

Astérisque

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Astérisque, tome 49 (1977), p. 115-116

<http://www.numdam.org/item?id=AST_1977__49__115_0>

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SPECTRUM OF TWO-DIMENSIONAL DYNAMICAL SYSTEMS

WITH

COMPLETELY POSITIVE ENTROPY

B. Kamiński

One of interesting classes of two - dimensional dynamical systems are the systems with completely positive entropy. Some properties of these systems have been discovered by J.P. Conze in [1]. One of the most interesting properties of these systems is their spectrum.

Conze has shown that if (X, μ, G) is a system in which G is generated by a pair of commuting automorphisms being K - system then (X, μ, G) has a countable infinite Lebesgue spectrum.

Recently we have shown in [2] that any system (X, μ, G) with completely positive entropy has a countable infinite Lebesgue spectrum.

In order to obtain our result we shall make use of some results appearing in [1] and [2].

Let $\{T, S\}$ be an ordered pair of generators of G .
Lemma 1 [1]. There exists a measurable partition ζ of X such that

- (1) $T^k S^l \zeta \subseteq \zeta$, where $(k, l) \prec (0, 0)$,
- (2) $\bigvee_{(k, l) \in \mathbb{Z}^2} T^k S^l \zeta = \varepsilon$,
- (3) $H(\zeta | \zeta_{T, S}^-) = H(\zeta | S^{-1} \zeta) = h(G)$.

Let σ be a measurable T - invariant partition of X and let $\bar{\sigma}$ be a Pinsker closure of σ .

Lemma 2 [2]. There exists a measurable partition ζ of X such that

- (4) $\sigma \subseteq T^{-1} \zeta \subseteq \zeta$,
- (5) $\bigvee_{n=0}^{\infty} T^n \zeta = \varepsilon$,
- (6) $\bigwedge_{n=0}^{\infty} T^{-n} \zeta = \bar{\sigma}$.

Using these lemmas and the separability of $L^2(X, \mu)$ one can construct a countable transfinite sequence $\{\sigma_\alpha\}_{\alpha < \alpha_0}$ of measurable partitions of X with properties :

- (7) $\sigma_0 = \mathcal{E}$,
- (8) $\sigma_{\alpha+1} \leq \sigma_\alpha$,
- (9) $T\sigma_\alpha = S\sigma_\alpha = \sigma_\alpha$,
- (10) if α is a limit ordinal number then $\sigma_\alpha = \bigwedge_{\beta < \alpha} \sigma_\beta$,
- (11) G has countable Lebesgue spectrum in $L^2(\sigma_\alpha) \ominus L^2(\sigma_{\alpha+1})$, $\alpha < \alpha_0$
- (12) $L^2(X, \mu) \ominus \mathbb{1} = \bigoplus_{\alpha < \alpha_0} (L^2(\sigma_\alpha) \ominus L^2(\sigma_{\alpha+1}))$.

Two last properties imply the desired result.

Now, let us suppose $h(G) < \infty$.

Conze and independently Katznelson and Weiss have proved

Lemma 3 ([1] , [3]) . If G is aperiodic and $h(G) < \infty$ then there exists a measurable partition ξ of X such that $H(\xi) < \infty$ and $\bigvee_{(k,l) \in \mathbb{Z}^2} T^k S^l \xi = \mathcal{E}$.

Using this result one can prove that for G having completely positive and finite entropy we have $\sigma_1 = \mathcal{V}$

The last property means that an arbitrary ordered pair $\{T, S\}$ of generators of such groups is a K - system in the sense of Conze.

It would be very interesting to explain if the assumption $h(G) < \infty$ is essential .

References

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