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A SPECIAL CLASS OF TWO DIMENSIONAL EXPONENTIAL-BESSEL SERIES OF FOX'S H-FUNCTION

Abstract: In this paper, we present a special class of two-dimensional Exponential-Bessel series of Fox's H-function [2], and present one two-dimensional series of this class.

The following formulae are required in the proof:

The integral [1, p.46, (1)]:

$$(1.1)\qquad \int_0^{\Pi} e^{-2iux}(\sin x)^{w_1-1} H^{m,n}_{p,q}\left[z(\sin x)^{-2h} \middle| \begin{matrix} (a_p, e_p) \\ (b_q, f_q) \end{matrix} \; dx \right]$$

$$= \frac{e^{-iu}}{2^{w_1-1}} H^{m+1,n}_{p+2,q+2}\left[z\, 2^{2h} \middle| \begin{matrix} (a_p, e_p), (\frac{w_1+1}{2} \pm u, h) \\ (w_1, 2h), (b_q, f_q) \end{matrix} \right],$$

where

$$h > 0,\ \sum_{j=1}^{p} e_j - \sum_{j=1}^{q} f_j \equiv A \le 0,\ \sum_{j=1}^{n} e_j - \sum_{j=n+1}^{p} e_j + \sum_{j=1}^{m} f_j - \sum_{j=m+1}^{q} f_j \equiv B > 0,$$

$$|\arg z| < \tfrac{1}{2} B\,\Pi,\ \mathrm{Re}\, w_1 - 2h \max_{1\le j\le n}[\mathrm{Re}(a_j - 1)/e_j] > 0.$$

The integral [4, p. 94, (2.2)]:

$$(1.2)\qquad \int_0^\infty y^{w_2-1}\sin y\; J_v(y)\; H^{m,n}_{p,q}\left[zy^{2k} \,\middle|\, \begin{matrix}(a_p,e_p)\\(b_q,f_q)\end{matrix}\right] dy$$

$$= 2^{w_2-1}\sqrt{\pi}\; H^{m+1,n+1}_{p+4,q+1}\left[2^{2k}z \,\middle|\, \begin{matrix}\left(\frac{1-w_2-v}{2},k\right),(a_p,e_p),\left(1+\frac{v-w_2}{2},k\right),\\ \left(1-\frac{v+w_2}{2},k\right),\left(\frac{1+v-w_2}{2},k\right)\\ \left(\tfrac{1}{2}-w_2,2k\right),(b_q,f_q)\end{matrix}\right],$$

where $k>0,\ A\le 0,\ B>0,\ |\arg z|<\frac{1}{2}B\,\Pi,\ \mathrm{Re}(w_2+v)+2k\min_{1\le j\le m}[\mathrm{Re}\,b_j/f_j]>0.$

The orthogonality property of the Bessel functions [3, p. 291, (6)] :

$$(1.3)\qquad \int_0^\infty x^{-1} J_{a+2n+1}(x)\; J_{a+2m+1}(x)\, dx$$

$$=\begin{cases} 0, \text{ if } m\neq n\,; \\ (4n+2a+2)^{-1}, \text{ if } m=n,\ \mathrm{Re}\,a+m+n>-1.\end{cases}$$

2. <u>TWO -DIMENSIONAL EXPONENTIAL-BESSEL SERIES.</u> The two-dimensional Exponential-Bessel series to be established is

$$(2.1)\qquad (\sin x)^{w_1-1}\, y^{w_2}\sin y\; H^{m,n}_{p,q}\left[z(\sin x)^{-2h}\, y^{2k} \,\middle|\, \begin{matrix}(a_p,e_p)\\(b_q,f_q)\end{matrix}\right]$$

$$= 2^{1-w_1+w_2} \sum_{r=-\infty}^{\infty} \sum_{s=0}^{\infty} (a+2s+1)\, e^{2ir(x-\Pi/2)}\, J_{a+2s+1}(y)$$

$$\times H^{m+2,n+1}_{p+6,q+2}\left[2^{2(h+k)} z \left| \begin{matrix} \left(-\frac{w_2+a+2s}{2}, k\right), (a_p, e_p), \left(\frac{w_1+1}{2} \pm r, h\right), \\ \left(1+\frac{a+2s+1-w_2}{2}, k\right), \\ \left(1-\frac{a+2s+1+w_2}{2}, k\right), \left(1+\frac{a+2s-w_2}{2}, k\right) \\ \\ (w_1, 2h), (1/2-w_2, k), (b_q, f_q) \end{matrix} \right. \right]$$

valid under the conditions of (1.1), (1.2) and (1.3).

Proof. Let

$$(2.2) \quad f(x,y) = (\sin x)^{w_1-1}\, y^{w_2}\, \sin y\, H^{m,n}_{p,q}\left[z(\sin x)^{-2h}\, y^{2k} \left| \begin{matrix} (a_p, e_p) \\ (b_q, f_q) \end{matrix} \right. \right]$$

$$= \sum_{r=-\infty}^{\infty} \sum_{s=0}^{\infty} C_{r,s}\, e^{2irx}\, J_{a+2s+1}(y).$$

Equation (2.2) is valid, since $f(x, y)$ is continuous and bounded variation in the region $0 < x < \Pi$, $0 < y < \infty$.

Multiplying both sides of (2.2) by $y^{-1} J_{a+2v+1}(y)$ and integrating with respect to y from 0 to ∞, then using (1.2) and (1.3). Now multiplying both sides of the resulting expression by e^{-2iux} and integrating with respect to x from 0 to Π, then using (1.1) and the orthogonality property of exponential functions, we obtain the value of $C_{r,s}$. Substituting this value of $C_{r,s}$ in (2.2), the expansion (2.1) is obtained.

Note : On applying the same procedure as above, we can establish three other forms of two-dimensional expansions of this class with the help of alternative forms of (1.1) and (1.2).

Since on specializing the parameters Fox's H-function yields almost all special functions appearing in applied mathematics and physical sciences. Therefore, the result presented in this paper is of a general character and hence may encompass several cases of interest.

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