is trivial. Compare [3] 5.5.

4.3 <u>Playing with the objective</u>. An objective γ is said to be <u>equivalent</u> to an objective δ if $S(X)_\gamma = S(X)_\delta$ for every subset X of V. An objective γ is said to be <u>trivial</u> if V is the γ-concentrator of the zero co-weight. Every non-trivial objective is equivalent to an objective $\gamma = (C_*, c_*, q)$ with $C_1, \ldots C_r$ a strictly increasing sequence of subvarieties and $C_r \neq V$, and with $c_1, \ldots c_r$ a strictly increasing sequence of rational numbers and $c_1 = 1$. The influence of the choice of the norm q is discussed in [2] section 7 and [3] 4.10.

4.4 <u>An example</u>. We restrict ourselves to the linear situation. So V is a G-module and $C_1, \ldots C_r$ are submodules. In [3] section 6 we gave examples with $r = 1$ and $C_1 = \{0\}$. Here we give an example with $r = 1$ and $C_1 \neq \{0\}$, and an example with $r = 2$ and $\{0\} = C_1 \neq C_2$.

Let G be the symplectic group Sp(4). Let V be the G-module $F \oplus \underline{\underline{g}}$, where $F = K^4$ with basis $e_1, \ldots e_4$ and the action of G on F is the classical representation; $\underline{\underline{g}}$ is the Lie algebra of G with the adjoint representation. The invariant alternating form β on F is chosen such that $\beta(e_i, e_j) \neq 0$ if and only if $i + j = 5$. The vectors $e_1, \ldots e_4$ are weight vectors of a maximal torus T of G. Let the Borel group B of G be the stabilizer of the flag $F_1, \ldots F_4$ with $F_j := \Sigma_{i=1}^{j} Ke_i$. Let P be the stabilizer of F_1 in G, and let $\underline{\underline{p}}$ be the Lie algebra of P. Let $x \in \underline{\underline{g}}$ be the element with $x e_4 = e_1$ and $\mathrm{Ker}(x) = F_3$. We claim that the subspaces $X_1 := F_1 \oplus \underline{\underline{p}}$ and $X_2 := F_1 \oplus Kx$ and $X_3 := F_2 \oplus Kx$ are closures of blades in V, so that theorem 3.4 applies.

We use two different objectives γ and δ, both with the norm q which is given by $q(\lambda) = a^2 + b^2$ if $\lambda(t) = \mathrm{diag}(t^a, t^b, t^{-b}, t^{-a})$. Let μ and ν be the co-weights given by

$$\mu(t) = \mathrm{diag}(t^2, 1, 1, t^{-2}), \quad \nu(t) = \mathrm{diag}(t^2, t, t^{-1}, t^{-2})$$

Then we have $G(\mu) = P$ and $G(\nu) = B$. We use the norm q to identify M(T) and M(T)*. Then the weights π_1 and π_2 of e_1 and e_2 in F, respectively, form an orthonormal basis of M(T) and we have $\mu = 2\pi_1$ and $\nu = 2\pi_1 + \pi_2$. The simple roots are $\alpha = \pi_1 - \pi_2$ and $\beta = 2\pi_2$.

In the notation of 2.7 we have

$R(F_1) = \{\pi_1\}$, $R(F_2) = \{\pi_1, \pi_2\}$, $R(\underline{\underline{p}}) = \{0, \pm\beta, \alpha, \alpha+\beta, 2\alpha+\beta\}$, and $R(Kx) = \{2\alpha+\beta\}$. These weights and co-weights are shown in the diagram below. The dominant chamber is shaded there.

The objective γ is of rank one with $C_1 = \underline{\underline{g}}$ and $c_1 = 2$. The objective δ is of rank two with $C_1 = \{0\}$, $C_2 = F$, $c_1 = 1$, $c_2 = 4$. We have $X_1 = V(\mu)_\gamma$ and $X_2 = V(\mu)_\delta$ and $X_3 = V(\nu)_\delta$. If $\lambda \in M(T)$ and $q(\lambda) < q(\mu)$ then $V(\lambda)_\gamma \subset \underline{\underline{g}}$. This proves that $X_1 \setminus p$ is a γ-blade with optimal co-weight μ and Kempf group P. The set GX_2 consists of the pairs (f,y) such that $Kf + Im(y)$ has dimension at most one. Therefore $X_3 \setminus GX_2$ is non-empty and equal to $X_3 \setminus (F_2 \cup X_2)$. If λ is a weakly δ-balanced dominant co-weight in M(T) with $q(\lambda) < q(\nu)$, then $\lambda = \mu$ or $V(\lambda)_\delta \subset F$. This proves that $X_2 \setminus F_1$ and $X_3 \setminus (F_2 \cup X_2)$ are δ-blades with optimal co-weights μ and ν and Kempf groups P and B, respectively. It follows that $\dim(GX_1) = 11$, that $\dim(GX_2) = 5$, that $\dim(GX_3) = 7$. If $char(K) = 0$, then X_1 and X_2 are P-resolute and X_3 is B-resolute.

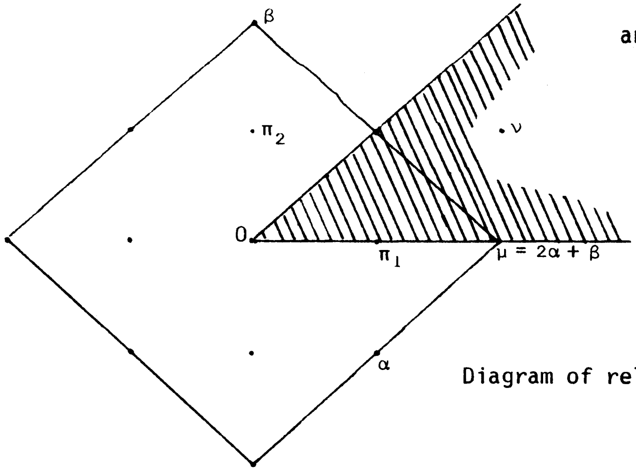

Diagram of relevant weights and co-weights.

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