Astérisque

ANTHONY MANNING

Topological entropy and homology

Astérisque, tome 40 (1976), p. 141-142
<http://www.numdam.org/item?id=AST_1976__40__141_0>

© Société mathématique de France, 1976, tous droits réservés.

L'accès aux archives de la collection « Astérisque » (http://smf4.emath.fr/Publications/Asterisque/) implique l'accord avec les conditions générales d'utilisation (http://www.numdam.org/conditions). Toute utilisation commerciale ou impression systématique est constitutive d'une infraction pénale. Toute copie ou impression de ce fichier doit contenir la présente mention de copyright.

NUMDAM
Article numérisé dans le cadre du programme
Numérisation de documents anciens mathématiques
http://www.numdam.org/

Société Mathématique de France
Astérisque 40 (1976) p.141-142

TOPOLOGICAL ENTROPY AND HOMOLOGY

Anthony Manning

Let X be a compact metric space and $f:X \to X$ a continuous map.

Definition. (Bowen)

A (k,δ) spanning set for f is a finite set $Y \subset X$ s.t. $\forall x \in X\ \exists y \in Y$ s.t. $d(f^j x, f^j y) < \delta$ for $0 \leq j < k$.

$h(f,\delta) = \limsup_{k\to\infty} \frac{1}{k} \log \text{card (minimal } (k,\delta) \text{ spanning set)}$

The topological entropy is defined as :

$$h(f) = \sup_{\delta>0} h(f,\delta) .$$

$h(f)$ thus describes the exponential growth rate of the number of types of f-orbits, or roughly speaking the amount f mixes up the points of X. $h(f) = \sup_\mu h_\mu(f)$ where μ ranges over all normalized Borel invariant measures (Dinaburg).

Conjecture. (Shub)

Let $f:M \to M$ be a diffeomorphism of a smooth compact manifold M. Then $h(f) \geq \log \text{sp}\, f_*$ where $\text{sp}\, f_*$ is the largest modulus of any eigenvalue of the induced map of homology $f_* : H_*(M;R) \to H_*(M;R)$.

$H_i(M;R)$ is a finite dimensional real vector space describing the i dimensional structure of M, f_* describes the action of f on this structure and so the conjecture says that a diffeomorphism that mixes the i dimensional structure of M for some i must also have at least the corresponding amount of entropy I.e. the homology of f detects some of the dynamics of f .

The conjecture has been proved for diffeomorphisms satisfying Axiom A and the no cycle property (Shub and Williams). Hence it holds for a C^0 open and dense set of diffeomorphisms of M . For C^0 maps the inequality fails even on the manifold S^2 . However, any C^0 map of M satisfies $h(f) \geq \log|\lambda|$ for each eigenvalue λ of f_* in $H_1(M;R)$ the first homology group (Manning).

For eigenvalues in higher groups differentiable hypotheses would usually be required. In this direction Misiurewicz and Przytycki showed recently that any C^1 map $f:S^2 \to S^2$ satisfies $\exp h(f) \geq |\deg(f)|$. (Added in proof: they now have this result for C^1 maps of any compact orientable manifold.)

REFERENCE

A. Manning, Topological Entropy and the First Homology Group, Dynamical Systems - Warwick 1974, Lecture Notes in Mathematics vol. 468, Springer, (1975).

Mathematics Institute
University of Warwick
Coventry, Warwickshire
CV4 7AL GB