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Cats' tales

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Consider the following: there are cats in some of the places of a two-sided infinite sequence. Then start, step by step, this process: at each step each cat jumps, independently of the others, with probability $\frac{1}{2}$ to each of the two neighbouring places. If two cats land on the same place, they disappear (imagine each second cat to be an anti-cat).

Define $A \equiv \{\ 0$ is visited an infinite number of times $\}$.

Question: $p(A) = ?$

(there are some versions of this problem, that can be handled in a very similar way).

The answer depends, of course, on the initial distribution of the cats.

We can immediately get $p(A) = 1$ and $p(A) = 0$ in the cases of odd and even number of cats, respectively.

Denote by $i(n)$ the initial number of cats in the block $1,\ldots,n$, and suppose the negatives are initially empty. Then in the infinite cats case in which $\frac{i(n)}{n} \to 0$ simple examples can be found for which $p(A) = 1$, as well as others for which $p(A) = 0$.

The general case $\overline{\lim}\, \frac{i(n)}{n} > 0$ is unsolved yet, but there is a large class for which the answer can be proved to be $p(A) = 1$. This class contains, as a typical sub-class, those sequences in which there is some n such that there are infinitely many n_k's such that the block $n,\dots,n+2n_k$ is, in the beginning, symmetric with respect to reflection about $n+n_k$, and contains an odd number of cats (the sequence in which all the naturals are initially occupied $(\frac{i(n)}{n} \equiv 1)$ is, of course, contained in this subclass).

The proof to the last claim is rather long, but its basic idea is the same as that in the following proof of $p(A)$ being 1 when there is one cat only.

Suppose the cat is in the n'th place. By symmetry, there is probability $\frac{1}{2}$ that $2n$ is visited before 0. If that happens, then there is probability $\frac{1}{2}$ that $4n$ is visited before 0, and so on. But $(\frac{1}{2})^{\infty} = 0$, so 0 will a.s. be visited, so it will a.s. be visited an infinite number of times.

In the case of finite number of cats, a similar method can be applied to the n-dimensional analogous problem ($p(A)$ found, as is known, to vanish for $n > 2$), but I don't know how to treat the general n-dimensional infinite cats problem (excluding some special cases).

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