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AVERTISSEMENT

Ce volume contient la plupart des exposés faits au Congrès d'Analyse des Systèmes qui s'est tenu à Bordeaux en Septembre 1978.

Les mots "Analyse des Systèmes" sont à la mode et recouvrent n'importe quoi. En préparant cette introduction j'ai essayé en quelques pages de décrire ce qu'on allait trouver. Je n'y suis pas parvenu et je me suis aperçu que j'essayais de justifier a postériori par des "raisons scientifiques" le choix des participants au Congrès. En réalité je dois confesser que ce sont surtout des raisons personnelles (relations nouées dans d'autres Congrès, amitié, intérêt esthétique...) qui m'ont guidé.

Il est quand même important de dire que dans ce qui suit ne sont abordés que certains aspects de l'Analyse des Systèmes. On peut affirmer que pour l'essentiel il s'agit d'Analyse Mathématique des Systèmes, que ce qui touche à l'Analyse des Systèmes gouvernés par des équations aux dérivées partielles ou stochastiques est absent et qu'il n'y a pas un mot d'analyse numérique. Par contre, l'utilisation de certains outils : Géométrie algébrique, Groupes de Lie, Combinatoire... pourra étonner en Mathématiques Appliquées.

A ce sujet, il faut dire qu'on ne trouvera pratiquement pas d'applications. Ces mathématiques s'appliquent-elles ? C'est un sujet (controversé) qui n'est pas abordé. Inversement, on peut affirmer que les problèmes (mathématiques) abordés sont généralement très directement issus de questions qui se posent réellement en Automatique. Ceci explique une terminologie bizarre que le mathématicien jugera sans doute agaçante et même obscure. Je pense que la lecture du ler chapitre du livre de LEE et MARKUS : Foundations of Optimal Control Theory (Wiley, New-York, 1967) est suffisante pour avoir une idée de tous les concepts de base.

Je remercie tous les auteurs de ce volume et les participants au Congrès de Bordeaux. Sans eux rien n'aurait été fait et je les remercie particulièrement de l'ambiance chaleureuse qu'ils ont créée.

Je remercie Annie POLZIN qui a dactylographié le texte à partir de manuscrits souvent difficiles à lire et Yves ROUCHALEAU qui m'a considérablement aidé dans le travail d'édition.

Le Congrès s'est déroulé dans les locaux et avec les moyens matériels de l'U.E.R. de Mathématiques et Informatique de l'Université de Bordeaux I. Que Geneviève CASTAIGNEDE, la secrétaire de l'U.E.R. dont j'ai toujours apprécié la compétence et les services, trouve l'expression de ma reconnaissance.

Claude LOBRY

TABLE DES MATIÈRES

ABSTRACTS

A. BACCIOTTI - Processus Optimaux.

We study minimal time control problems in the "initial control" cases. The Pontryagin maximum principle is established for this kind of problem. Sufficiency is considered with hypothesis like those of dynamic programming.

B. BONNARD, V. JURDJEVIC, I. KUPKA, G. SALLET - Transitive vector fields on Lie groups.

A subset $\mathcal{F}$ of the Lie algebra $\underline{g}$ of a Lie group G is said transitive iff the semi-group generated by the set $\exp(\mathbb{R}^+ \mathcal{F})$ is transitive on G (synonyum of Completely controlable). In this paper a sufficient (in some way the best possible) condition is given for a system to be transitive on semi-simple Lie groups (JURDJEVIC-KUPKA). For a given $X \in \underline{g}$ does it exists a $Y \in \underline{g}$ such that $\{X, Y\}$ is transitive on $\underline{g}$. The answer is yes if X is compact (and a evident condition on X) for the semi-simple Lie groups of interior type (SALLET). The answer is yes for the killing vector fields on $\mathbb{R}^{2n}$, from the semi-direct product of $\mathbb{R}^{2n}$ and the symplectic group $sp(n,\mathbb{R})$ (BONNARD).

R. BROCKETT - Recursive Estimation.

A relationship is established between the existence of recursive estimators for a random process and properties of a Lie algebra generated from the operators appearing in the conditional density equation. This leads to a necessary condition for the existence of a finite dimensional nonlinear estimator expressed in terms of VOLTERRA kernels, and throws new light on the classical KALMAN filter in the linear case.

C. BYRNES - Groups and Realizations.

A unifying presentation is given of several problems of linear realization theory : the relationship between constant rank HANKEL matrices and minimal realizations and between symmetric transfer functions and symmetric

minimal realizations.

C. CONTE and A.M. PERDON - Canonical Forms.

We determine in this paper a class of open subsets of the set of completely reachable pairs (F, G) of given size over which there exists a local continuous canonical form.

P. CROUCH - Realizations of a single Volterra kernel.

The input-output map of a nonlinear analytic system can be expanded in a suitable domain as a Volterra series. The Volterra kernels, each of which defines a term in the series, can be expressed in terms of the system data. In this paper these expressions are used to realize a single term in the series, again directly in terms of the system data. This will lead in later work to a synthesis algorithm for systems with finite Volterra series.

A. FERFERA - Systèmes en série.

This paper shows that the series composition of two analytic systems is analytic, but that the composition of two bi-linear systems need not be. A counter-example is given, as well as a necessary condition on a system for its composition with any other bi-linear system to be bi-linear. This is done within the frame work of non commuting formal power series.

M. FLIESS - Équations Différentielles.

Let : $$\dot{q}(t) = A_o(q) + \sum_{i=1}^{n} u_i(t)\, A_i(q)$$
$$y(t) = b(q) \quad [q(0) \text{ given}]$$

be a non-linear differential system, where $A_o, A_1, \ldots, A_n$ are analytic vector fields and b is an analytic function. The output y is a causal analytic functional of the inputs $u_1, \ldots, u_n$, i.e. y is given by the following non-commutative power series

$$g = b|_o + \sum_{\nu \geq 0} \sum_{j_o, \ldots, j_\nu = 0} A_{j_o} \ldots A_{j_\nu}\, b|_o\, x_{j_\nu} \ldots x_{j_o} .$$

This is used to derive the Volterra series expansion in closed form. The proofs are essentially of algebraic nature and generalize W. Gröbner's works on Lie series.

E. FORNASINI - 2-D Systems.

Image processing leads us to the study of 2-D systems, hence to that of linear systems over partially ordered time sets. Using the formalism of non commutative power series, we give an asymptotic stability criterion for 2-D systems. We then study the realization problem for linear systems over free group time sets.

G. GOODMANN - Titre et textes non parvenus.

M. HAZEWINKEL - Invariants and moduli.

Previous results concerning invariants, fine moduli spaces and canonical forms are extended to the case of linear systems over a differential field, applying to a wide range of linear systems with time-varying coefficients. The relevant notions of differential algebraic geometry are introduced.

H. HERMES - Evolution Equations.

Lie algebraic methods are now classic in the study of control systems governed by ordinary differential equations. We show how this method can be extended in order to study controllability problems for systems governed by partial differential equations.

R. HIRSCHORN - Nonlinear Inverses.

A nonlinear control system is invertible if the associated input-output map is injective for nonlinear systems of the form $\dot{x} = A(x) + \sum_{k=1}^{m} u_i B_i(x)$, $y = C(x)$ which evolve on a real analytic manifold we obtain sufficient conditions for invertibility and construct systems which act as inverse systems. In the case of single-input systems our conditions are necessary and sufficient for invertibility. For invertible systems we construct nonlinear systems which act as left-inverses for the original systems.

A. ISIDORI - Output regulation of a special class of bilinear systems.

Texte non parvenu.

B. JAKUBCZYK - Nonlinear Realization.

This paper gives a realization theory for nonlinear input-output maps, i.e. gives necessary and sufficient conditions for the existence of such realizations,

proves that any two minimal such realizations are diffeomorphic and outlines a construction procedure. The framework is that of C^k-manifolds.

V. JURDJEVIC - (cf. B. BONNARD).

A. KRENER - Boundary Value.

The standard linear time varying system is modified by replacing the initial conditions on the state by boundary conditions, the result is an noncausal system. The concepts of input/output map, dual system, weighing pattern controllability, observability, minimal realization and linear quadratic regulation for such systems are discussed.

I. KUPKA - Accessibility Theory.

In this talk I want to discuss several problems that appear in the theory of accessibility of polysystems. Since very little seems to be known about these problems, I will indicate some ideas about possible solutions. The talk will consist of two parts ; the first one will consider left or right invariant polysystems on Lie groups the second one points out some pathologies in the structure of accessibility sets and discusses the stability of transitivity or non transitivity. The proofs of all the results mentionned will appear elsewhere.

I. LANDAU - Modèles Bi-linéaires.

This article presents a synthesis of several of the authors' papers concerning the bi-linear system representation of distillation columns and the identification of such models. Discussed are the practical implications of this approach, as well as some of the remaining open problems.

S. ŁOJASIEWICZ - Maximum Principle.

We consider a nonlinear control system described by differential inclusion. We give sufficient conditions for the characterization of the boundary trajectories of the system by PONTRYAGIN-HAMILTON equations and for the convexity of attainable sets over a certain time interval (we also give the best and computable estimates for the length of this interval). We apply them to obtain the sufficiency of the maximum principle for a time-optimal problem.

S. MITTER - Filtering Theory.

In this paper we describe certain remarkable connections that exist between mathematical developments in quantum field theory (and euclidean field theory) and filtering theory (and in general, system theory). Roughly speaking, the Kalman filter is the mathematical analog of a free quantum field (in a precise sense) and the study of nonlinear filtering is the analog of the study of interacting quantum fields. Due to lack of space we only sketch this theory in this paper and the details of this work will be presented elsewhere. Some of these ideas were presented in an earlier paper of this author.

G. SALLET - (cf. B. BONNARD).

A. SEC - Sur les trajectoires de Moment Maximal.

Texte non parvenu.

E. SONTAG - Polynomial Systems.

This paper studies the class of quasi-reachable realizations of a fixed polynomial response. This class is described as a complete lattice. Various subclasses are explored in detail with respect to the induced order, and examples are given.

H. SUSSMANN - Semi-groupes.

We state a theorem about the infinitesimal generators of transformation semi-groups F of an analytic variety M satisfying several assumptions, the most important of which is hypo-analyticity. The conclusion of this theorem is that we can always associate with such an F an infinitesimal generator which is piecewise analytic. We apply this theorem to prove the existence of an optimal closed loop, control law for a class of problems which is piecewise analytic.

R. TARRES - Stochastic Control.

We study the minimization problem for an integral discounted functional on a set of non explosive, non constrained diffusions. Using dynamical programming we characterize the optimal cost. We then consider the behaviour of the problem when the discount rote vanishes.

J. WILLEMS - Invariant Subspaces.

In this paper we define two new concepts in the context of continuous time finite dimensional time-invariant linear systems. They are those of 'almost' [A(mod)B)-] invariant ' and 'almost controllability' subspaces. It is shown that there exists a supremal almost invariant and a supremal controllability subspace contained in any given subspace of the state space, and an algorithm for their computation will be given. A feedback characterization of these subspaces is derived. The paper ends with an application of these ideas to the disturbance decoupling problem.

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